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New Zealand Vertical Datum 2009

ABSTRACT

Until the recent implementation of the New Zealand Vertical Datum 2009 (NZVD2009), heights in New Zealand (NZ) were referenced to one of 13 disparate local vertical datums. The local datums were based on tide-gauge based estimates of mean sea level (MSL) that were transferred by precise levelling along the major roads. The local datums have limited spatial coverage due to the steep NZ topography. The heights of the benchmarks have generally not been verified since they were established 30 to 50 years ago. The regional and fragmented nature of the local datums means that they do not integrate well with ellipsoidal heights from Global Navigation Satellite Systems (GNSS) or the New Zealand Geodetic Datum 2000 (NZGD2000). NZVD2009 has therefore been developed as a national vertical datum for NZ and its continental shelf. It uses the normal-orthometric height system because gravity observations are not available at many benchmarks and a gravimetric quasigeoid as its reference surface rather than a tide-gauge determined MSL. This provides a height system that can be accessed at all locations within NZ and is compatible with both GNSS and NZGD2000.

INTRODUCTION

To enable heights to be consistently referenced among different datasets, they either need to be held in terms of a common datum, or the relationship between the different datums needs to be known. Until recently in New Zealand (NZ), heights were typically referred to one of 13 local vertical datums (LVDs) that were based on local estimates of mean sea level (MSL) determined at different times between 1926 and 1977. The LVDs were known to be offset from each other, but the magnitude of the offset was often uncertain. With the increased use of Global Navigation Satellite System (GNSS) positioning and the need for consistency with the New Zealand Geodetic Datum 2000 (NZGD2000), the use of 13 separate LVDs is undesirable, and

the establishment of a new vertical datum for NZ is warranted.

A vertical datum can be defined by the selection of a height system and a reference surface. This paper presents the major types of height system and reference surface that could be used for a new NZ vertical datum. From this theoretical context, the existing LVDs and their limitations are investigated and used as a basis for selecting the new datum, New Zealand Vertical Datum 2009 (NZVD2009).

HEIGHT SYSTEMS AND VERTICAL DATUMS

Contrary to common perception, the concept of 'height' is not straightforward. For example, there are several different

height systems that can be defined. Most (but not all) relate to the Earth's gravity field, or an approximation of it (e.g. Featherstone and Kuhn, 2006). Gravity-based systems give heights that can predict or approximate the flow of fluids (i.e. so that fluids flow from a higher point to a lower one). Other height systems can be defined that are independent of the Earth's gravity field, and can give the appearance of fluids flowing 'uphill'. This section compares the major height systems that have been proposed over the years.

Geopotential Numbers

Strictly, all natural or physical height systems must be based on geopotential numbers, C . A geopotential number is the difference in potential from a reference equipotential surface, W_0 , (usually the geoid) to the potential at the point of interest, W_p (Heiskanen and Moritz, 1967), such that:

$$C = W_0 - W_p \quad \text{Equation 1}$$

Geopotential numbers are measured in geopotential units (GPU), where $1 \text{ GPU} = 10 \text{ m}^2\text{s}^{-2}$. Because they do not have units of length, geopotential numbers are less intuitive to non-technical users. They accurately predict the flow of water (water will flow from a higher geopotential number to a lower one), and they exhibit the property of holonomy (Sanso and Vaníček, 2006). Holonomic height systems provide a theoretical zero misclosure regardless of

the levelling route taken. Conversely, non-holonomic height systems will give a levelling misclose, even if the levelling were errorless, as a result of the approximations made when modelling the gravity field. Furthermore, geopotential numbers cannot be directly observed as there is no instrument that can measure gravity potential. Instead they are practically determined using geopotential differences that are derived from precise levelling and gravity observations (e.g. Torge, 2001).

Dynamic Heights

To overcome the intuitive problem with geopotential numbers not being expressed in units of length, the dynamic height, H^{dyn} , was proposed by Helmert (1884). This is obtained by dividing the geopotential number by a constant gravity value, g_0 , often chosen to be the value of normal gravity at 45°N/S . The dynamic height is given by:

$$H^{\text{dyn}} = \frac{C}{g_0} \quad \text{Equation 2}$$

Dynamic heights are very simple to compute (if the geopotential number is known), and because they retain the attributes of the geopotential number, they predict the flow of fluids correctly and give a holonomic zero levelling loop closure. The unit of length changes depending on the gravity constant that is used, so it is generally not the same as an international standard (SI)

metre. The dynamic height does not have a geometrical meaning because it is purely a physical quantity (Jekeli, 2000). These heights are typically obtained by applying a dynamic correction to precisely levelled height differences. These corrections can be very large if g_0 is not representative of the region concerned.

Orthometric heights

The most common type of height that is claimed to be used is orthometric height, H^{ortho} . The orthometric height is defined as the length of the curved plumbline from a point P , to its intersection with the geoid at P_0 , as shown in Figure 1 and is given by:

$$H^{\text{ortho}} = \frac{C}{\bar{g}} \quad \text{Equation 3}$$

where \bar{g} is the integral mean value of gravity along the plumbline (Figure 1). It should, however, be noted that the term "orthometric" is often applied to a range of height definitions, but although these are related to the Earth's gravity field, they do not have the strict definition given here. As such, many height systems that purport to be orthometric do not actually provide true orthometric heights.

To correctly determine \bar{g} , the exact path of the plumbline through the Earth and the gravitational acceleration at all points along that plumbline need to be known. This requires knowledge of gravity variations (cf Strange, 1982) or the mass-density distribution (cf Allister and Featherstone, 2001) through the topography. Because this information is not available, it is not possible to observe or compute a true orthometric height.

Approximate orthometric heights

To overcome the problem of not being able to determine \bar{g} exactly, a number of approaches have been developed to approximate it. Each approximation results in a different kind of orthometric height which is normally named after its proponent.

