

**LINZS25000**

**Standard for New Zealand  
Geodetic Datum 2000**

# TABLE OF CONTENTS

TERMS, DEFINITIONS AND SYMBOLS.....	3
FOREWORD.....	6
1 INTRODUCTION AND SCOPE.....	9
1.1 Introduction.....	9
1.2 Scope.....	9
1.3 Intended use of standard .....	9
2 NEW ZEALAND GEODETIC DATUM 2000.....	9
2.1 When this standard is compulsory .....	9
2.2 NZGD2000 parameters.....	9
3 NZGD2000 DEFORMATION MODEL.....	10
3.1 Deformation calculation .....	10
4 COORDINATE TRANSFORMATIONS .....	11
4.1 Transformations from NZGD1949 – NZGD2000.....	11
4.1.1 The datum reference ellipsoid parameters .....	11
4.1.2 Types and accuracy of transformations .....	11
4.1.3 Defining three parameter transformation.....	12
4.1.4 Defining seven parameter transformation .....	12
4.1.5 Defining grid file transformation.....	12
4.2 Transformations from NZGD2000 – NZGD1949.....	13
4.2.1 Defining three parameter transformation.....	13
4.2.2 Defining seven parameter transformation .....	13
4.2.3 Defining grid file transformation.....	14
4.3 Transformations from WGS84 – NZGD2000.....	14
4.3.1 The datum reference ellipsoid parameters .....	14
4.3.2 Null transformation.....	14
4.4 Transformation from CIGD1979 – NZGD2000.....	15
4.4.1 The datum reference ellipsoid parameters .....	15
4.4.2 Type of transformation .....	15
4.4.3 Defining seven parameter transformation .....	15
4.5 Transformation from NZGD2000 – CIGD1979.....	16
4.5.1 Type of transformation .....	16
4.5.2 Defining seven parameter transformation .....	16
APPENDIX A: IMPLEMENTATION OF TRANSFORMATIONS.....	17
A.1 Introduction.....	17
A.2 Geographic coordinates to Cartesian coordinates.....	17
A.3 Three parameter transformation .....	17
A.4 Seven parameter transformation .....	17
A.5 Cartesian coordinates to geographic coordinates .....	18

## TERMS, DEFINITIONS AND SYMBOLS

<b>Term/Abbreviation</b>	<b>Definition</b>
Cartesian coordinates	values representing the location of a point in relation to three orthogonal intersecting straight lines called axes. For the purpose of this standard the intersection of all three axes coincides with the centre of mass of the Earth so that the z-axis is aligned with the Earth's mean pole of rotation; the x-axis passes through 0° longitude on the equator; and the y-axis passes through 90° longitude on the equator such that it completes a right-handed rectangular coordinate system. Cartesian coordinates are expressed as (X,Y,Z) triplets with units of metres.
CIGD1979	Chatham Island Geodetic Datum 1979
coordinate	any one of a set of numbers used in specifying the location of a point on a line, in space, or on a given plane or other surface (for example, latitudes and longitudes are coordinates of a point on the Earth's surface)
coordinate reference system	a type of coordinate system based on orthogonal axes intended to provide a consistent and standardised reference for many users to reliably specify and share information on the location of points in space
coordinate system	a system for allocating coordinates to points in space in some specified way in relation to designated axes, planes, or surfaces. The simplest coordinate system consists of orthogonal coordinate axes, known as a Cartesian coordinate system.
coordinate transformation	parameters to enable change of coordinates from one coordinate system to another coordinate system
datum	a particular type of reference system in which coordinates are defined in relation to a particular reference surface by means of distances or angles, or both
deformation model	a mathematical model that predicts the Earth's movements due to crustal deformation
ellipsoid	a surface formed by the rotation of an ellipse about a main axis. For the purposes of this standard, the ellipsoids used are oblate to match the general shape of the Earth. An oblate ellipsoid is one in which the semi-minor axis of the ellipse is the axis of revolution.

