

# A Proposal for Vertical Datum Development in New Zealand

*OSG Technical Report 10*

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## Foreword

Land Information New Zealand (LINZ) (Toitu te Whenua) was established in July 1996. It is a government department with roles and responsibilities in the following key areas:

<b>Regulatory Responsibilities</b>	<b>LINZ Regulatory Groups</b>
National spatial reference system and cadastral survey infrastructure	Office of the Surveyor-General
Topographic and hydrographic information	National Topographic/Hydrographic Authority
Land Titles	Office of the Registrar-General of Land
Setting rules for rating valuations	Office of the Valuer-General
Crown Property	Office of the Chief Crown Property Officer (Crown Property)
Assisting the government address land related aspects of Treaty of Waitangi issues	Office of the Chief Crown Property Officer (Crown Property)

The main role of the department is a regulatory one, to set guidelines and standards and manage contracts for carrying out the day to day business associated with each of the key areas.

LINZ also offers a range of services to customers related to land titles, survey plans and Crown property. Land Titles and Survey services are carried out by the Operations Group based in LINZ regional offices throughout New Zealand.

The LINZ overarching objective is to be recognised as a world leader in providing land and seabed information services.

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# ***A PROPOSAL FOR VERTICAL DATUM DEVELOPMENT IN NEW ZEALAND***

## **1 Introduction**

New Zealand Geodetic Datum 1949 (NZGD49) was introduced to meet the needs for an integrated horizontal spatial reference system and complete topographical and cadastral mapping database coverage of New Zealand. Since its introduction in 1949 it has played a significant role in the subsequent development of New Zealand. The demand for accurate heights has generally been less than for the horizontal component. The former government departments of Lands and Survey (L&S) and the Ministry of Works and Development (MWD) did invest significant resources into the upgrading and establishment of new orthometric height networks to support primarily engineering projects such as irrigation and hydroelectric schemes. These height networks were normally based on the determination of mean sea level at one tide gauge and have not been adjusted together into a single national network. As a consequence New Zealand has multiple height datums and since normal gravity rather than gravity observations were used to reduce the spirit levelling differences the heights are actually normal-orthometric heights.

The Surveyor-General, Land Information New Zealand (LINZ), has implemented a new geodetic datum, New Zealand Geodetic Datum 2000 (NZGD2000) to replace NZGD49 (OSG 1998a). The new datum provides an accurate geometric reference system for New Zealand and its areas of territorial responsibility. NZGD2000 is a geometric datum, that is the coordinates of defining stations are computed in terms of geocentric cartesian coordinates (XYZ) that are then converted to latitude, longitude and ellipsoidal height in terms of the Geodetic Reference System 1980 (GRS80) reference ellipsoid (Pearse, 2000). Ellipsoidal heights do not take into account the changes in gravitational potential and therefore cannot be used to predict the direction that water will flow.

With the establishment of NZGD2000 it is an appropriate time to look at the heighting needs for New Zealand. With the ability of new technologies such as the Global Positioning System (GPS) to determine accurate ellipsoidal height differences over large areas more cost effectively than conventional spirit-levelling it is necessary to consider the need for orthometric heights or whether ellipsoidal heights are sufficient. There is also a need to consider the continuing effects of ground deformation and the requirements (both internal and external to LINZ) of New Zealand's national height datum for the foreseeable future.

The primary business drivers that require this investigation into the state of the current New Zealand height datums and the future requirements are contained within the New Zealand Geodetic Strategic Business Plan (OSG 1998b). The relevant goals are:

- To provide a cost-effective system that can generate orthometric heights of points in terms of a nationally accepted system to an acceptable and defined accuracy.
- To support multiple vertical datums and authoritative transformation of coordinates to an acceptable and defined accuracy.
- To enhance the automated cadastral system and extension to include the seabed.
- To consider the implementation of a four-dimensional datum.
- To develop a height system to a defined accuracy that enables the generation of orthometric heights from spheroidal heights.

This report looks at the options for vertical datum development in New Zealand.

## 2 Definitions

Dynamic Height	the geopotential number divided by a constant reference gravity, often chosen to be the normal gravity value.
Ellipsoidal Height	the height measured from the ellipsoid surface. Also referred to as the spheroidal height.
Equipotential Surface	a surface passing only through points at which the specified potential has the same (arbitrarily chosen) value.
Geopotential Model	a set of spherical harmonic coefficients of the Earth's external gravitational potential.
Geopotential Number	the negative potential difference between the point and the geoid measured in geopotential units.
Geoid	the equipotential surface that approximates mean sea level globally.
Geoid Height	the height measured from the geoid.
Geoid Model	a mathematical model that attempts to represent the shape of the geoid.
Gravitational Potential	the ability of a body to attract another body.
Gravity	the force that attracts two bodies together (generally thought of as the attraction of an object to the centre of the Earth). The resultant of gravitational potential and centrifugal potential.
Gravity Observations	the measured value of gravity at a point determined either by relative or absolute methods.
Mean Sea Level	(MSL) the average height of the local sea surface that has been recorded by a sea level recorder which has ideally been operating correctly with calibration for more than 19 years.
Normal Gravity	the value of gravity computed from a reference ellipsoid, such as GRS80, rather than from Gravity Observations.
Normal Height	the geopotential number divided by the mean normal gravity along the normal plumb line.
Orthometric Height	the geometrical distance between the geoid and the point measured along the plumb line.
Spirit Levelling	measurement of height differences between points in terms of the local gravity vector at each set up. The name comes from the act of levelling the instruments using a spirit bubble.
Vertical Datum	a set of defining parameters (such as the value of MSL) and observations.

### 3 Current Status

#### 3.1 Tide Gauges

Historically within New Zealand tide gauges have been installed in harbours by Port Authorities for use in the prediction of tide tables (Blick *et al.*, 1997). These tide gauges produced data of varying quality and over different observation periods. The digital data available from the National Topographic/Hydrographic Authority (NTHA) Tides and Sea Level database for the Standard Ports (as at July 1999) is summarised in Table 1. As can be seen from Table 1 the available data varies significantly between ports, from 1 year up to almost 100 years and very rarely is continuous. It should be noted that only some of these Standard Port gauges have been used as origins for the levelling networks as described in section 3.2.

Standard Port	Dates
Auckland	1904-1999
Bluff	1983-1989, 1994, 1998
Dunedin	1899-1990, 1996, 1998
Gisborne	1984-1991, 1996
Lyttelton	1903-1988, 1995-1999
Marsden Point	1984-1985, 1989, 1992, 1996-1997
Napier	1979-1994
Nelson	1987-1990, 1996, 1998
Onehunga	1996, 1998
Picton	1990-1991
Port Taranaki	1984-1993, 1995-1998
Tauranga	1984-1991, 1993-1998
Timaru	1987-1994
Wellington	1944-1999
Westport	1982-1986, 1995-1998
Whangarei	1988-1990, 1992, 1996-1997

Table 1 : Digital Data available for the Standard Ports as at July 1999

Other than for tide predictions the data has also been used to investigate the long term sea level trends at the Auckland, Wellington, Lyttelton and Dunedin gauges (Hannah, 1988). This report found that there was a general rise in sea level of approximately 1.2 mm/yr.