The approximation of Helmert (1890) is based on the Poincaré-Prey relationship

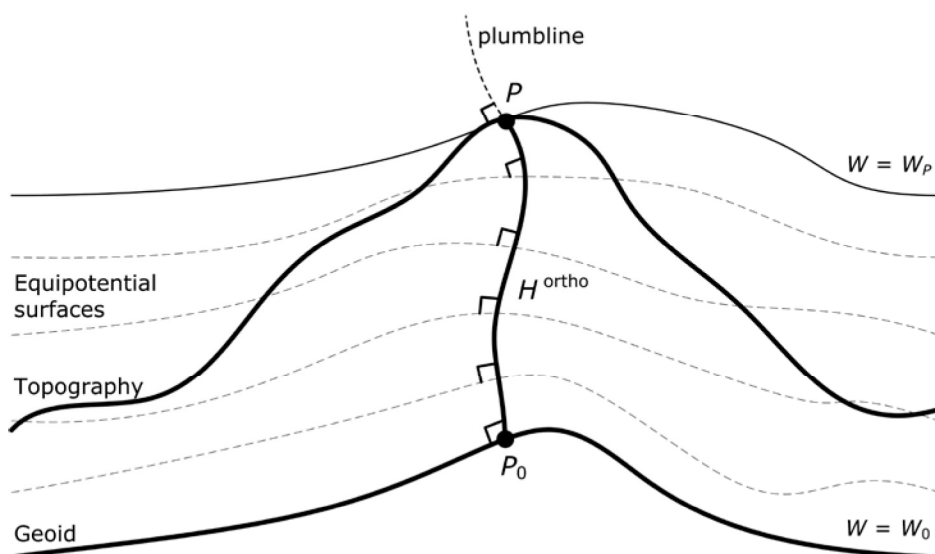


Figure 1. The orthometric height (adapted from Featherstone and Kuhn, 2006).

for integral mean gravity (Heiskanen and Moritz, 1967) and the Bouguer shell gravity expression that accounts for the topographic mass above the geoid but neglects the terrain effects. The Bouguer shell accounts for the effect of the terrain by approximating it as a shell at a constant height. The terrain effects are a result of the difference between the actual topography and the shell, and are modelled by a terrain correction.

Helmert-orthometric heights are computed either from a geopotential number or by the application of an orthometric correction to precise levelling observations. Both methods require surface gravity observations at the points of interest. These heights can be quite different from their *true* orthometric counterparts due to the large corrections to precise levelling observations that are necessary (Featherstone and Kuhn, 2006). Nevertheless Helmert-orthometric heights are probably the most common type of 'orthometric' height in actual use; for example in Belgium, Denmark, Finland, Italy and Switzerland (EUREF, 2006).

Other approximations such as Neithammer (1932) and Mader (1954) include corrections for the terrain effect and as such give a closer approximation of the true orthometric height than Helmert heights. However their computational complexity has seen them used less frequently in practice. More recent approaches (e.g. Tenzer et al, 2005) give a close agreement to true orthometric heights, but as a result of their recent development have not yet been implemented in practice.

Normal Heights

The normal gravity field is the gravity field defined by an Earth-fitting ellipsoid that contains the total mass of the Earth (including its atmosphere), and rotates at a constant angular velocity more or less equivalent to that of the Earth (Moritz, 1980). The normal gravity field can be used to define a height that avoids assumptions about the shape and density of the topographic masses needed to compute \bar{g} .

The normal height, H^N , was proposed in 1954 by Molodensky (cited in Molodensky

et al 1962). It replaces \bar{g} in Equation 3 (which was measured along the plumbline) with normal gravity, $\bar{\gamma}$, measured along the curved ellipsoidal normal (of the reference ellipsoid) hence (Jekeli, 2000):

$$H^N = \frac{C}{\bar{\gamma}} \quad \text{Equation 4}$$

The change from a physical to a geometrical gravity field also means that H^N is measured between the ellipsoid, Q_0^N , and a surface called the telluroid, Q (Figure 2), not between the geoid, P_0 , and the topographic surface, P , used for orthometric heights (Figure 1). The telluroid is a surface whose normal potential at every point Q is equal to the actual potential to every corresponding point P . The distance from the telluroid to the topographic surface is the height anomaly, ζ .

Because normal heights have no physical meaning (being defined by a gravity model), they are not as applicable to the real Earth as the orthometric (and Helmert-orthometric) height (Featherstone and Kuhn, 2006). Additionally, while they cannot predict fluid flows universally, they nevertheless give a reasonable approximation in many situations. Like the other heights described above, normal heights can be computed by applying a correction to spirit levelled height

differences if there are suitably dense gravity measurements along the levelling route.

It is common to illustrate the H^N relation in Figure 2 in reverse so that the height anomaly, ζ , also becomes the distance between the quasigeoid and the topographic surface (shown in Figure 2 as H^{N-O}). Note that the height anomaly, ζ , and quasigeoid height, ζ , are the same, but different terminology is used to reflect the different conceptualisations (Featherstone and Kuhn, 2006). This means that H^N and normal-orthometric heights, H^{N-O} , (see section below) are geometrically the same. As such both can be compatible with GNSS ellipsoidal heights when they are derived from the quasigeoid.

Normal-orthometric heights

Many countries, NZ included, do not have gravity observations along all the precise levelling routes, so the computation of the geopotential numbers necessary for (approximate) orthometric or normal heights is not strictly possible. To overcome this limitation, the normal-orthometric height, H^{N-O} , was developed (e.g. Rapp, 1961; Heck, 2003). In this system the geopotential number, C , is replaced with the spheropotential number, C' , which is wholly derived from the normal gravity

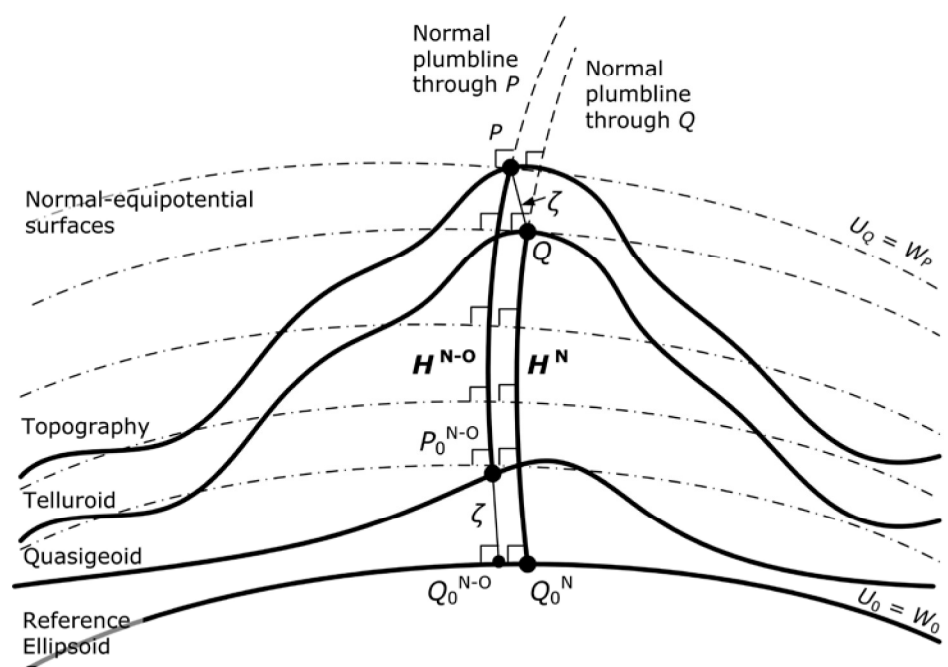


Figure 2. The normal and normal-orthometric heights (from Featherstone and Kuhn, 2006).

field. The normal-orthometric height is defined as the distance from the quasigeoid to the surface of the Earth along the curved ellipsoidal normal (Figure 2) and is given by:

$$H^{N-O} = \frac{C'}{\gamma} \quad \text{Equation 5}$$

The consequence of not using surface gravity observations is that while normal-orthometric heights are easy to compute, they are even less likely to predict fluid flows correctly than normal heights. In practice, normal-orthometric heights are obtained by applying a normal-orthometric correction (NOC) to precisely levelled height differences (e.g. Heck 2003).