ellipsoid flattening	a measure of the flattening of an ellipsoid in relation to a sphere, it is the ratio of the length of half the short axis (semi-minor axis) and half of the long axis (semi-major axis) subtracted from one
geodetic datum	a datum used to depict coordinates in relation to a model of the surface of the Earth. It is often implemented by the definition of a coordinate reference system and a reference ellipsoid
geodetic system	a system that enables positions on the surface of the Earth to be determined by reference to a mathematical model that describes the size and shape of the Earth as defined in s 4 of the Cadastral Survey Act 2002
geographic coordinate	the quantities of latitude and longitude (and optionally height) that define the position of a point on the surface of the Earth. Geographic coordinates are often expressed in units of degrees, minutes, and seconds
GNSS	Global Navigation Satellite System
GRS80	Geodetic Reference System 1980 – an ellipsoid adopted by the International Association of Geodesy as the recommended best-fit ellipsoid for the Earth, Moritz (2000)
IERS	International Earth Rotation and Reference Systems Service
IGS	International GNSS Service
inverse flattening	the inverse of the ellipsoid flattening term, used by convention in ellipsoidal calculations
ITRF	International Terrestrial Reference Frame - a realisation of an ITRS through a global set of space geodetic observing sites. An ITRF comprises geodetic Cartesian coordinates and velocities for a set of fiducial points.
ITRF96	International Terrestrial Reference Frame 1996 (Boucher et al 1998) – the 1996 realisation of the ITRF
ITRS	International Terrestrial Reference System – an Earth-fixed reference system (one that rotates with the Earth) that is defined by a three-dimensional geocentric coordinate reference system. The system is realised by the IERS as the ITRF.
LINZ	Land Information New Zealand
national survey control system	a system used to determine the position of points, features, and boundaries in cadastral surveys, other surveys, and land information systems (as defined in s 4 of the Cadastral Survey Act 2002)
NZGD2000	New Zealand Geodetic Datum 2000

NZGD1949	New Zealand Geodetic Datum 1949
semi-major axis	semi-diameter of the longest axis of an ellipsoid; this is the axis measured in the equatorial plane for an oblate ellipsoid
WGS84	World Geodetic System 1984 (NIMA, 2000)
<b>Symbol</b>	<b>Definition</b>
$a$	semi-major axis of reference ellipsoid
$e^2$	squared eccentricity of reference ellipsoid
$f^{-1}$	inverse flattening of reference ellipsoid
$\phi$	latitude
$\lambda$	longitude
$h$	height
$R_x, R_y, R_z$	coordinate transformation rotation parameters
$T_x, T_y, T_z$	coordinate transformation translation parameters
$X, Y, Z$	Cartesian coordinates in a geocentric coordinate reference system
$\Delta s$	coordinate transformation scale change parameter

## FOREWORD

Section 7(1)(a) of the Cadastral Survey Act 2002 makes it a function and duty of the Surveyor-General to maintain a national geodetic system. The Surveyor-General has issued this standard to meet that obligation.

A national geodetic system and its associated national survey control system are fundamental components of a nation's infrastructure. The unique property of a geodetic system is its ability to allow the integration of multiple geographically dependent data sources into a single geographic reference frame.

A fundamental element of a geodetic system is a national geodetic datum. A method is required to enable spatial data held in different datums to be translated into a single consistent datum - the national datum. This method is called a transformation. Transformation to the national datum enables the consistent measurement and location of features in the physical world. The presence of a national datum and the appropriate transformations ensures that diverse spatial datasets can be corrected, correlated, and compared within a consistent framework.

### Purpose of standard

The purpose of this standard is to:

- (a) define the New Zealand Geodetic Datum 2000 (NZGD2000),
- (b) detail the parameters that define NZGD2000,
- (c) define the NZGD2000 deformation model, which allows the transformation of coordinates from the reference epoch 2000.0 to any other time, and
- (d) define the transformation parameters between NZGD2000 and other historic and commonly used geodetic datums in New Zealand.