Otago University and the Institute of Geological and Nuclear Sciences (GNS) have a joint project to monitor the stability of the four sea level recorders at Auckland, Wellington, Lyttelton and Dunedin using GPS receivers. The project aim is to determine whether any of the recorders are moving with respect to sea level (i.e. is the wharf moving, the local ground deforming or is sea level changing).

The National Institute for Water and Atmospheric (NIWA) has installed a network of ten sea level recorders for monitoring global sea level change. In contrast to many of the Standard Port sites, which are located in ports, these sites are located at open ocean sites so as to reduce the effects of topography such as harbours and or hydrography such as adjacent river flows.

Away from the mainland of New Zealand gauges are operated by Civil Defence on the Chatham Islands, and by Victoria University at Cape Roberts in the Ross Dependency.

Issues and recommendations with respect to access and availability of sea level data are contained in Blick *et al.* (1997).

### 3.2 Levelling Networks

Historically sea level data obtained from tide gauges was analysed by the predecessor organisations to LINZ (L&S, DOSLI) to determine mean sea level at that site. Once determined the height of mean sea level was then used as the zero height (datum) to which a local geodetic levelling network was referenced.

Independent tide gauges around New Zealand provide the datum origin for the associated geodetic levelling networks. The tide gauges used to provide origins for the first order levelling networks are shown in Table 2 with their locations shown in Figure 1. Not all the tide gauges used for datum origins are the same gauges used for tidal predictions.

Datum Name	Connected to Triangulation at	Observation Period of MSL	In Tides and Sea Level database
Auckland	Mt Eden	1909 - 1923	Yes – digitally
Bluff	The Bluff	1918 - 1934	No
Dunedin	Flagstaff	Unknown	Yes – digitally
Gisborne		1926	No
Lyttelton	2nd order station	1918 - 1933	Yes – digitally
Moturiki		1949 - 1953	Some on Microfilm
Napier	Bluff Hill No. 2	Unknown	unknown
Nelson	Botanical Hill	1939 - 1942	No
One Tree Point		1960 - 1963	No
Tararu		1922 - 1923	No
Taranaki	3rd order station	1918 - 1921	No
Wellington	Kelburn	1909 - 1946	Last 2 years - digitally

Table 2 : Mean Sea Level (MSL) datum used for the precise levelling networks. Compiled from Gilliland (1987) and Lee (1978)

Data analysed to determine mean sea level at each tide gauge was often from different observation periods (primarily between 1920-1970), varied in duration (normally 3 years but ranges from 1 to 37 years) and is of differing quality. There was also no

attempt to correct for the differences in sea surface topography at each gauge as this effect was unknown at the time.

The tide gauges were operated by Port Authorities who made the data available primarily for use in the prediction of Tide Tables. Up until 1990 regular check levelling was made by L&S and DOSLI to verify the vertical relationship between the tide gauges and adjacent benchmarks. Since approximately 1990 there have been no major levelling projects undertaken to either extend or monitor the current levelling networks.

For remote areas, such as Fiordland and Antarctica, the department established temporary (secondary) tide gauges for the collection of data and analysed the data to establish a levelling datum.

As reported in Blick et al. (1997) none of the tide gauges used to provide an origin for the respective vertical datum have had their mean sea level height redefined by LINZ (or DOSLI) and no benchmarks have had their height changed due to the predicted rise in mean sea level.

The first order levelling networks (as at 1990) are shown in Figure 1 with some of the tide gauges that have been used to define the origin of local vertical datums. Not all of these gauges are still operating nor is their data recorded in the NTHA Tides and Sea Level database. There has been no attempt to adjust together the spirit levelling data from each network to form a national (or island based) height datum. Offsets between the overlapping mean sea level based networks have been computed by DOSLI at common stations. These offsets can be significant with one of the larger offsets being 0.23 m between the Napier Datum 1962 and the Wellington Datum 1953.

The spirit levelling data was reduced using normal gravity and not observed gravity values (Gilliland 1987). Normal gravity is the gravity computed from a reference ellipsoid such as GRS80. Therefore the heights published in terms of each of the height datum are normal-orthometric heights. The use of normal gravity means that the height of a station is dependent on the levelling path taken due to the differences between the actual gravity and the modelled (normal) gravity.

Overall there is a fairly disjointed situation for the levelling networks within each of the main islands and nationally as a whole.



Figure 1 - New Zealand Precise levelling network and associated tide gauges as at 1990 (adapted from NZNCGG, 1991). Note the Gisborne tide gauge is located north of Napier at the 178 Longitude.

### 3.3 Gravity Data

A summary by observation method of the gravity data available for New Zealand is presented in this section. Some or all of these sources could be used in the computation of a national gravimetric geoid model.

#### 3.3.1 Land Data

The former New Zealand Department of Scientific and Industrial Research (DSIR) collected land based free-air gravity observations in the past. This data is now held by GNS but it is unclear if GNS has the mandate to maintain the data long term. What data is contained within their data holdings (e.g. raw uncorrected observations or only reduced data) and what file storage format is used has not been ascertained. The density of the observations as reported by Gilliland (1988) is 1 station per 7.5 km<sup>2</sup> nationally. As can be seen from Figure 2 the density does vary across the country with less data having been collected in areas of rugged terrain.

Issues with the free-air anomalies are:

- the heights of the observation points were often determined by barometric levelling with an accuracy estimated to range between 2 and 20 m.
- the coordinates of the observation points were often scaled off maps
- there appears to be erroneous data within the record, e.g. the data off the Canterbury coast (see Figure 2)

The land gravity data, as far as the author is aware, has not been used to compute a national geoid model but it has been incorporated into the global geopotential model – EGM96 (Lemoine *et al.*, 1998).

Between approximately 1976 and 1990 GNS had a program to make precise gravity measurements at first order benchmarks. Benchmarks at approximately 5 km spacing were selected and though few of the repeat measurements have been made there are approximately 1140 benchmarks that have been surveyed.

#### 3.3.2 Sea Data

DSIR has also collected gravity observations in the past from ship based instruments. The density along the coastal fringe varies considerably but in general it reduces the further you are away from the main ports. NIWA could also hold gravity data that may supplement the DSIR data.

Satellite altimetry is not able to collect data up to the coastline but does provide uniform data in open oceans. Even when the sea data is combined there is a coastal strip around the country where there are very few gravity observations.