The quasigeoid is a surface that results from the approximations and assumptions about the structure and composition of the Earth that are made under Molodensky's theory. The quasigeoid is identical to the geoid over the oceans and is typically within a few decimetres of it over most land areas, but the difference can reach nearly 3 m in extreme cases (Flury and Rummel, 2009). The maximum difference in NZ is approximately 0.5 m at Aoraki/Mt Cook (Amos and Featherstone, 2003). At heights less than 250 m, where most NZ settlements are located, the difference is less than 3 cm.

Ellipsoidal heights

The ellipsoidal height, h , is the distance from the reference ellipsoid to the Earth's surface along the ellipsoidal surface normal as shown in Figure 3. Unlike the heights discussed in the sections above, it is defined independently of the Earth's gravity field, i.e. it is a purely geometric quantity. Consequently, ellipsoidal heights

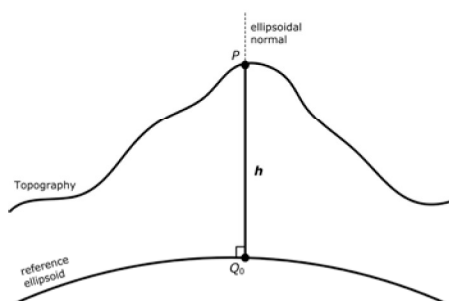


Figure 3. The ellipsoidal height.

are generally poor at predicting fluid flows. They are however relatively easy to define mathematically and as such are the type of height obtained from GNSS receivers.

Relationships between Height Systems

The different types of height described above can be related using the reference surfaces that are consistent between them. These relationships are summarised in Figure 4. Algebraically, ellipsoidal heights are related to orthometric heights by the geoid height, N :

$$H^{\text{ortho}} = h - N \quad \text{Equation 6}$$

and, normal-orthometric heights by the quasigeoid height, ζ :

$$H^{N-O} = h - \zeta \quad \text{Equation 7}$$

Vertical datum definition

To realise a vertical datum, it is necessary to select a type of height system and a compatible reference surface. Once these choices have been made, and the observed height differences corrected for systematic errors affecting their observation (e.g. Vaníček et al., 1980), a vertical datum can be realised point-wise by performing a least-squares adjustment of the corrected height differences to minimise the impact

of random errors, and to account for the impact of random and systematic errors in the levelling loops (e.g. Sansò and Vaníček, 2006).

The type of height system chosen normally depends on the data that was available to the agency responsible at the time of datum definition (or the system can be chosen and the necessary data then acquired). For example, if gravity observations are unavailable, then only the normal-orthometric or ellipsoidal height systems can be used. The choice of reference surface is guided by the choice of height system, i.e. orthometric heights use the geoid; normal-orthometric heights the quasigeoid and ellipsoidal heights the ellipsoid. In gravimetric systems the reference surface is normally defined so that it approximates MSL and therefore provides heights that are broadly consistent with it.

If a vertical datum is defined by fixing MSL at a single tide-gauge point it can result in the 'zero height' departing from MSL at other locations in the datum – because MSL is not a truly level surface. An alternative practice that has been adopted in a number of countries (e.g., Australian Height Datum, AHD, [Roelse et al., 1975]; Canada CGVD28 [Kingdon et al., 2005]) is to constrain multiple tide-gauge MSL values

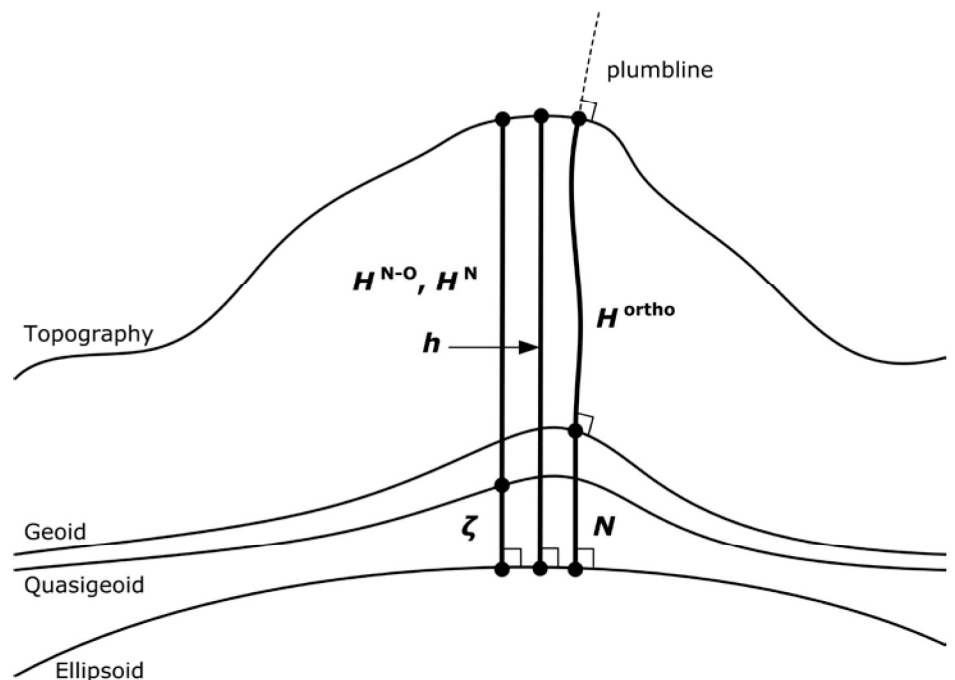


Figure 4. Summary of the relationship between orthometric, normal, normal-orthometric and ellipsoidal heights.

to zero in the precise levelling adjustments. This approach gives a vertical datum with a “zero level” that is close to the observed MSL at all locations, but it does not represent an equipotential or “level” surface. For example, the Australian AHD adjustment fixes 30 tide-gauges to “absorb” the effect of a 70 cm sea level “slope” around the Australian coast (Featherstone and Kuhn, 2006). If a vertical datum is not defined in relation to an equipotential surface, it is more difficult to determine its relationship to other vertical datums.