### Rationale for new standard

Land Information New Zealand (LINZ) is tasked with meeting the Government's desired economic, social, and environmental outcomes in relation to its mandated subject areas. Accordingly, end outcomes, intermediate outcomes, objectives, and sub-objectives have been developed to clearly articulate the regulatory framework for each subject area.

A risk-based approach is then used to determine the optimum level of intervention. If there is a high risk of not achieving an objective or sub-objective, then, generally, a higher level of intervention is required. Similarly, a low risk of not achieving an objective or sub-objective means a low level of intervention is necessary. The desired intervention is then developed to manage the identified risks and thereby achieve the relevant sub-objectives, objectives and, therefore, the outcome.

This standard has been developed to mitigate the risk of not achieving the following end outcome and objectives.

End outcome	Objectives
A single common reference system that underpins the efficient operation of the cadastral, hydrography, and topography systems and meets directed government needs	<ul style="list-style-type: none"> <li>• Common preferred geodetic datums and projections are used by Managers of geospatial data</li> <li>• National datasets and positioning systems accurately relate to global datasets</li> <li>• Data in terms of historical datums are able to be converted into the preferred datums</li> <li>• Coordinates are maintained up to date to reflect movements of the Earth</li> </ul>

## Brief history of standard

In 1998, LINZ introduced NZGD2000 as the official national geodetic datum for New Zealand. This superseded the New Zealand Geodetic Datum 1949 (NZGD1949).

This standard supersedes the following documents:

LINZ 1997, *Transformation parameters from WGS84 to NZGD49, Policy SG97/3v1.1*, Office of the Surveyor-General, LINZ, Wellington

LINZ 1999, *New Zealand Geodetic Datum 2000, Policy 98/3, Version 2*, Office of the Surveyor-General, LINZ, Wellington

LINZ 1999, *New Zealand Geodetic Datum 2000 coordinates (computed at 5 January 1999), Policy 99/1*, Office of the Surveyor-General, LINZ, Wellington

LINZ 2000, *Transformation parameters from NZGD49 to NZGD2000, Policy 2000/1*, Office of the Surveyor-General, LINZ, Wellington

LINZ 2000, *Transformation parameters from Chatham Islands 1979 to NZGD 2000, Policy 2000/6*, Office of the Surveyor-General, LINZ, Wellington

## Committee responsible for standard

The expert committee responsible for reviewing this standard consisted of the following representatives:

Business Group (Company)	Name
Office of the Surveyor-General (LINZ)	Don Grant Matt Amos
National Topographic and Hydrographic Authority (LINZ)	Dave Mole
Regulatory Frameworks & Processes (LINZ)	Amanda Thompson Michelle Gooding Ruth Willis

Customer Services (LINZ)	Graeme Blick John Ritchie Chris Kinzett
Antarctica New Zealand	Dean Peterson
GNS Science	John Beavan
New Zealand Cartographic Society	John Beavan
University of Otago – School of Surveying	Paul Denys

## References

The following documents were used in the development of this standard:

Boucher, C., Altamimi, Z. & Sillard, P. 1998, *IERS Technical Note No. 24: Results and analysis of the ITRF96*, International Earth Rotation Service, Paris.

Crook C.C. and Pearse M.B. 2001, *OSG Technical Report 13: Development of transformation parameters from NZGD1949 to NZGD2000*, Office of the Surveyor-General, LINZ, Wellington.

Junkins, D.R. & Farley, S.A. 1995, *NTv2 National Transformation Version 2 Users Guide*, Geodetic Survey Division, Geomatics, Canada.

Lee, L.P. 1952, *The geodetic datum 1949*, New Zealand Survey Draughting Journal, vol 1, pp.320-324.

LINZ 2000, *OSG Technical Report 5: Realisation of the New Zealand Geodetic Datum 2000*, Office of the Surveyor-General, LINZ, Wellington.

McCarthy, D.D. (ed) 1996, *IERS Conventions (1996), IERS Technical Note 21*, International Earth Rotation Service, Paris.