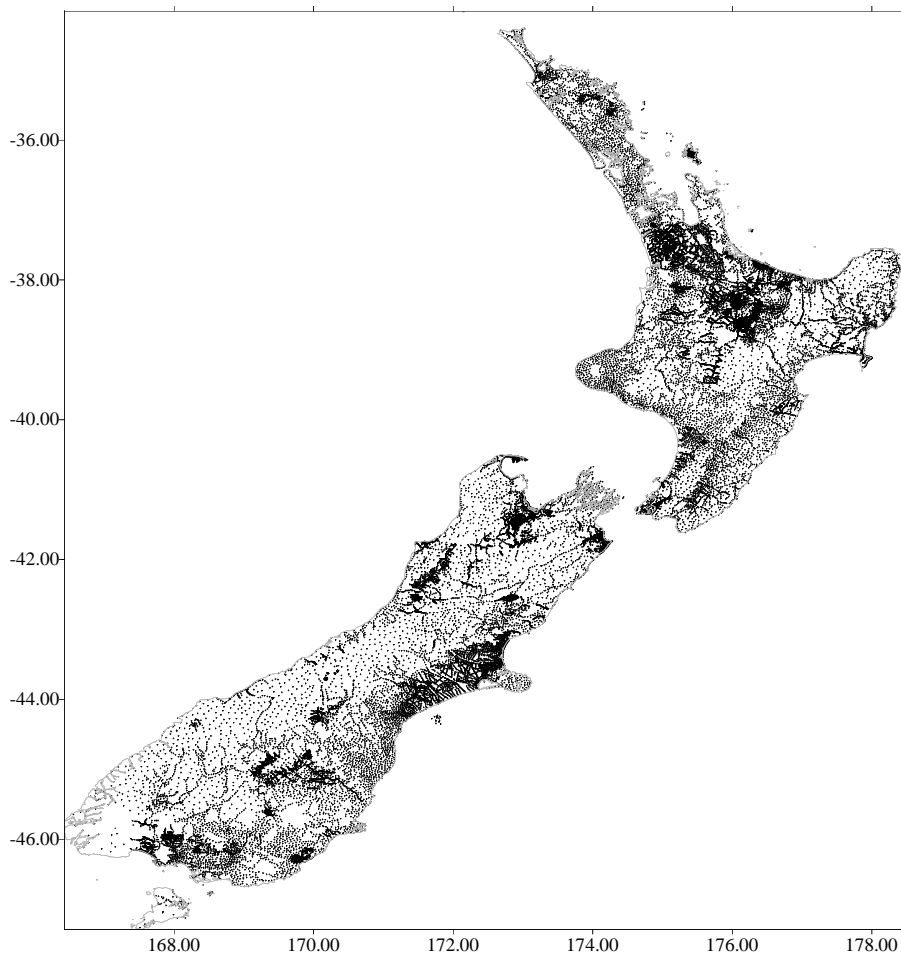


Figure 2 : Distribution of Land Based Gravity Observations (Pearse 1998)

### 3.3.3 *Absolute Data*

The above land and sea data has been collected using relative gravity observations and were referenced to the International Gravity Standardisation Network 1971 (IGSN71).

In recent years the research work of Roger Bilham (University of Colorado, Boulder) has resulted in a network of 16 absolute gravity stations in the South Island. There has been no absolute gravity observed in the North Island as part of this research and the author is not aware of any other projects that have collected absolute gravity data in the North Island.

### 3.4 *Digital Elevation Data*

LINZ holds the digital elevation data as 20m contour vectors in the topographic database. The database is not able to output a regular grid of spot heights that would be required for computing the terrain effect in any high-resolution geoid model. To obtain a suitable digital elevation model LINZ will probably have to contact one of the recognised Topographic Data Distributors.

## 4 Requirements for Heights

This section on the requirements for heights is an assessment of the requirements for both LINZ and external organisations. The assessment is based on the experience of the author and through personal communications with LINZ staff and external users.

### 4.1 LINZ Requirements

#### 4.1.1 Office of the Surveyor-General (OSG)

##### Background

Under Section 11 (1) (a) of the Survey Act 1986, 'Functions and duties of Surveyor-General' one of the functions and duties shall be -

*(a) To administer, co-ordinate, and arrange for the maintenance and extension of geodetic control networks and traverse, precise levelling or other precision measurements forming the National Survey Control System, and to arrange for the maintenance of the salient permanent reference marks governing or providing subsidiary controls for title surveys.*

This requires LINZ to maintain a height datum which forms the basis for determination of cadastral coastal boundaries in accordance with this and other legislative requirements and for other general survey and mapping.

Blick *et al.* (1997) report over 20 laws and regulations which make reference to sea level and tides for the purposes of describing areas of land. The Surveyor-General and LINZ therefore have the responsibility on behalf of the crown to provide a vertical datum and the relationship between it and sea level to support these Acts. These Acts do not state the accuracy to which the vertical datum and the relationship to sea level need to be known, which is normal practice in Acts.

##### Requirements

- Heights that are expressed in metre units and are related to local mean sea level.
- Geopotential based heights across the country to allow cadastral survey measurements to be reduced to mean sea level.
- Heights which reflect changes in gravitational potential (eg orthometric heights) which ellipsoid heights (e.g. NZGD2000 heights) are unable to provide.
- A model/method to convert between different tide heights (e.g. Lowest Astronomical Tide, Mean Sea Level).
- A height datum that can be used for origin of strata titles and other cadastral boundaries that are defined in terms of hydro controlled lakes.

- Accuracy of 0.25 m or better relative to the chosen reference surface (e.g. MSL) though for local heights the relative accuracy between marks is often required to an order of magnitude higher (e.g. 0.02 m) or more.

### Issues

- Is the national tidal model (constituents) able to determine/check sea level to sufficient accuracy for defining cadastral boundaries or is a local determination of sea level required?
- The accuracy to which heights are required will vary depending on the application and the local land use and topography (e.g. the accuracy of heights required for the sea level correction of measured horizontal distances is probably two or three orders of magnitude less demanding than those required for coastal cadastral boundaries).
- There needs to be a program to monitor the changes in the reference surface and the related monuments. The methods used should be able to distinguish between changes in the sea surface and land subsidence or uplift.

#### 4.1.2 *National Topographic/Hydrographic Authority (NTHA)*

### Background

Topographic survey and mapping is usually carried out in terms of a horizontal datum and mean sea level is the origin for heights. This system, where mean sea level represents zero height, allows positive contours for height on land and negative contours (or depth) off shore. This approach serves many practical purposes and is probably the way most of the public perceive heights.

For Nautical Charting the origin for depths is Lowest Astronomic Tide (LAT) but for heights (clearances) it is Highest Astronomic Tide (HAT). The accuracy required varies depending on the depth of the water. For special order (generally less than 30 m, e.g. harbour channels) the depths need to be accurate to 0.25 m or better at the 95% confidence interval.

When LINZ was established on 1 July 1996 part of the hydrographic purchases on behalf of the Crown included the responsibility for providing tidal information/data/services for the following clearly identified needs and roles:

- as a member of the IHO (International Hydrographic Organisation), New Zealand (represented by LINZ) should comply with Technical resolution A6 of the IHO.
- as a member of IMO (International Maritime Organisation), New Zealand (represented by LINZ and the Maritime Safety Authority) is required to make tide tables available in advance for maritime safety. See the International Convention for the Safety of Life at Sea (SOLAS) Chapter 5 Reg. 20.

- to fulfil the Shipping (Nautical Publications) Regulations 1988.

These needs and roles therefore require sea level to be measured at sufficient density and quality throughout New Zealand's area of territorial responsibility to chart the area and be able to predict tides at any location within that area.

Depending on the length of time that sea level has been observed, ideally 18.6 years, the tidal constituents can predict tides to within 0.2 m in height and 10 minutes of the event at the 95% confidence interval. The tidal constituents enable the conversion between the different tide heights (LAT, HAT, MHWS, MSL, etc) but their accuracy is dependent on the length of data used to determine them and also any changes in the coastline or sea floor (e.g. dredging channels and reclaiming land).

### **Requirements**

- Heights which are expressed in metre units.
- Tidal constituents able to predict tide heights and times to 0.2 m and 10 minutes respectively.
- Accuracy of 4 m or better in terms of MSL for Mapping.
- Accuracy of 0.25 m or better for Charting.

### **Issues**

- Ability to monitor the long-term stability of the sea level recorders.
- Variety of data formats provided from port authorities and the need to purchase some data (e.g. NIWA data).
- Identifying and recording heights of rocks/islands which could be used in defining the areas of New Zealand's territorial claim.