NEW ZEALAND'S HEIGHT SYSTEMS

Local vertical datums

Until recently a nationally consistent gravimetric vertical datum was not available for heighting in NZ. Instead, heights were typically referenced to one of 13 major local vertical datums (LVDs; Table 1). Each of the NZ LVDs is based on a determination of MSL at different tide-gauges over a range of time intervals (normally at least three years) and epochs (primarily 1920 – 1970).

LVD heights are in terms of the normal-orthometric height system. These have been incorrectly referred to as orthometric heights in the LINZ geodetic database, and in many publications (e.g. Gilliland, 1987; DoSLI, 1989; Reilly, 1990).

Many smaller or special-purpose datums have also been defined over the years. A

significant number of these (e.g. Tekapo, Karapiro and Maraetai) were defined with respect to other existing datums for specific hydro-electric power projects. Others (e.g. Deep Cove, Tikinui, and Chatham Island) were defined from short periods (e.g. several months) of tidal data and are only used for local purposes.

Tide gauges

Historically, the tide gauges used in NZ have been established in harbours and rivers by local port authorities for use in the prediction and verification of tide tables. Data from these gauges was analysed by Land Information NZ (LINZ) and its predecessor agencies (Department of Survey and Land Information – DoSLI; Department of Lands and Survey – L&S) to determine MSL at each site. This MSL value was then used as the zero height for the LVD to which a local levelling network was referenced.

The NZ tide gauges are generally in locations that are less-than-optimal for vertical datum definition purposes (Figures 5 and 6). They are frequently situated in harbours or rivers (within a few kilometres of the coast), whereas the ideal locations are either offshore or on the open coast to minimise the non-linear tidal effects that occur near the coast (e.g., Pugh, 2004). This means that the observed MSL will not necessarily be representative of the region that the datum is

expected to cover (e.g., Hipkin, 2000; Cross et al., 1987; Merry and Vaníček, 1983).

The Dunedin-Bluff 1960 datum is a notable anomaly in Table 1. Unlike the other LVDs, it was defined by fixing the height of a benchmark in Balclutha in terms of the Dunedin 1958 datum, and a



Figure 5. NZ North Island precise levelling networks, tide gauges and LVD junction points



Figure 6. NZ South Island precise levelling networks, tide gauges and LVD junction points.

Table 1. Major levelling datum origins and periods of MSL observation used to define them.

Local vertical datum	Observation period	Duration
One Tree Point 1964	1960 – 1963	3 years
Auckland 1946	1909 - 1923	14 years
Moturiki 1953	1949 - 1952	3 years
Gisborne 1926	1926	1 year
Napier 1962	<i>Unknown</i>	<i>Unknown</i>
Taranaki 1970	1918 - 1921	3 years
Wellington 1953	1909 - 1946	37 years
Nelson 1955	1939 - 1942	3 years
Lyttelton 1937	1918 - 1933	15 years
Dunedin 1958	1918 - 1937	19 years
Dunedin-Bluff 1960	<i>None</i>	-
Bluff 1955	1918 - 1934	16 years
Stewart Island 1977	1976 - 1977	3-5 tides

benchmark in Invercargill in terms of the Bluff 1955 datum. The Stewart Island 1977 datum is not defined by a 'long-term' tide gauge derived estimate of MSL. Rather, it has a 'zero' based on a value of MSL determined from three temporary tide gauges established around Stewart Island/Rakiura using observations over three to five successive (but not simultaneous) tides. The Stewart Island approach, furthermore, was based on trigonometric heights that could be in error by 0.2-0.3 metres. Consequently, the resulting MSL could be in error by 0.5 metres from the long-term trend.

Precise levelling networks

First-order precise levelling in NZ (accuracy standard of $\pm 2\text{ mm}\sqrt{k}$, where k is the levelled distance in km) has historically been the method for precise height transfer in NZ. Reciprocal trigonometric ($\pm 0.1\text{--}0.2$ m) and barometric levelling (± 15 m) has also been used to increase the density of the precise levelling networks. However, due to their lower accuracy, trigonometric and barometric levelling are not generally considered part of the NZ precise height network.

There currently exists more than 16,000 km of two-way first-order precise levelling that has been observed since the 1960s to give the coverage shown in Figures 5 and 6 (e.g., Gilliland, 1987). These networks were observed in a piece-meal fashion and the large loop around the South Island (Figure 6) was only completed in the late 1980s. Each local vertical datum (LVD) has been defined using a least-squares adjustment to give heights for its constituent marks.

It can be seen from Figures 5 and 6 that the levelling coverage is not uniform over NZ. Some areas, such as the central North Island (Figure 5) in the vicinity of the Moturiki tide gauge, have a very strong network configuration, but other areas, notably the south-west of the South Island (Figure 6), are particularly sparse.

The irregular coverage has a great deal to do with the topography over which the levelling runs traverse and the lack of roads in the sparser areas along which precise

levelling lines are placed for stability and access reasons. The South Island levelling lines that transect the Southern Alps/Kā Tiritiri o te Moana (Figure 6) are limited to the three mountain passes over them. It is not practicable (or in some cases possible) to obtain a denser precise levelling network in these remote areas due to the steep and rugged topography.

New Zealand geodetic datum 2000

The current official geodetic datum for NZ is NZGD2000. It is a three-dimensional geocentric datum that uses the GRS80 ellipsoid (Moritz, 1980) and is aligned to the ITRF96 (Boucher et al., 1998) global reference frame. To incorporate the effects of tectonic deformation, NZGD2000 uses a horizontal deformation and velocity model to 'correct' observations for the effects of deformation from the time of acquisition to the datum's reference epoch (1 January 2000). No vertical deformation model is used in NZGD2000. NZGD2000 uses ellipsoidal heights in terms of the GRS80 ellipsoid (LINZ, 2007). The use of ellipsoidal heights is becoming increasingly popular amongst users of the survey control system, even though they do not refer to the Earth's gravity field.

PROBLEMS WITH THE CURRENT DATUMS

Sea level variability

Sea level observed at tide gauges can vary on annual, inter-annual and inter-decadal cycles, hence the particular epoch of data used will affect the determined level of MSL (Bell et al., 2000). Ideally sea level observations would be analysed from a full

18.6 year metonic cycle, however for the NZ LVDs this has seldom been achieved (Table 1).