Moritz, H. 2000, *Geodetic Reference System 1980*, Journal of Geodesy, 74(1), pp. 128-133.

NIMA 2000, *NIMA TR 8350.2 Department of Defence World Geodetic System 1984: Its definition and relationship with local geodetic systems*, Third Edition, 3 January 2000, US National Imagery and Mapping Agency, Washington DC.

Pearse, M.B. 2000, *OSG Technical Report 14: Transformation parameters from Chatham Islands 1979 to NZGD2000*, Office of the Surveyor-General, LINZ, Wellington.



# 1 INTRODUCTION AND SCOPE

## 1.1 Introduction

The methodology adopted when computing coordinates of the initial stations defining NZGD2000 is contained in LINZ 2000, *OSG Technical Report 5: Realisation of the New Zealand Geodetic Datum 2000*<sup>1</sup>.

## 1.2 Scope

This standard defines the:

- (a) New Zealand Geodetic Datum 2000 (NZGD2000),
- (b) parameters that define NZGD2000,
- (c) NZGD2000 deformation model, which allows the transformation of coordinates from the reference epoch 2000.0 to any other time, and
- (d) transformation parameters between NZGD2000 and other historic and commonly used geodetic datums in New Zealand.

## 1.3 Intended use of standard

Any person using NZGD2000 must comply with this standard.

# 2 NEW ZEALAND GEODETIC DATUM 2000

## 2.1 When this standard is compulsory

Spatial data provided to and supplied by LINZ must conform to this standard when it is referenced by coordinates in terms of NZGD2000.

## 2.2 NZGD2000 parameters

The parameters for NZGD2000 are:

- (a) the GRS80 equipotential ellipsoid, described in Moritz (2000) (see table below), as the reference ellipsoid,

GRS80 Ellipsoid Parameters (Moritz 2000)		
Ellipsoid	Semi-major axis (metres)	Inverse flattening
GRS80	6378137	298.257222101

<sup>1</sup> The positions and velocities in ITRF96 of IGS sites MADR, SANT, ALGO, YELL, GOLD, FAIR, TIDB, YAR1, HART, TROM, and KOSG, were constrained when computing the coordinates for NZGD2000.

- (b) the ITRS and the parameters defining ITRF96, as defined in Boucher et al (1998) following the conventions in McCarthy (1996),
- (c) the latitude, longitude, and optional ellipsoidal height at 1 January 2000 (2000.0),<sup>2</sup>
- (d) the IGS sites constrained to their ITRF96 coordinate values computed at the mean epoch of each survey using their ITRF96 velocities, and
- (e) the NZGD2000 deformation model (described in paragraph 3 below) to compute NZGD2000 coordinates.

## 3 NZGD2000 DEFORMATION MODEL

### 3.1 Deformation calculation

The model defines the deformation in terms of the displacement of a point at a specified time relative to the nominal latitude and longitude of that point at 2000.0. The deformation parameters are as follows:

- (a) The deformation model must be consistent with ITRF96 velocities.
- (b) The current version and definition of the NZGD2000 deformation model must be used to transform coordinates from the reference epoch 2000.0 to any other time.
- (c) The current version must be used to transform coordinates from any other time to the NZGD2000 reference epoch (2000.0).

#### Commentary:

##### 1. NZGD2000 deformation model

The NZGD2000 deformation model:

- ensures that epoch 2000.0 coordinates can be generated from observations made at other times; and
- allows users to generate coordinates for times other than the reference epoch.

To convert NZGD2000 coordinates to a reference frame other than ITRF96 (eg ITRF2005), or vice versa, a three, six, or seven parameter transformation is required in addition to the deformation model.

##### 2. How the model was calculated

The model was computed from repeated surveys in New Zealand and was calculated by interpolating and extrapolating observed deformation. Its accuracy is constrained by the observational data available to calculate it. As new surveys and analyses become available, the model will be upgraded and new versions released.