## **4.2 Requirements by External Organisations**

The following sub-sections list some of the height requirements of some external organisations to highlight the variety. It is not intended to be a complete list.

### **4.2.1 Maritime Transport**

#### **Background**

For safe navigation of cargo ships and other vessels entering and leaving harbours Port Authorities require both predicted and real time tidal information. Shipping companies require a detailed knowledge of tidal ranges and times in advance to plan shipping movements to maximise cargo loading. It is therefore in the interest of Port Companies to provide sea level data to the NTHA to ensure accurate tidal predictions are available for publication in National Maritime Publications (eg Admiralty Tide Tables, New Zealand Nautical Almanac).

Currently Nautical Charts have LAT as the datum for depths. Shipping companies then use the tidal constituents to compute keel clearances at the time they intend entering or leaving a port. With the increasing use of GPS for navigation it is possible that charts (especially digital charts) could show the ellipsoidal height of features at LAT. Shipping companies could still use the tidal constituents to compute clearances but instead of them being added to LAT they are added to the ellipsoidal height. With GPS monitoring in real time the ellipsoidal height of the ships keel clearances could be determined as the vessel sails along.

In addition to shipping companies there is a need for accurate chart datums and predictions to support search and rescue services and defence requirements (e.g. beach landings).

### **Requirements**

- Heights which are expressed in metre units.
- Accuracy of 0.25 m or better for both heights shown on charts and the tidal predictions.

### **Issues**

- Monitoring of differences between predicted and actual tides.
- Use of GPS for determining keel clearances such as negative ellipsoidal heights still having a positive LAT height.
- There are similar clearance issues for land and air transport to the marine situation though they do not have the tidal fluctuations.

#### **4.2.2 *Scientific Research***

### **Background**

Organisations like NIWA, GNS and universities will have a range of projects that require different levels of accuracy and can utilise different heighting methods. For example Earth Deformation monitoring is normally concerned with the relative changes in heights of points and therefore can easily work with ellipsoidal heights. However, when monitoring the movement of liquids then heights that can account for changes in the geopotential are required.

### **Requirements**

- Heights which are expressed in metre units.
- Depending on the project either ellipsoidal heights or a geopotential based height.
- Consistent system across the country.
- Accuracy of 0.005 m or better.

## Issues

- A range of accuracy requirements from millimetre to metres.

### 4.2.3 *Engineering Projects*

#### Background

Engineering Projects that involve the transportation of fluids are one of the main users of the current levelling networks. Whether the projects involve piping liquids, artificial canals or existing waterways changes in gravity across the project area will affect the project design. Not all engineering projects will require heights which reflect changes in gravity (e.g. radio transmitter lines of sites and measuring stockpiles).

#### Requirements

- Heights that are expressed in metre units and are related to local mean sea level.
- Consistent system across the country.
- Depending on the project either ellipsoidal heights or a geopotential based height.
- Accuracy of 0.005 m or better.

## Issues

- A range of accuracy requirements from millimetre to metres.
- Often can require very high relative accuracy requirements but not absolute (e.g. use of a local datum origin of negative 100 m to avoid negative design heights).

### 4.2.4 *Public*

#### Background

In general the public would expect heights to be measured from mean sea level and have values that are expressed in length units (e.g. metres).

Few would know about the complexities of actually determining mean sea level though they expect heights to be accurate with little regard for the error sources.

#### Requirements

- Heights that are expressed in metre units and are related to local mean sea level.
- Consistent system across the country.

- Reliable and accurate tide predications of times and ranges.
- Accuracy of 0.5 m or better.

### Issues

- With the increased use of GPS they will need to better understand the difference between ellipsoidal heights and mean sea level.
- There is an expectation that heights are determined within a consistent datum so that hazard mitigation (flood risk assessment) and insurance costs can be assessed effectively.

### 4.3 Required height datum characteristics in New Zealand

Table 3 summarises the characteristics required of heights in New Zealand based on the previous anticipated user requirements presented in Sections 4.1 and 4.2.

	Survey	Maps	Charts	Maritime	Science	Engineering	Public
Represents changes in gravity	yes	yes	yes	yes	yes	yes	yes
Zero height approximates Mean Sea Level	yes	yes	no	No	yes	yes	yes
Single point accuracy better than 0.5 m	yes	no	yes	Yes	yes	yes	yes
Single point accuracy better than 0.05 m	yes	no	no	yes	yes	yes	no
Relative accuracy between points better than 0.05 m	yes	no	yes	yes	yes	yes	no
Has metre units	yes	yes	yes	yes	yes	yes	yes

Table 3: Characteristics required of a height datum in New Zealand

## 5 Height System Assessment

There are a number of different options that a national height datum can be based on. This section firstly looks at those options before summarising which option has been adopted in other countries before then assessing their suitability against the New Zealand requirements.

### 5.1 Height System Options

There are a number of different height reference systems available in geodesy, with some being able to detect which direction water will flow due to the height being referred to a natural equipotential surface, while others are simply mathematical approximations of the Earth. A brief summary of different height reference systems

that are commonly used in geodesy is contained in the following sub-sections that were adapted from Heiskanen and Moritz (1967) and Vanicek and Krakiwsky (1986).

### 5.1.1 *Geopotential Number*

Since only one equipotential surface ( $W_i$ ) passes through any point ( $P_i$ ), the gravity potential represents one possible way of defining a unique vertical position. The negative potential difference between the point  $P_i$  and the geoid is referred to as the geopotential number,  $C_i$  (Vanicek and Krakiwsky, 1986, eqn. 16.85).

$$C_i = -(W_i - W_o) = \int_P^{P_i} g \, dl \quad (1)$$

The geoid is the equipotential surface ( $W_o$ ) that approximates mean sea level.  $g$  is the value of gravity at  $P_i$  and  $dl$  is the continuous function of height difference along the terrain from a point on the geoid.  $dl$  is approximated in reality by  $\delta l$ , being the observed (levelled) height difference.  $C$  is measured in geopotential units (gpu), where 1 gpu = 1 kGal m.

Advantages:

- independent of levelling path used.
- points with the same height are on the same potential surface.
- no hypothesis needed about the composition of the Earth's interior.
- independent of reference ellipsoid or reference gravity.

Disadvantages:

- not expressed in length units.
- requires the potential at the geoid but the geoid is not a tangible surface.

The geopotential number is used by height datums of the world (eg. NAVD 88) as the basic quantity for the definition and computation of the vertical reference system.

### 5.1.2 *Dynamic Height*

To overcome the intuitive problem of geopotential numbers not being expressed in length units, the dynamic height ( $H^D$ ) was developed. Dynamic height is obtained by dividing the geopotential number by a constant reference gravity, often chosen to be the value of the current normal gravity ( $\gamma_o$ ) formula adopted by the IAG (Section 4.2.1) at a standard latitude (usually  $45^\circ$ ,  $\gamma_o$ ) (Heiskanen and Moritz, 1967, eqn. 4-9).

$$H_i^D = \frac{C_i}{\gamma_o} \quad (2)$$

Dynamic heights are numerically about 2% less than orthometric heights.

## Advantages:

- independent of levelling path used.
- points with the same height are on the same potential surface.
- no hypothesis needed about the composition of the Earth's interior.
- expressed in length units.