Analysis of sea level observations by LINZ (Rowe, 2006, pers. comm.) has shown that variations in the observed MSL can differ from the long-term average by 10 cm over a three-year period. Figure 7 shows the monthly sea level trends for the Wellington tide gauge (Rowe, 2006, pers. comm.). Given that a number of vertical datums have been defined by only three years of sea level observations (cf. Table 1), it is very likely that they refer to a MSL that is not representative of the long-term average. For example if MSL was defined from data indicated by either of the horizontal lines in Figure 7 rather than the full set, the resulting MSL could be offset from the long-term average by over 50 mm. Based on the very limited data available (e.g., Figure 7), an offset of 5-10 cm could readily be attributed to the choice of epoch for the shorter duration definitions (e.g., One Tree Point 1946, Moturiki 1953, Gisborne 1926, Napier 1962, Taranaki 1970, Nelson 1955, Stewart Island 1977; cf. Table 1).

Local vertical datum offsets

Due to the factors described above, the 13 LVDs are offset from each other. Where two or more vertical datums abut or overlap, it is possible to estimate the offset that exists between the datums at that point. This offset will be affected by systematic observation and reduction errors along the route of the precise levelling and any deformation that has occurred since the levelling was carried out. The consequence of this is that when vertical datums join at multiple places, the observed offsets will also differ.

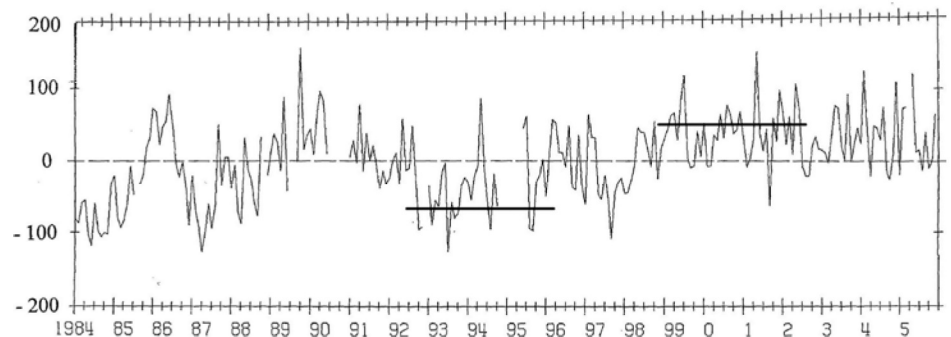


Figure 7. Monthly sea level observations for Wellington tide-gauge from LINZ records, 1984 – 2006 (mm).

Observed (post-adjustment) NZ LVD offsets have been obtained from the LINZ geodetic database by comparing the heights of marks that are located at the junction points of the adjacent datums. The offsets are shown in Table 2; the junction points are shown on Figures 5 and 6. The Taranaki-Moturiki offset observed at AHBB (-0.455 m) is abnormally large compared to the other offsets. This is probably due to mark movement between the observations of the respective levelling lines, however it was not possible to confirm or disprove this hypothesis from analysis of the precise levelling records.

Vertical deformation

The Earth's surface in the NZ region experiences relative movements that deform its shape (e.g., earthquakes). The horizontal movements are reasonably well known (e.g. Beavan and Haines, 1997; Beavan, 1998; Walcott, 1984), but the vertical

movements are not. Regional studies show that areas within the Taupo Volcanic Zone are subsiding by up to 10 mm/yr (Otway et al., 2002). Local subsidence of up to 8.5 m has been reported from the Wairakei area (38° 37' S, 176° 06' E) due to geothermal energy draw-off for electricity generation (Bevin et al., 1984).

Uplift rates of the Southern Alps/Ka Tiritiri o te Moana (cf. Figure 5) are in the order of 10 mm/year due to the interaction of the Pacific and Australian tectonic plates along the Alpine Fault (e.g., Beavan et al., 2004; Walcott, 1984; Wellman, 1979). These subsidence and uplift rates have a slow but continuous effect on the heights of stations.

Earthquakes and associated co-seismic, post-seismic and inter-seismic deformation often have the largest short-term effect on heights. Important NZ examples include: subsidence of up to 2 m from the Edgecumbe earthquake

of 1987 (Beanland et al., 1990); uplift of 2.7 m from the Inangahua earthquake of 1968 (Lensen and Otway, 1971); uplift of 2.4 m and subsidence of 0.9 m from the Napier earthquake of 1931 (Henderson, 1933); and uplift of 1.3–2.1 m in Wellington Harbour and up to 6.4 m near Turakirae Head from the 1855 Wairarapa earthquake (Begg and McSaveney, 2005).

Although the evidence for uplift is not conclusive everywhere, and some areas will have undergone subsidence, it is still useful to evaluate its potential effect. Given that most of the NZ LVDs were defined about 50 years ago, and assuming an average linear uplift rate of 0.2 mm/year, this could mean that the heights of some benchmarks and datum origins may have risen by up to 10 cm between then and 2010.

Inconsistent with GNSS/NZGD2000

Over the past 10 years, LINZ has physically surveyed many control marks to determine their coordinates and ellipsoidal heights in terms of the NZGD2000 geodetic datum. Because no official relationship between NZGD2000 and the LVDs has been defined, it has been difficult for users to consistently integrate GNSS observations with LVD-based heights.

Common approaches to integrate heights have been to use a global gravity model (GGM) such as EGM96 (Lemoine et al. 1998) or EGM2008 (Pavlis et al. 2008) as a transformation surface, or to observe a GNSS height on a LVD benchmark. The GGM approach is based on two assumptions: that the LVD origins are coincident with the GGM/geoid; and that the GGM accurately models the Earth's gravity field in the area of interest. GGMs model the geoid globally with a relatively coarse spatial resolution. EGM96 has a resolution of approximately 39 km in NZ (Amos 2007), and EGM2008 13 km, therefore the actual geoid (or MSL) variations in the geoid at smaller scales to this will not be detectable from the models. The accuracy (1 σ) of EGM96 over NZ has been estimated from GPS-levelling as 0.6 metres (Amos 2007) and 0.1 metres for EGM2008. When assessing the suitability

Table 2. Offsets determined from height differences at junction points of LVDs (metres).

