

Conversion between Latitude and Longitude and New Zealand Map Grid

OSG Technical Report 4.2
Office of the Surveyor-General

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Foreword

Land Information New Zealand *Toitu te Whenua* (LINZ) was established in July 1996. It is a government department with roles and responsibilities in the following key areas:

Regulatory Responsibilities	LINZ Regulatory Groups
National spatial reference system and cadastral survey infrastructure	Office of the Surveyor-General
Topographic and hydrographic information	National Topographic/Hydrographic Authority
Land Titles	Office of the Registrar-General of Land
Crown Property	Property Regulatory Group

The main role of the department is a regulatory one, to set guidelines and standards and manage contracts with the private sector for carrying out the day to day business associated with each of the key areas.

LINZ also offers a range of services to customers related to land titles, survey plans and Crown property. The Operations Group based in LINZ regional offices throughout New Zealand carries out these services.

LINZ overarching objective is to be recognised as a world leader in providing land and seabed information services. For further information see our Internet site on <http://www.linz.govt.nz>

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This technical report is issued under the authority of the Surveyor-General who is responsible for setting geodetic standards, specifications, and guidelines in accordance with departmental policy and statutory requirements.

Version 1.1 corrected the conversion formulae to require north latitude as positive and included example coordinates in both systems. The version numbering is all that has changed between Report 4.1 and Report 4 Version 1.1. Version 4.2 has changed the number of decimal places shown for latitudes and longitudes in the test data.

Any comments or amendments should be forwarded to the Surveyor-General, Land Information New Zealand.

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Surveyor-General

CONVERSION BETWEEN LATITUDE AND LONGITUDE AND NEW ZEALAND MAP GRID

1 Conversion between latitude and longitude and New Zealand Map Grid

The New Zealand Map Grid (NZMG) is a projection that is used to convert latitudes and longitudes to easting and northing coordinates used for most mapping of New Zealand. The projection is unique to New Zealand. It was designed by Dr W. I. Reilly (1973) to minimize the scale error over the land area of the country - it cannot be sensibly used elsewhere. The conversion is calculated using a complex polynomial formula. Familiarity with complex arithmetic is assumed in this description.

Note that the projection applies to latitudes and longitudes referenced to the New Zealand Geodetic Datum 1949 (NZGD49). Coordinates from other datums, such as the WGS84 datum commonly used by GPS receivers, must be converted to NZGD49 before these formulae can be applied.

2 Conversion from latitude and longitude to NZMG

The conversion requires the latitude ϕ and longitude λ expressed in decimal degrees (north and east are positive). The conversion to easting and northing requires the following steps. Note that the constants and coefficients used in these formulae are listed below.

Calculate $\Delta\psi$ and $\Delta\lambda$ using

$$\Delta\phi = (\phi - \phi_0) \times 3600 \times 10^{-5}$$

$$\Delta\psi = \sum_{n=1}^{n=10} A_n \Delta\phi^n$$

$$\Delta\lambda = (\lambda - \lambda_0) \times \pi/180$$

Calculate a complex polynomial function of the complex number $\theta = \Delta\psi + i\Delta\lambda$ using the formula

$$z = \sum_{n=1}^{n=6} B_n \theta^n$$

Express the complex number z in terms of its real and imaginary parts $z = x + iy$ and calculate the easting E and northing N from x and y as

$$E = (y \times a) + E_0$$

$$N = (x \times a) + N_0$$

3 Conversion from NZMG to latitude and longitude

The conversion from NZMG to latitude and longitude involves an iterative approximation to reverse step 2 above. This starts with the easting E and northing N. The steps to calculate latitude and longitude are as follows. Note that the constants and coefficients used in these formulae are listed below.

Derive the complex number z as

$$z = (N - N_0)/a + i(E - E_0)/a$$

Determine the complex number θ as a series of approximations θ_0 θ_1 θ_2 and so on. The first approximation is

$$\theta_0 = \sum_{n=1}^{n=6} C_n z^n$$

Successive approximations are obtained by applying the formula

$$\theta_{i+1} = \left[z + \sum_{n=2}^{n=6} ((n-1)B_n \theta_i^n) \right] / \left[\sum_{n=1}^{n=6} (nB_n \theta_i^{n-1}) \right]$$

Two applications of this formula will give millimetre accuracy.

Express θ in terms of its real and imaginary parts $\theta = \Delta\psi + i\Delta\lambda$ and calculate the latitude ϕ and longitude λ as follows

$$\Delta\phi = \sum_{n=1}^{n=9} D_n \Delta\psi^n$$

$$\phi = \phi_0 + (\Delta\phi \times 10^5/3600)$$

$$\lambda = \lambda_0 + (\Delta\lambda \times 180/\pi)$$

4 Constants and coefficients

$a = 6378388$ metres is the semi-major axis of the International spheroid

$E_0 = 2510000$ metres is easting of the NZMG origin

$N_0 = 6023150$ metres is the northing of the NZMG origin

$\phi_0 = -41.0$ degrees is the latitude of the NZMG origin

$\lambda_0 = 173.0$ degrees is the longitude of the NZMG origin

Coefficient Real part Imaginary part

A_1	0.6399175073	
A_2	-0.1358797613	
A_3	0.063294409	
A_4	-0.02526853	
A_5	0.0117879	
A_6	-0.0055161	
A_7	0.0026906	
A_8	-0.001333	
A_9	0.00067	
A_{10}	-0.00034	
B_1	0.7557853228	0.0
B_2	0.249204646	0.003371507
B_3	-0.001541739	0.041058560
B_4	-0.10162907	0.01727609
B_5	-0.26623489	-0.36249218
B_6	-0.6870983	-1.1651967
C_1	1.3231270439	0.0
C_2	-0.577245789	-0.007809598
C_3	0.508307513	-0.112208952
C_4	-0.15094762	0.18200602
C_5	1.01418179	1.64497696
C_6	1.9660549	2.5127645
D_1	1.5627014243	
D_2	0.5185406398	
D_3	-0.03333098	
D_4	-0.1052906	
D_5	-0.0368594	
D_6	0.007317	
D_7	0.01220	
D_8	0.00394	
D_9	-0.0013	

5 Test Data

Provided below are three test points showing their NZMG coordinates and their NZGD49 Latitude and Longitude.

NZMG E,N:	2487100.638 metres	6751049.719 metres
NZGD49 Lat/Long:	-34.44406632 degrees	172.73919371 degrees
NZMG E,N:	2486533.395 metres	6077263.661 metres
NZGD49 Lat/Long:	-40.51240908 degrees	172.72310554 degrees
NZMG E,N:	2216746.425 metres	5388508.765 metres
NZGD49 Lat/Long:	-46.65129456 degrees	169.17206243 degrees

References

Reilly, W.I. (1973): A Conformal Mapping Projection with Minimum Scale Error. *Survey Review Vol XXII, no 168, pp 57-71.*