## Disadvantages:

- height can be wrongly interpreted as the geometrical distance between the geoid and  $P_i$ , when  $P_i$  not at the reference latitude.
- its dependant on chosen reference gravity.
- it requires the potential at the geoid.

The dynamic height difference ( $\Delta H^D$ ) between two points  $P_i$  and  $P_j$  can be expressed as a summation of the levelled height difference ( $\Delta l_{ij}$ ) plus a correction referred to as the dynamic correction ( $DC_{ij}$ ) (Vanicek and Krakiwsky, 1986, p. 370)

$$\Delta H^D = \Delta l_{ij} + DC_{ij} = \Delta l_{ij} + \sum_{k=i}^j \frac{g_k - \gamma_0}{\gamma_0} \delta l \quad (3)$$

The dynamic correction can be very large because gravity varies from the equator to pole by about 5000 mGal (Heiskanen and Moritz, 1967). For a levelling line with a difference in height of 1000 m at the equator,  $g \cong 978.0$  Gal, computed with  $\gamma_0 = \gamma_{45^\circ} = 980.6$  Gal then  $DC = -2.7$  m.

### 5.1.3 Orthometric Height

The orthometric height ( $H_i$ ) of a point  $P_i$  is defined as the geometrical distance between the geoid (P) and the point  $P_i$ , measured along the plumb line (Vanicek and Krakiwsky, 1986, p. 371).

$$H_i = \frac{1}{\bar{g}} \int_P^{P_i} g \, dl = \frac{C_i}{\bar{g}} \quad (4)$$

where  $\bar{g}$  is the mean value of gravity along the plumb line between the geoid (P) and the point  $P_i$ . Since it is not practically possible to measure  $g$  along the plumb line between P and  $P_i$ , some assumption has to be made as to the behaviour of the density of the Earth in this region. Heiskanen and Moritz (1967) show that a density error of  $0.6 \text{ g/cm}^3$ , which corresponds to the maximum variation of rock density in practice, introduces an error in  $H_i = 1000\text{m}$  of only  $0.025$  m.

There are a number of different approaches to approximate  $\bar{g}$ , each of which result in a different kind of orthometric height usually referred by the name of the proponent, eg, Helmert, Niethammer or Mader. The Helmert orthometric height is one of the

most commonly used in practice (Vanicek and Krakiwsky, 1986, p. 371) and is defined as:

$$H_i^H = C_i / g_i^H = C_i / (g_i + 0.0424H_i) \quad (5)$$

where  $g_i$  is the gravity at  $P_i$  on the Earth's surface (in Gal). The numerical coefficient (0.0424) follows directly from the use of Poincare-Pray's gravity gradient considered to be constant along the plumb line between the geoid and terrain, thus allowing  $\bar{g}$  to be directly computed for the midpoint of the plumb line of  $P_i$ .  $H_i$  is the observed height in km.

Advantages:

- it is independent of levelling path used.
- its expressed in length units.

Disadvantages:

- it requires observed gravity data at the Earth's surface.
- that a hypothesis is needed about the composition of the Earth's interior.
- that points with the same orthometric height are not necessarily on the same equipotential surface, especially at high altitudes because of the uncertainty of the Earth's density and equipotential surfaces not being parallel to each other.

The orthometric height difference ( $\Delta H$ ) between two points  $P_i$  and  $P_j$  can be expressed as a summation of the levelled height difference ( $\Delta l_{ij}$ ) plus a correction referred to as the orthometric correction ( $OC_{ij}$ ) (Heiskanen and Moritz, 1967, p. 168)

$$\Delta H = \Delta l_{ij} + OC_{ij} \quad (6)$$

where

$$OC_{ij} = \sum_{k=i}^j \frac{g_k - \gamma_0}{\gamma_0} \delta l + \frac{\bar{g}_i - \gamma}{\gamma_0} H_i - \frac{\bar{g}_j - \gamma}{\gamma_0} H_j \quad (7)$$

In reality, as mean gravity along the plumb line is unknown,  $OC$  is approximated by equations based on normal gravity (See Rapp, 1961). Therefore in theory, since the levelling differences have been corrected using an orthometric correction based on normal gravity rather than observed gravity the heights should be referred to as normal orthometric heights. In practice if  $g_i - \gamma_i$  is 10 mGal, an error of only 0.001 m in 100 m of measured height difference ( $\delta l$ ) will result. This is trivial unless it accumulates systematically (Bomford, 1980).

#### 5.1.4 Normal Height

The normal height ( $H^N$ ) of a point  $P_i$  was proposed in 1954 by Molodenskii et al. (1962) to overcome the problem in orthometric heights of having to determine the mean value of gravity along the plumb line. The normal height is obtained by

dividing the geopotential number by the mean normal gravity along the normal plumb line of  $P_i$  (Vanicek and Krakiwsky, 1986, p. 372):

$$H_i^N = \frac{C_i}{\bar{\gamma}_i} \quad (8)$$

where  $\bar{\gamma}_i$  is computed between the reference ellipsoid surface and the telluroid. The telluroid is the surface whose height above the geocentric reference ellipsoid is the same as the height of the terrain above the geoid (Vanicek and Krakiwsky, 1986, p. 117).

Advantages:

- it is independent of levelling path used.
- it is expressed in length units.
- it does not require observed gravity data at the Earth's surface.
- there is no hypothesis needed about the composition of the Earth's interior.

Disadvantages:

- it is dependant on the chosen reference gravity and ellipsoid.
- that points with the same normal height are not on the same equipotential surface.

### 5.1.5 *Ellipsoidal Height*

The ellipsoidal height ( $h$ ) is the distance along the normal to the reference ellipsoid between  $P_i$  and the surface of the ellipsoid.

Advantages:

- it is independent of the levelling path used.
- it is expressed in length units.
- it does not require observed gravity data at the Earth's surface.
- there is no hypothesis needed about the composition of the Earth's interior.
- it can be directly measured in terms of a geocentric reference ellipsoid using satellite techniques (i.e. Satellite Altimetry or GPS).

Disadvantages:

- it is dependant on reference ellipsoid.
- that points with the same ellipsoidal height bear no common relationship with the actual gravity field of the Earth..

### 5.1.6 *Summary of Vertical Reference Systems*

Out of the five different vertical reference systems described above, the ellipsoidal height is least able to predict the direction water will flow. A reference ellipsoid can be chosen to minimise the difference between the ellipsoid surface and the geoid for a portion of the Earth, but a geocentric reference ellipsoid surface (i.e. WGS84) can vary by up to 100 m from the geoid. This variation makes ellipsoidal heights unsuitable for topographic mapping that uses MSL as the basis for heights and for use in general. With the introduction of space based techniques, such as Satellite Altimetry, Satellite Laser Ranging and GPS, absolute ellipsoidal heights in terms of the Earth's geocentre are attainable in a globally consistent reference system. These ellipsoidal heights are useful, since being a directly determinable quantity, to monitor the relative change of a station with respect to the chosen reference system (eg. monitoring crustal deformation).

The geopotential number is of great scientific importance since it uniquely defines an equipotential surface, and is the most direct result from spirit levelling (Heiskanen and Moritz, 1967). However, it is not a height in a geometrical or practical sense.