Mark	Vertical datum 1	Vertical datum 2	Offset
ABHL	One Tree Point 1964	Auckland 1946	+0.206
AGD8	Auckland 1946	Moturiki 1953	-0.069
ABTE	Auckland 1946	Moturiki 1953	-0.075
ABV5	Auckland 1946	Moturiki 1953	-0.067
ABX2	Gisborne 1926	Moturiki 1953	-0.075
AD2J	Napier 1962	Gisborne 1926	+0.166
AEVR	Napier 1962	Moturiki 1953	+0.099
AE54	Napier 1962	Taranaki 1970	+0.046
AE54	Taranaki 1970	Wellington 1953	+0.191
AE54	Napier 1962	Wellington 1953	+0.237
AHBB	Taranaki 1970	Moturiki 1953	-0.455
B48K	Taranaki 1970	Moturiki 1953	-0.014
AEXF	Taranaki 1970	Moturiki 1953	-0.019
AEXF	Taranaki 1970	Wellington 1953	+0.102
AEXF	Moturiki 1953	Wellington 1953	+0.121
AEJ5	Nelson 1955	Lyttelton 1937	+0.014
AP5E	Nelson 1955	Lyttelton 1937	+0.039
ADHE	Nelson 1955	Lyttelton 1937	-0.086
ADCK	Nelson 1955	Lyttelton 1937	-0.076
B4A2	Lyttelton 1937	Dunedin 1958	-0.054
AE7N	Lyttelton 1937	Dunedin 1958	-0.087
ADP2	Dunedin-Bluff 1960	Dunedin 1958	-0.019
AB9T	Dunedin-Bluff 1960	Bluff	-0.001

of using a GGM to transform heights, the accuracy of the GGM in relation to the local gravity field (as well as the accuracy of the input heights) needs to be considered. This method will not give heights in terms of a LVD. The resultant heights will be in relation to the average level of the sea as defined by the GGM.

To convert a GNSS (ellipsoidal) height to a LVD normal-orthometric height, the common approach has been to physically survey a benchmark with a LVD height using GNSS, and therefore geometrically determine the difference between NZGD2000 and the LVD. This offset can then be used to transform heights in the vicinity of the benchmark. Because this approach utilises a single offset to model the geoid surface, the accuracy of the transformation degrades with increasing distance from the benchmark. If a number of marks are surveyed over a small area (generally up to 5 km x 5 km), then an inclined plane can be used to model the offset and thereby extend the coverage of the transformation. The primary limitation of this approach is that it is attempting to model an irregular surface with a point or plane. Larger regions can be better modelled if additional regression coefficients are used to define a non-planar surface.

No single reference system for large applications

The existing LVDs work suitably for tasks that are wholly contained within a single LVD where there is ready access to the benchmarks. Where LVDs abut or overlap, benchmarks may have heights in relation to more than one datum. This can introduce user confusion where the difference between two or more heights is not understood or user error where the difference between datums is not noted. The LVDs are not suitable for applications that span more than one datum and which require all heights to be consistently recorded.

This confusion is exacerbated when each of the datums purports to depict mean sea level. Where the offset between datums is small (or the user does not recognise that the heights are in terms of different datums) it is likely

that the datum relating to a particular height could be mistaken. Having height data in a number of datums also makes the integration of different datasets difficult, especially over large areas. To avoid these problems a national datum needs to be defined so that all heights can be referred to it.

Unsuitability of levelling-based datum

Internationally, the most common method of establishing a vertical datum has been to determine MSL at a tide gauge and then transfer the level to benchmarks in the hinterland by precise levelling. Precise levelling is a labour intensive and expensive method of transferring heights that only provides heighted benchmarks along the levelling routes. Since NZ does not have an extensive road network over many parts of the country (cf. Figures 5 and 6) it is not possible to efficiently implement a national vertical datum based on precise levelling alone.

One approach to modernise the LVDs would be to determine updated estimates of MSL at the original tide-gauges and re-adjust the existing precise levelling observations in terms of the 13 LVDs. Alternatively, the precise levelling observations could be readjusted to form single networks in the North, South and Stewart Islands. In both cases the levelling observations that would be readjusted are typically 30 to 50 years old. Without physically re-observing large parts of the height network (an untenable task due to its high cost) the 'new' heights would typically move at the decimetre level and thereby cause more confusion by doubling the number of datums in an area. The resultant heights would not be any more accurate or reliable than the existing LVD heights so it is expected that their uptake by users would be low.

NEW ZEALAND VERTICAL DATUM 2009

The New Zealand Vertical Datum 2009 (NZVD2009) was officially released in September 2009 and is formally defined in the standard: LINZS25004 (LINZ, 2009). It is the first time that a single

vertical datum has been implemented across NZ and its continental shelf. The notable feature of NZVD2009 is that it uses a gravimetric geoid as its reference surface rather than the conventional tide-gauge estimate of MSL.

NZVD2009 is a world first implementation of a geoid-based national vertical datum. The concept of using a geoid as the reference surface in NZ's vertical datum was initially proposed in 2001 (Grant and Blick, 2001). Since 2001, the approach has gained in popularity and it is being proposed as the basis for a number of modernised vertical datums (e.g. United States [Childers et al, 2009], Canada [Véronneau et al, 2006]).

The key parameters for NZVD2009 are:

Table 3. NZVD2009 parameters.

Attribute	Value
Height system	Normal-orthometric
Reference surface	New Zealand Quasigeoid 2009
Normal gravity field	GRS80
Reference ellipsoid	GRS80

The New Zealand Quasigeoid 2005 (NZGeoid05; Amos 2007) was published by LINZ in 2005 together with a set of offsets that could be used to transform NZGD2000 ellipsoidal heights to the 13 LVDs. NZGeoid05 was intended for use as a transformation surface not as a datum in its own right, and heights should not be referred to as being in terms of 'NZGeoid05'.

Height system

NZVD2009 retains the normal-orthometric height system based on the GRS80 normal gravity field (LINZ, 2009). This choice was made because of the lack of gravity observations on NZ's precise levelling marks. The use of the GRS80 normal gravity field makes NZVD2009 consistent with NZGD2000 which also uses the GRS80 reference system (LINZ, 2007). This differs from the LVD normal-orthometric height system which referred to the GRS67 normal gravity field.

Where NZVD2009 heights are determined by precise levelling from an existing benchmark,

the normal orthometric correction (NOC) as defined in Equation 8 should strictly be applied to the height differences.

$$\text{NOC} = -\frac{f^*}{R} H_{\text{avg}} \sin 2\phi \cos \alpha \delta s$$

Equation 8

Where: f^* GRS80 normal gravity flattening constant (0.005 302 440 112)

R GRS80 mean Earth radius (6,371,000 m)

H_{avg} average normal-orthometric height of benchmarks (m)

ϕ mid-latitude latitude of benchmarks

α azimuth between benchmarks

δs horizontal distance between benchmarks (m)

The magnitude of the NOC is 0.83 mm when evaluated over 1 km at 1000 m altitude (45° S, 1 km north-south levelling line with 20 change points 50 m apart). At a more typical average height of 200 m the NOC is 0.17 mm. This means that application of the NOC should take into account the length of the levelling line and the accuracy of the resulting NZVD2009 heights.