<sup>2</sup> It is not a requirement of NZGD2000 that coordinates of any point remain constant over time. Coordinates of points at times other than the reference epoch will generally change due to tectonic motion. Coordinates at the reference epoch may be updated as required to account for new observations or localised ground movements.

### 3. How the model is published

The current version of the deformation model is published on the LINZ website [www.linz.govt.nz](http://www.linz.govt.nz). It is provided as a number of gridded and triangulated data sets defining different aspects of the overall deformation model. The model may include constant velocities and localised offsets (for example, due to earthquakes). Each dataset is defined for a specific spatial extent and valid time range.

## 4 COORDINATE TRANSFORMATIONS

The following section defines transformation parameters between NZGD2000 at the reference epoch 2000.0 and commonly used geodetic datums in New Zealand. The transformations defined are:

- (a) NZGD1949 and NZGD2000,
- (b) WGS84 and NZGD2000, and
- (c) CIGD1979 and NZGD2000.

Transformations in the reverse direction are also covered in this section.

### 4.1 Transformations from NZGD1949 – NZGD2000

The transformations available to transform horizontal coordinates between NZGD1949 (Lee 1952) and NZGD2000 are:

#### 4.1.1 The datum reference ellipsoid parameters

The datum reference ellipsoid parameters that must be utilised are:

Reference ellipsoid parameters			
Datum	Ellipsoid	Semi major axis (metres)	Inverse flattening
NZGD1949	International 1924	6378388	297
NZGD2000	GRS80	6378137	298.257222101

#### 4.1.2 Types and accuracy of transformations

One of the following three transformations must be used:

- (a) three parameter transformation with a nominal accuracy of 5 m,
- (b) seven parameter transformation with a nominal accuracy of 4 m, or
- (c) grid file transformation with a nominal accuracy of 0.1 – 1 m.

### 4.1.3 Defining three parameter transformation

- (a) When utilising the three parameter transformation to transform from NZGD1949 to NZGD2000, the parameters that must be used are:

Transformation from NZGD1949 to NZGD2000		
Translation (metres)		
$T_x$	$T_y$	$T_z$
54.4	-20.1	183.1

Formulas to transform coordinates between datums are provided in Appendix A.

- (b) It is a sufficient compliance with (a) if alternative formulas are used that give the same results as the formulas in Appendix A within a tolerance of 1 mm.

### 4.1.4 Defining seven parameter transformation

- (a) When utilising the seven parameter transformation to transform from NZGD1949 to NZGD2000, the parameters that must be used are:

Transformation from NZGD1949 to NZGD2000						
Translation (metres)			Rotation (seconds of arc)			Scale change (parts per million)
$T_x$	$T_y$	$T_z$	$R_x$	$R_y$	$R_z$	$\Delta s$
59.47	-5.04	187.44	-0.470	0.100	-1.024	-4.5993

Formulas to transform coordinates between datums are provided in Appendix A.

- (b) It is a sufficient compliance with (a) if alternative formulas are used that give the same results as the formulas in Appendix A within a tolerance of 1 mm.

### 4.1.5 Defining grid file transformation

The distortion grid file on the LINZ website [www.linz.govt.nz](http://www.linz.govt.nz) must be used when utilising the grid file transformation between NZGD1949 and NZGD2000.

#### Commentary:

#### Transformations from NZGD1949 – NZGD2000

1. It is not possible to transform heights between NZGD1949 and NZGD2000 as heights are not defined in the NZGD1949 datum.
2. Coordinates generated by NZGD1949 to NZGD2000 transformations will be affected by:
  - any error in the original NZGD1949 coordinates plus

- any additional error introduced by the transformation.
3. The choice of which transformation option to utilise is dependent on the accuracy level required (Crook and Pearse, 2001).
- | Transformation type | Nominal accuracy (metres) |
|---------------------|---------------------------|
| three parameter     | 5                         |
| seven parameter     | 4                         |
| grid file           | 0.1 – 1                   |
4. The grid file of distortions (conversion/transformation) values has the following characteristics:
- Because NZGD1949 does not have a height component, the grid file converts only the horizontal coordinates and does not transform heights.
  - The grid file has been computed at a 20 km interval.
5. At the time of producing this standard, the grid file was in the Canadian grid format, known as National Transformation Version 2 (NTv2) described in Junkins and Farley (1995).