The difference between dynamic, normal or orthometric height is only in the choice of scaling the geopotential number. Dynamic heights are not suitable as practical heights due to the large dynamic corrections. Normal heights have less obvious physical and geometrical meaning than orthometric heights, due to the dependence on the reference ellipsoid used, but can be easily computed rigorously. Orthometric heights are the natural height above the geoid and thus have an unequalled geometrical and physical significance. However, orthometric heights have relatively involved computations due to requiring surface gravity data, unless Helmert orthometric heights are used (Heiskanen and Moritz, 1967, p. 172.)

Table 4 compares the different vertical reference systems described in the preceding sections. Considering the geometrical and physical significance of orthometric heights, this system is recommended for the development of any national vertical datum in New Zealand. It should be remembered that, due to the definition of orthometric heights, points on the same equipotential surface (except on the geoid) do not generally have the same orthometric height; water may appear to flow "up hill" - from a lower to higher orthometric height (Vanicek and Krakiwsky, 1986). Though detecting this phenomenon in reality is considerably more difficult than when ellipsoidal heights are used.

	Geopotential Number	Dynamic Height	Orthometric Height	Normal Height	Ellipsoidal Height
Independent of levelling path	yes	yes	yes	yes	yes
Hypothesis required for Earth's crust density	no	no	yes	no	no
Dependant on reference ellipsoid	no	no	no	yes	yes
small corrections to levelling data	no	no	yes	yes	no
same height on same potential surface	yes	yes	no	no	no
units are in metres	no	yes	yes	yes	yes

Table 4 : Comparison of vertical reference systems (adapted from Brouwer and De Min, 1994)

The problem with all the heights discussed, except ellipsoidal height, is that the position of the geoid is not directly measurable or is it tangible.

## 5.2 *International Examples*

International experience is worth looking at to see what direction and definition problems other countries have needed to overcome in the development of their height systems. The following examples build on the work of Hannah (2000) and covers Australia, North America, Europe and South America.

One thing that almost all of these countries have done is adjust together their levelling data to provide a national height system. The author is unsure whether each South American country has a single national levelling datum. Though these national adjustments may have technical imperfections they have served the vast number of users successfully. Some of the technical imperfections are that they did not take account of sea surface topography between the defining tide gauges and they used normal orthometric height corrections.

With perhaps the exception of Australia, all the countries face the effects of either tectonic deformation or post-glacial rebound. The Nordic countries have developed models for the rate of post-glacial rebound and scientific users apply this correction to their heights but it is often ignored by the non-scientific user.

The USA has recently completed the North American Vertical Datum 1988 (NAVD88) which superseded the National Geodetic Vertical Datum 1929. Zilkoski *et al.* (1992) report that the new adjustment was constrained by one tide gauge benchmark and observed gravity data was used to compute orthometric heights from the geopotential numbers using the Helmert approach (See equation 5).

South American countries have a project, called SIRGAS, to modernise the horizontal and vertical datums across the continent. They recently completed their new geocentric datum for the continent and are now working on options for their primary vertical datum. In addition to the ellipsoidal height component of the geocentric datum the Working Group on the Vertical Datum is likely to recommend the adoption of normal heights. Initially they are concentrating on the computation of geopotential numbers at the GPS stations and the tide gauge stations (Luz, private communication)

Some European countries (e.g. France and Germany) use normal heights for their national vertical reference system, as no gravity observations or assumptions about the Earth's density are required. While other European countries (e.g. Denmark) use geopotential numbers or orthometric heights (e.g. Britain).

From these limited examples of the International situation it can be seen that there is no clear guidance for New Zealand on which height system they should adopt. There also appears to be no agreed upon approach from scientific and governmental organisations on the best approach.

### **5.3 *Implementation in NZ***

This section outlines the current status, any additional data required and an indication of the stages in computing each type of height system described in Section 5.1.

#### **5.3.1 *Ellipsoidal heights***

With the implementation of the new geodetic datum, NZGD2000, ellipsoidal heights are available at each station in the NZGD2000 network that have been observed with GPS. Currently though the velocity model associated with NZGD2000 has a zero height velocity across the entire country. This is a result of the GPS observations generally only being repeated across more than three years at the zero and first orders stations, with the lower order stations normally being occupied only within a single year. Combined with the larger vertical error component in GPS observations and the relatively small rates of vertical motion (0-10 mm/yr in comparison to the horizontal motion of 50 mm/yr) there is insufficient data to reliably model the vertical velocity across New Zealand.

Further investigations are needed into the long-term effects of not modelling the vertical motion on the NZGD2000 network.

#### **5.3.2 *Geopotential Numbers***

Neither LINZ nor its predecessor organisations computed or recorded geopotential numbers. To compute geopotential numbers rigorously one needs gravity data observations (equation 1). As was mentioned in section 3.3.1 LINZ does not hold gravity observations, they would need to be obtained from GNS, and it is uncertain whether the raw observations have been stored or only the reduced free-air anomalies. If the raw data has been stored then it is possible that more rigorous reduction of the

data could be possible now with the improvements in computer resources. If computing orthometric heights rigorously then one needs to compute the geopotential number as shown in equation 4.

### 5.3.3 *Dynamic heights*

The former departments L&S and DOSLI used to compute dynamic heights. The dynamic heights were originally intended for use by engineers but they were very rarely used so in the late 1980's they were abandoned. Rather than being computed as shown in equation 2 the dynamic heights were derived from the orthometric height by applying a dynamic height correction as given in equation 8.16.3 of Lee (1958).

### 5.3.4 *Orthometric heights*

As was outlined in section 3.2 the orthometric heights in New Zealand are regional networks based on tide gauge readings and levelling differences reduced using normal gravity. In addition to the levelling networks established by L&S and DOSLI there are networks that were installed by the former Ministry of Works and Development primarily to support hydroelectric schemes. Some of these networks, particularly in the Otago Land District, have not been recorded in the LINZ geodetic records.

To develop a national, or at least an island based, consistent orthometric height network one has to decide whether to:

- search out the original levelling height difference data and perform a rigorous orthometric height reduction adjustment (taking into account observed gravity), or
- take the existing heights from the normal-orthometric height levelling networks and model the differences between each and a new consistent network.

One issue with going back to the old height differences is that they may no longer represent the actual relationship. For this reason and considering that users have been adopting the existing heights it is probably more efficient and useful to just compute the differences between the existing networks and a new consistent network.

Non-technical users who are using GPS for positioning are now driving the need for geoid models. They require a simple and accurate method to convert their ellipsoidal heights to orthometric heights.

### 5.3.5 *Normal heights*

As normal heights are computed by dividing geopotential numbers by a mean normal gravity (equation 8) then the same problems and issues as given for geopotential numbers in section 5.3.2 also apply. The only new issue is which normal gravity value you choose to divide the geopotential numbers by.

### 5.3.6 *Geoid heights*

New Zealand does not have a high-resolution national geoid height model. Gilliland (1990) produced a gravimetric geoid for New Zealand on a 0.25° grid by combining gravity data and the OSU81 (Rapp, 1981) GGM to degree and order 180. However without any ellipsoidal heights no comparison using equation 9 could be made.

Mackie (1982) determined geoid heights at 18 stations, distributed across New Zealand, by comparing Doppler derived WGS72 ellipsoidal heights with spirit levelled orthometric heights.

Mackie's work did not use local gravity data, though when Gilliland compared his results with the results of Mackie, after reduction to a common datum and the removal of biases, a Root Mean Square (rms) of geoid heights of less than 1.3 m was obtained. At the time this was a reasonable result given observation error in Doppler height, but with present day techniques it should be possible to obtain results at least an order of magnitude better.