New Zealand quasigeoid 2009

NZGeoid2009 (Figure 8) is a regional gravimetric quasigeoid computed over the extent of NZ's continental shelf (160°E – 170°W, 60°S – 25°S). Although NZGeoid2009 is technically a quasigeoid, the more common term 'geoid' and symbology (N) is used to avoid confusion when describing it in relation to NZVD2009.

The NZGeoid2009 was computed by the Western Australian Centre for Geodesy (Claessens et al, 2010) following the same general procedure that was used for its predecessor NZGeoid05 (Amos, 2007; Amos and Featherstone, 2009). It is based on the EGM2008 global gravity model up to degree and order 2160 and has been enhanced with 40,737 terrestrial gravity observations across NZ, marine anomalies from the DNSC08 global model (Andersen et al, 2008), and a 1.8" grid (~56 m) digital

elevation model to correct for the effect of the topography on the gravity field.

The model was computed using a remove-compute-restore approach using Stokes integration with a deterministically modified integration kernel (Featherstone et al, 1998) with $L = 40$ and $\psi_0 = 2.5^\circ$ (L is the spherical harmonic degrees removed from the kernel and ψ_0 the integration cap radius). A detailed description of the computation process for NZGeoid2009 is provided in Claessens et al (2010).

Across the NZ mainland the "height" of NZGeoid2009 above the GRS80 ellipsoid varies from 0 m at the south of Rakiura/Stewart Island to approximately 40 m at the north of the North Island. This change is generally in a north-south direction, with some local variations around topographic and geological features. It is published (and was computed) on a 1' x 1' grid (~1.9 km in NZ) which means that localised variations in the geoid that are smaller than this will not be represented in the model.

Datum offsets

The accuracy of NZGeoid2009 in relation to the LVDs was estimated by comparisons with geometrically determined geoid values at control marks where both ellipsoidal and normal-orthometric heights had previously been observed. In NZ there are 1,422 suitable GPS-levelling points that are unequally spread among the 13 LVDs (Figure 9, Table 4). The spatial coverage of the GPS-levelling points is not uniform, and large gaps exist in some areas, notably the south-west of the South Island, north-west Nelson and East Cape. Furthermore, many of the points are located in topographically flat terrain rather than the mountains where the geoid surface is expected to be more variable.

The results of the GPS-levelling comparisons on a datum-by-datum basis are shown in Table 4 (LINZ 2009). All of the offsets are significantly non-zero, and in most cases they agree (within statistical limits) with the offsets observed at the LVD junction points

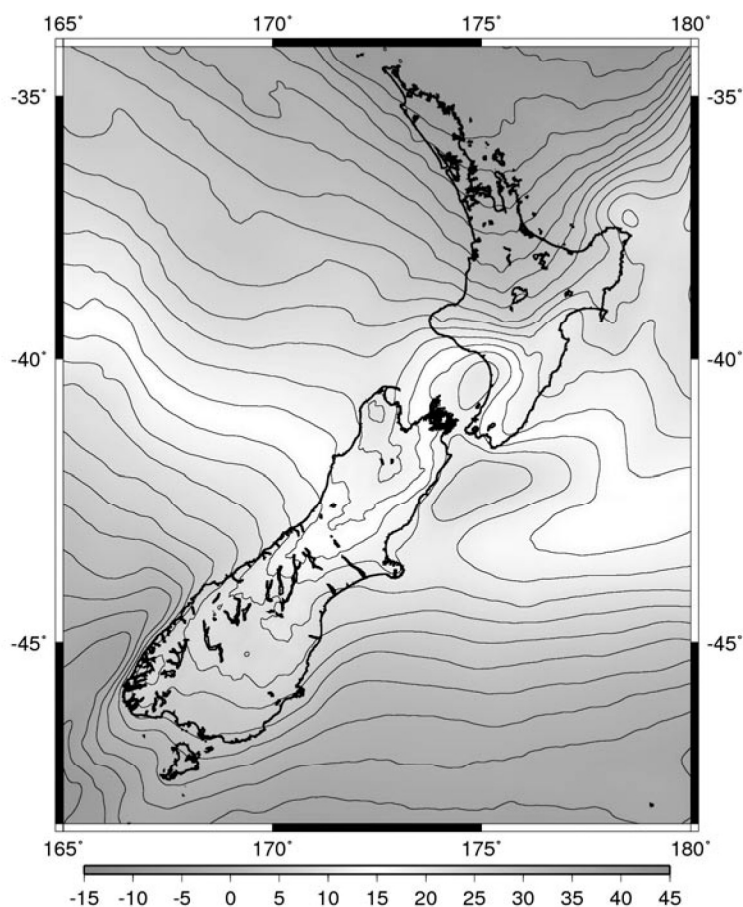


Figure 8. New Zealand Quasigeoid 2009 (NZGeoid2009) relative to the GRS80 ellipsoid (two metre contours).

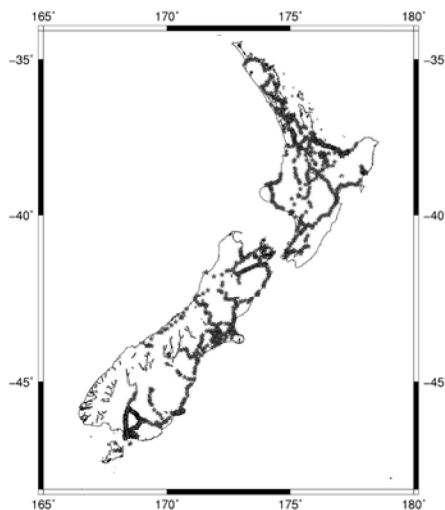


Figure 9. 1422 GPS-levelling points.

(Table 2). The standard deviation of the Stewart Island 1977 datum is larger than the others because it has been determined from five relatively low accuracy GPS-levelling points. The Lyttelton 1937 offset is probably higher due to the very long levelling lines that were used to establish the initial normal-orthometric heights (e.g. Lyttelton to Haast via Arthurs Pass). All of the offsets are positive as a matter of coincidence rather than planning. This shows that the NZGeoid2009 is consistently 'higher' than the LVDs.

An estimate of the overall accuracy of the NZGeoid2009 can be found from the standard deviations at all 1422 GPS-levelling points once their respective offset biases have

been removed. This gives an overall standard deviation for NZVD2009 of 0.062 m.

Transformations

Heights can be transformed between NZVD2009, NZGD2000 and the NZ LVDs using Equations 9 to 12 (LINZ, 2009). The relationship between the heights is shown schematically by Figure 10.