## 4.2 Transformations from NZGD2000 – NZGD1949

### 4.2.1 Defining three parameter transformation

- (a) When utilising the three parameter transformation to transform from NZGD2000 to NZGD1949, the parameters that must be used are:

Transformation NZGD2000 to NZGD1949		
Translation (metres)		
$T_x$	$T_y$	$T_z$
-54.4	20.1	-183.1

Formulas to transform coordinates between datums are provided in Appendix A.

- (b) It is a sufficient compliance with (a) if alternative formulas are used that give the same results as the formulas in Appendix A within a tolerance of 1 mm.

### 4.2.2 Defining seven parameter transformation

- (a) When utilising the seven parameter transformation to transform from NZGD2000 to NZGD1949, the parameters that must be used are:

Transformation from NZGD2000 to NZGD1949						
Translation (metres)			Rotation (seconds of arc)			Scale change (parts per million)
$T_x$	$T_y$	$T_z$	$R_x$	$R_y$	$R_z$	$\Delta s$
-59.47	5.04	-187.44	0.470	-0.100	1.024	4.5993

Formulas to transform coordinates between datums are provided in Appendix A.

- (b) It is a sufficient compliance (a) if alternative formulas are used that give the same results as the formulas in Appendix A within a tolerance of 1 mm.

### 4.2.3 Defining grid file transformation

The grid file transformation only exists for the conversion of coordinates from NZGD1949 to NZGD2000. To convert coordinates from NZGD2000 to NZGD1949, the current grid file on the LINZ website [www.linz.govt.nz](http://www.linz.govt.nz) must be used in conjunction with the procedure set out below:

- Use the NZGD2000 coordinates in the grid file to determine an estimate of the NZGD1949 to NZGD2000 correction.
- Subtract this correction from the NZGD2000 coordinates to determine initial estimates of NZGD1949 coordinates.
- Use the estimated NZGD1949 coordinates to determine a more accurate NZGD1949 to NZGD2000 correction.
- Subtract this correction from the original NZGD2000 coordinates to determine the final NZGD1949 coordinates.

## 4.3 Transformations from WGS84 – NZGD2000

### 4.3.1 The datum reference ellipsoid parameters

The ellipsoid parameters for WGS84 and NZGD2000 are:

Transformation from WGS84 to NZGD2000			
Datum	Ellipsoid	Semi major axis (metres)	Inverse flattening
WGS84 (NIMA 2000)	WGS84	6378137	298.257223563
NZGD2000	GRS80	6378137	298.257222101

### 4.3.2 Null transformation

A null transformation must always be used to convert coordinates from WGS84 to NZGD2000 and from NZGD2000 to WGS84.

**Commentary:**

**Limitations of WGS84 coordinates**

WGS84 is currently the reference system being used by the Global Positioning System (GPS). It is geocentric and globally consistent within  $\pm 1$  m. Current geodetic realisations of ITRS are geocentric and internally consistent at the few centimetre level while still being metre level consistent with WGS84. This means WGS84 coordinates may be inconsistent with NZGD2000 coordinates at the sub-metre level.

There is a small difference in the flattening term in the reference ellipsoids used by WGS84 (NIMA 2000) and by ITRS. The difference in coordinates at the Earth's surface is minimal (less than 1 mm) and so for most purposes is irrelevant.

## 4.4 Transformation from CIGD1979 – NZGD2000

### 4.4.1 The datum reference ellipsoid parameters

The ellipsoid parameters that must be used to transform from CIGD1979 to NZGD2000 are:

Transformations from CIGD1979 to NZGD2000			
Datum	Ellipsoid	Semi major axis (metres)	Inverse flattening
CIGD1979	International 1924	6378388	297
NZGD2000	GRS80	6378137	298.257222101

### 4.4.2 Type of transformation

When transforming horizontal coordinates between CIGD1979 and NZGD2000, a seven parameter transformation must be used.