The possibility of increased accuracy has come about with the improvement in global geopotential models and the accuracy of ellipsoidal heights, primarily from GPS, at which geoid heights can be tested.

### 5.3.7 *Tidal Connections*

The project between GNS and the University of Otago to monitor sea level recorder stability is an important part of a new vertical datum. Ideally all sea level recorders should have collocated GPS receivers operating continuously to monitor the stability of the gauge. This scenario would then provide a direct measurement of the ellipsoidal height of sea level at any time. Once the sea level data has been analysed it also provides the direct measurement of the local geoid height. Note that this local mean sea level is not likely to coincide with the global mean sea level. However for most users this is the offset they require between their ellipsoidal height and sea level and not a globally averaged value.

Where it is not feasible to install permanent GPS tracking stations high quality ellipsoidal heights need to be determined for at least two marks near the sea level recording stations. A connection (both horizontally and vertically) needs to be made between these marks and the recording station. These "witness marks" will allow episodic GPS campaigns to check the stability of the site.

For the actual sea level recorders it would be highly desirable to have instruments that can produce the same quality data and output in a consistent format. These instruments will also require calibrating each year to ensure the data represent the actual sea level.

## 6 Transforming between height systems

To transform between ellipsoidal heights ( $h$ ) and orthometric heights ( $H$ ), in theory, you only need to know the geoid height ( $N$ ) and you can apply the simple relationship of (Heiskanen and Moritz, 1967):

$$h = H + N \quad (9)$$

Equation 9 assumes two things. Firstly that you know the error estimates and biases in all three heights and that they all are in terms of a common reference system. Secondly that the mark to which these heights refer has not moved during the time you measured each of the three components.

In New Zealand the ellipsoidal heights are in terms of NZGD2000, the orthometric height will be in terms of any one of the levelling networks and currently LINZ uses EGM96 (Lemoine *et al.*, 1998) as the geoid model.

Depending on the level of accuracy you can find biases in all three of the height components and therefore it is more appropriate to express the situation as (Milbert and Smith, 1997):

$$h + \delta h = (H + \delta H) + (N + \delta N) \quad (10)$$

where the new terms indicate the bias in each height component.

It is reasonable to expect in the New Zealand situation with the rugged terrain that the geoid bias ( $\delta N$ ) will be largest, followed by the orthometric heights bias ( $\delta H$ ) due to the multiple orthometric height datum and the smallest would be the ellipsoidal height bias ( $\delta h$ ) due to the modern NZGD2000.

The NZGD2000 network has ellipsoidal heights for fourth order and higher marks. The NZGD2000 network also incorporated a significant number of existing marks that have orthometric heights thus it is possible to solve for  $N$  using equation 10. There are a number of different approaches that could be used and these are summarised in the following sub-sections. One of the challenges is to try and separate out the different biases.

### 6.1 Difference Model

If we take all the stations within the region of a levelling network that have NZGD2000 ellipsoidal heights and orthometric heights we can compute the difference between these two heights at each station. These irregularly spaced differences could then be converted into a regular grid of differences in a similar way to the Distortion Model developed to transform NZGD49 coordinates to NZGD2000 coordinates. Considering the way the levelling data has been adjusted it would be unlikely that the differences across a levelling network would be constant, there are likely to be at least slopes.

By rearranging equation 10 to represent this approach it can be seen in equation 11 the result is the combination of the geoid height and all three biases.

$$h - H = N + \delta H + \delta N - \delta h \quad (11)$$

Equation 11 should probably be solved for each of the orthometric height datum regions due to their overlapping. Marks heighted via either spirit levelling or vertical angles could be used in the computation as long as appropriate weights were assigned to each type of observation.

While this approach would probably be acceptable to users since it provides them with a simple way to convert their ellipsoidal heights into their local levelling datum it does not provide a nationally consistent height system. When working across the boundaries of two or more local height datum there will be discontinuities.

This difference/distortion model approach to transforming between heights has been suggested for use in the United States by Milbert and Smith (1997) and in Australia by Featherstone (1996).

## 6.2 *Geopotential Model*

Geoid heights can be generated using geopotential models. By combining an ellipsoidal height and a geoid height you can obtain an orthometric height (as can be seen from equation 9). The accuracy of this orthometric height is dependent on all three height biases. The next two sub-sections discuss whether to adopt a global model or develop a national geopotential model.

### 6.2.1 *Adopting a Global Geopotential Model*

Currently the geopotential model LINZ recommends using in New Zealand is the global model EGM96 (Lemoine *et al.*, 1998). EGM96 has a maximum degree and order,  $n_{\max}$ , of 360 which theoretically means it can recover features with a half wavelength of  $0.5^\circ \approx 55 \text{ km}$  ( $\approx 1/n_{\max}$ ). Given the topography of New Zealand it is most unlikely that EGM96 can model the geoid in New Zealand to sufficient accuracy.

Pearse (1998) reported these combined biases to be 0.5 m in the lower North Island and up to 1.0 m in the South Island. The main contribution to the bias in both islands is not understood but could be due to either the way each orthometric height datum has been defined or EGM not able to model the gravity field sufficiently.

### 6.2.2 *Developing a Regional Geopotential Model*

Local gravity observations can be used in conjunction with a global geopotential model to produce an improved geoid model for the New Zealand region (Pearse, 1998). There are a number of different techniques available to compute geoid heights from gravity observations, which includes Collocation, Fast Fourier Transform and RING Integration. They all use the remove and restore approach and produce results of similar quality.

The quality of the data used in the geoid model computation is more of a limiting factor in the accuracy of the results than the technique used. The data sets required are gravity observations (land, sea, air and space observations), a digital elevation model and a global geopotential model. Though there are some issues with the quality and density of the gravity observations (Section 3.3) the computation of a regional geopotential model using the existing data will improve the accuracy of the geoid model due to the addition of the higher frequency information.

Once an initial high-resolution geopotential model has been developed and tested, there will be a better understanding of whether any additional gravity observations will be required. If new observations are required then the Airborne observation method will need to be considered closely as it can provide a uniform high frequency data set over land and sea.

### 6.3 *A Geopotential Model and a Difference Model*

Once the preferred geopotential model has been chosen it can be used to compute  $N$  values. We already have ellipsoidal and orthometric heights, therefore equation 9 should hold. If we rewrite equation 11 so that all the terms with known values are on the left-hand side we can see in equation 12 that there remain the biases.

$$h - H - N = \delta H + \delta N - \delta h \quad (12)$$

As was first discussed in Section 6.1 we can compute a difference model. Having the geoid height component in its own right now the size of the differences/bias will be smaller. There still will need to be a difference model for each orthometric height datum region.

One advantage of this combined geopotential and difference model approach is that you can have a consistent national orthometric height datum based on the ellipsoidal and geoid heights. You also have a method to transform between the nationally consistent height datum (combined NZGD2000 ellipsoidal heights and geoid heights) and the regional spirit levelling datums.

It is important that users record with their final height the datum and method used to compute the height, that is the ellipsoidal height system, regional or national orthometric system, geoid model and difference model. For example an orthometric height computed from an NZGD2000 ellipsoidal height and an EGM96 geoid height. This will allow future users to change the geoid model used if an improved model is released without the need to re-observe the other height components.