To determine the value of the NZGeoid2009 at a point, the geoid grid needs to be bilinearly interpolated at the NZGD2000 (latitude/longitude) position of the height being transformed. The transformations to/from the LVDs do not take into account the change in the normal gravity field from GRS67 to GRS80 because its effect is typically sub-millimetre (Amos, 2007). The sign convention in Equations 9 to 12 was chosen to ensure that the LVD offsets were positive and therefore increase the likelihood that they would be implemented correctly.

The nominal accuracy of the transformations is a combination of the offset/NZGeoid2009 accuracy (from Table 4) and the accuracy of the original height. Care needs to be taken when combining heights that have been derived from different sources, such as transformed ellipsoidal heights and precisely levelled NZVD2009 heights. In this scenario it is possible that the heights may not be in terms of each other and

additional checks should be made to verify the relationship.

NZGD2000 to NZVD2009:

$$H_{\text{NZVD2009}} = h_{\text{NZGD2000}} - N \quad \text{Equation 9}$$

LVD to NZVD2009:

$$H_{\text{NZVD2009}} = H_A - o_A \quad \text{Equation 10}$$

Between LVDs:

$$H_B = H_A - o_A + o_B \quad \text{Equation 11}$$

LVD to NZGD2000:

$$h_{\text{NZGD2000}} = H_A + N - o_A \quad \text{Equation 12}$$

Where: H_{NZVD2009} NZVD2009 normal-orthometric height

H_A, H_B LVD A and B normal-orthometric heights

h_{NZGD2000} NZGD2000 ellipsoidal height

N NZ Geoid2009 value at the NZGD2000 position of h

o_A, o_B Offsets of LVDs A and B from Table 3

Use of NZVD2009

NZVD2009 provides, for the first time in NZ, a national height reference system

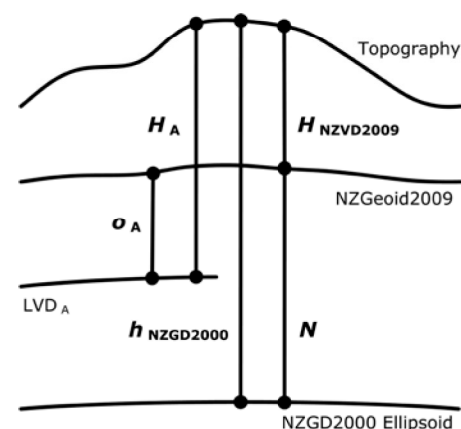


Figure 10. Schematic relationships between NZVD2009, NZGD2000 and LVD heights.

that can be used to consistently integrate geospatial datasets. Although it is the official national vertical datum, it will not formally replace the existing LVDs in the near future.

Table 4. Offsets from NZVD2009 to the 13 LVDs and their standard deviations (metres).

Local vertical datum	Number of points	Offset from NZVD2009	Standard deviation
One Tree Point 1964	51	0.06	0.03
Auckland 1946	137	0.34	0.05
Moturiki 1953	258	0.24	0.06
Gisborne 1926	61	0.34	0.02
Napier 1962	54	0.20	0.05
Taranaki 1970	70	0.32	0.05
Wellington 1953	78	0.44	0.04
Nelson 1955	111	0.29	0.07
Lyttelton 1937	251	0.47	0.09
Dunedin 1958	73	0.49	0.07
Dunedin-Bluff 1960	181	0.38	0.04
Bluff 1955	92	0.36	0.05
Stewart Island 1977	5	0.39	0.15

Instead, it will co-exist with them, so that existing datasets that cover distinct areas can continue to use a LVD as their height system, and also take advantage of the NZGD2000/GNSS height transformations that are provided with NZVD2009.

Unlike the LVDs, NZVD2009 is not explicitly tied to local MSL. The 'zero level' of the NZGeoid2009 surface is defined by the EGM2008 GGM that was used in its computation, and as such, NZVD2009 heights do not attempt to represent local MSL, although they typically occur within 0.5 metres of it. This means that, if the relationship between a NZVD2009 height and local sea level is required, then this will need to be quantified by physical inspection at the site in question. This requirement is actually no different for the LVDs since the MSL value for these heights is only applicable in the vicinity of the origin tide-gauges, and perhaps also only during the time period of the tide gauge observations (Table 1).

The new NZVD2009 heights for control marks that currently have ellipsoidal or normal-orthometric LVD heights will be obtainable from the LINZ geodetic database. In the first instance, however, these will be determined by transformation. This means that a mark that has a LVD height that was precisely levelled in 1973 will be assigned a NZVD2009 height derived from the 1973 height. Therefore, although the NZVD2009 height will be computed in 2010, it will in effect be a height from 1973 that in many cases has not been recently verified. As such, like with any survey mark, the published heights or coordinates should be verified in the field to ensure that they are reliable before they are used.

The NZVD2009 will not be a panacea for all height applications in NZ. The datum has been designed as a national datum for the consistent representation of geospatial and surveying data rather than for high-accuracy engineering projects. Where a particular application demands very precise heights, or where gradients/outfall levels are critical, it may be appropriate to use an application-specific datum that may also be related to NZVD2009.

SUMMARY

NZVD2009 provides for the first time a consistent height reference system that can be accessed across NZ and its offshore islands. Because gravity observations are not available at many of the control marks in NZ, the normal-orthometric height system has been retained. NZVD2009 is based on the NZGeoid2009 quasigeoid surface. Unlike most datums it is not directly connected to MSL at a tide gauge but it is generally within 0.5 metres of it.

The NZGeoid2009 is based on the EGM2008 GGM and is defined in relation to the GRS80 ellipsoid, therefore NZVD2009 normal-orthometric and NZGD2000 ellipsoidal heights can be efficiently transformed between the two systems. This relationship also allows GNSS derived ellipsoidal heights to be consistently related to NZVD2009.

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TECHNICAL REFERENCE INFORMATION

The following information is not strictly a part of this paper, but is included here for those readers who wish to obtain further information.

LINZS25004 Standard for New Zealand Vertical Datum 2009: <http://www.linz.govt.nz/geodetic/standards-publications/standards/index.aspx>.

LINZG25705 NZVD2009 Factsheet: <http://www.linz.govt.nz/geodetic/standards-publications/standards/index.aspx>.

LINZ Geodetic Database: <http://www.linz.govt.nz/gdb>

Coordinate Conversion on the LINZ website: <http://www.linz.govt.nz/coordinateconversion>

Further information about NZVD2009 and NZGeoid2009: <http://www.linz.govt.nz/nzvd>