### 4.4.3 Defining seven parameter transformation

- (a) When utilising the seven parameter transformation to transform from CIGD1979 to NZGD2000, the parameters (from Pearse, 2000) that must be used are:

Transformations from CIGD1979 to NZGD2000						
Translation (metres)			Rotation (seconds of arc)			Scale change (parts per million)
$T_x$	$T_y$	$T_z$	$R_x$	$R_y$	$R_z$	$\Delta s$
174.05	-25.49	112.57	0	0	-0.554	0.2263

Formulas to transform coordinates between datums are provided in Appendix A.

- (b) It is a sufficient compliance with (a) if alternative formulas are used that give the same results as the formulas in Appendix A within a tolerance of 1 mm.

**Commentary:**

**Transformations from CIGD1979 – NZGD2000**

The accuracy of transformed coordinates computed using these parameters will generally be no better than 2 m due to distortions in the CIGD1979 datum.

**4.5 Transformation from NZGD2000 – CIGD1979**

**4.5.1 Type of transformation**

When transforming from NZGD2000 to CIGD1979, a seven parameter transformation must be used.

**4.5.2 Defining seven parameter transformation**

- (a) When utilising the seven parameter transformation to transform from NZGD2000 to CIGD1979 the parameters (from Pearse, 2000) that must be used are:

<b>Transformations from NZGD2000 to CIGD1979</b>						
<b>Translation (metres)</b>			<b>Rotation (seconds of arc)</b>			<b>Scale change (parts per million)</b>
$T_x$	$T_y$	$T_z$	$R_x$	$R_y$	$R_z$	$\Delta s$
-174.05	25.49	-112.57	0	0	0.554	-0.2263

Formulas to transform coordinates between datums are provided in Appendix A.

- (b) It is a sufficient compliance with (a) if alternative formulas are used that give the same results as the formulas in Appendix A within a tolerance of 1 mm.



## Appendix A: Implementation of transformations

### A.1 Introduction

This Appendix provides formulas to convert geographic coordinates (latitude, longitude, and height) between datums using three or seven parameter transformations.

### A.2 Geographic coordinates to Cartesian coordinates

These formulas can be used to convert geographic coordinates (latitude, longitude, and height) into Cartesian coordinates (X, Y, Z):

$$X = (v + h) \cos \phi \cos \lambda$$

$$Y = (v + h) \cos \phi \sin \lambda$$

$$Z = [v(1 - e^2) + h] \sin \phi$$

where:

$$v = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

$$e^2 = 2f - f^2$$

### A.3 Three parameter transformation

To convert from datum 1 to datum 2 using the three parameter transformation, the following equation must be used.

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

### A.4 Seven parameter transformation

To convert from datum 1 to datum 2 using the Helmert seven parameter similarity transformation, the following equation must be used.

The rotation parameters must be converted from arc-seconds to radians before being used in this equation.

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + [1 + \Delta_S \times 10^{-6}] \begin{bmatrix} 1 & +R_Z & -R_Y \\ -R_Z & 1 & +R_X \\ +R_Y & -R_X & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

## A.5 Cartesian coordinates to geographic coordinates

These formulas can be used to convert Cartesian coordinates (X, Y, Z) into geographic coordinates (latitude, longitude, height).

$$\tan \lambda = \frac{Y}{X}$$

$$\tan \phi = \frac{Z(1-f) + e^2 a \sin^3 \mu}{(1-f)(p - e^2 a \cos^3 \mu)}$$

$$h = p \cos \phi + Z \sin \phi - a \sqrt{1 - e^2 \sin^2 \phi}$$

where:

$$p = \sqrt{X^2 + Y^2}$$

$$r = \sqrt{p^2 + Z^2}$$

$$\tan \mu = \frac{Z}{p} \left[ (1-f) + \frac{e^2 a}{r} \right]$$