## 7 Time varying heights

It is well known that the Earth's surface in the New Zealand region has relative movements that deform its shape. The horizontal movements are reasonably well

known (e.g. Beavan, 1998) but in comparison the vertical movements are not. There have been regional studies that show that areas within the Taupo Volcanic Zone are subsiding up to 10 mm/yr (Blick and Otway, 1995). Also the Southern Alps are subjected to uplift rates in the order of 10 mm/yr due to the interaction of the Pacific and Australian plates along the alpine fault (Walcott, 1984). These subsidence and uplift rates have a slow but continuous effect on the height of stations. Earthquakes however often have the largest short term effect on heights, for example subsidence of up to 2 m resulted from the Edgecumbe earthquake of 1987 (Beanland et al., 1990).

The effects of vertical changes can be more devastating than a large horizontal movement due to the possible influx of water immediately after an earthquake or through the increased risk of seasonal flooding.

The author is not aware of any national vertical velocity model that can be used for the transforming of heights observed at one epoch to that of another. Often the levelling networks have only been re-observed after noticeable height changes, such as earthquakes, rather than to monitor seasonal variations or long term trends. There is a need for LINZ to develop a national vertical velocity model based on these regional and irregular re-observations to assist in the comparison of historic heights.

In the same way that it is important to record the name and version number of each component used to derive a height, e.g. NZGD2000 ellipsoidal height or Wellington Datum 1953, it is also important to record the epoch/date of the observation of each component. In the future as understanding of the vertical velocity rates improve it will be possible to transform heights observed at one epoch to that of another. It may be many years away before such a vertical velocity model has sufficient accuracy but acknowledging that heights change now will assist in the possible move to a fully dynamic national datum as discussed in OSG (1998a).

## 8 Recommendations

The following recommendations are made:

1. **LINZ upgrade the existing height datums to a nationally consistent orthometric height datum.** This recommendation is based on the requirements outlined in Section 4 and the features of different types of heights in Section 5.1. The accuracy of the new orthometric heights should be 0.20 m or better at the 95% confidence level.
2. **LINZ develop a national geoid model.** The geoid model should build upon the global geopotential models by using local gravity data (Section 6.2.2). Once developed the geoid model will allow consistent conversion of NZGD2000 ellipsoidal heights to the nationally consistent orthometric height datum (recommendation 1).
3. **LINZ develop transformation grids to allow conversion between the new national orthometric datum and the existing regional levelling networks.**

These grids will allow users to convert between height datums in a similar way to the grid of distortions that allows conversion between NZGD49 and NZGD2000 coordinates (Section 6.1).

4. **LINZ develop and maintain a national vertical velocity model.** This velocity model will allow LINZ to provide height coordinates at a nominated epoch in a similar way to the NZGD2000 coordinates (Section 7).

## 9 Action Plan

An approach to meet the recommendations in section 8 is presented in this section.

### 9.1 *Recommendation One – Nationally Consistent Orthometric Height Datum*

#### 9.1.1 *Prepare a Policy Statement*

Prepare a LINZ Policy statement which states that a nationally consistent height datum is required, that the type of heights are to be orthometric heights and the standards (considering different orders) to which the new heights should be determined.

#### 9.1.2 *Determine the method of producing the new datum*

There are two different approaches that could be followed to produce the new height datum. Each approach will require a transformation grid to be computed for each of the existing levelling networks.

One approach is to collect together the original spirit levelling differences and reduce them rigorously into a single network of orthometric heights for each island. The assumption is that we know the sea surface topography well enough between the islands to ensure that each network is in terms of the same equipotential surface. This approach will use data that has been collected over approximately 40 years and therefore will be affected by the vertical deformation signal. Most of the levelling differences are not in digital files so there would be a significant resource required to extract the information from the paper records. This approach will not provide a geoid model directly but if LINZ computed a transformation grid between the NZGD2000 ellipsoidal heights and the new levelling adjustment this could be used in place of a geoid model.

The other approach is to compute a geoid model to apply to the NZGD2000 ellipsoidal heights. As the NZGD2000 ellipsoidal heights were derived from approximately 5 years of data the vertical deformation signal contained within them is negligible (see Section 5.3.1). This approach will provide a geoid model that will more closely follow international trends than the above approach. One probable disadvantage of this approach could be that the available gravity observations are not suitable to enable the height accuracy requirements to be met without new observations being made, possibly across the entire country.

Consideration needs to be given to which approach is more likely to meet the accuracy requirements and which is more cost and time efficient to compute. It is anticipated that following further investigations into the quality and availability of the levelling and gravity data, that computing a geoid model is likely to be the preferred approach.

### **9.1.3 *Publicity***

Develop a communication plan for informing the user groups and industry of the proposal for a new height datum with statements confirming the policy of a single vertical datum and availability of transformations.

## **9.2 *Recommendation Two – National Geoid Model***

### **9.2.1 *Prepare a Policy Statement***

Prepare a LINZ Policy statement that states that a national geoid model is required and the accuracy to which the geoid heights should be determined.

### **9.2.2 *Identify Existing Gravity data***

Obtain the existing sea and land based gravity data from GNS as well as information on the format of the digital files and the reductions applied to the observations. Approach NIWA to see if they hold any gravity data that could be used in the computation of a geoid model. Investigate the availability of satellite altimetry around New Zealand from international sources such as the Scripps Institution of Oceanography ([http://topex.ucsd.edu/marine\\_grav/mar\\_grav.html](http://topex.ucsd.edu/marine_grav/mar_grav.html)).

### **9.2.3 *Assess Existing Gravity data***

Validate the gravity data for any irregularities including the land data that plots off the Canterbury coast (Figure 2). Assess whether additional gravity data needs to be observed and what method (land based, airborne or satellite) would be most cost efficient if further data needs to be acquired.

### **9.2.4 *Obtain Digital Elevation data***

Contact LINZ topographic data distributors to determine the availability of gridded digital elevation data and the associated costs. The NTHA database only holds the contour information in vector format and can not produce a grid of spot heights.

### **9.2.5 *Computation and Testing of a Geoid Model***

Determine whether there would be more benefit to LINZ by employing a geoid computation expert at LINZ on a short-term contract or to contract someone to compute the geoid model at their work premises. If employing someone at LINZ then access to suitable computing resources should be acquired. Assess which computation method (RINT, FFT, and collocation) would best suit the available gravity data and the topography of New Zealand.

The relative accuracy of the geoid model could be tested by comparing points within a regional levelling network that have orthometric and NZGD2000 ellipsoidal heights.

### **9.2.6 *Publicise the Geoid Model***

Publish the geoid model, provide software to compute the geoid heights and document the development process.

## **9.3 *Recommendation Three – Transformation Grids***

### **9.3.1 *Prepare a Policy Statement***

Prepare a LINZ Policy statement that states that transformation grids between each of the levelling networks and the nationally consistent orthometric height datum are required.

### **9.3.2 *Compute Regional Grids***

Compute a distortion grid between each of the regional levelling networks and the nationally consistent orthometric height datum. Investigate what weighting is given to different orders of orthometric height data in the grid computation process (e.g. the same for first order benchmarks and trig heights?).

### **9.3.3 *Publicise Grids***

Publish the distortion grids for each of the regional levelling networks, provide software to simplify the use of the grids and document the development process. Also describe the history behind the old levelling networks, what will happen to existing heights and some of the difficulties in establishing mean sea level datums.

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