Using the Deformation Model to Generate NZGD2000 Coordinates – A Practical Example

A Practical Example

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Introduction

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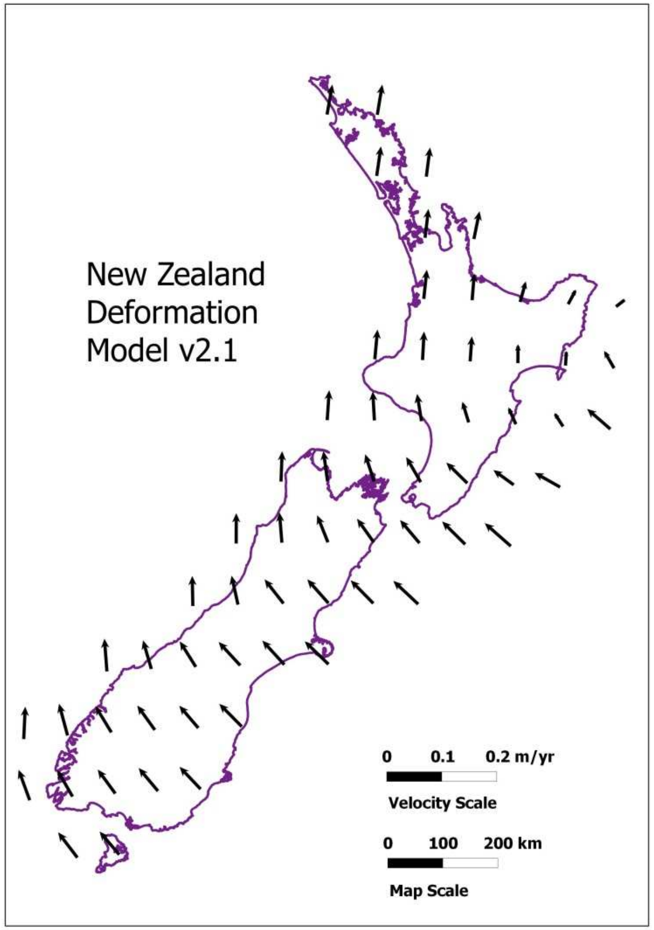
Why do we need a deformation model?

What is Precise Point Positioning?

How can the deformation model be used?

NZGD2000 Deformation Model



- NZGD2000 defined as ITRF96 at epoch 2000.0
- New Zealand is at the boundary of the Australian and Pacific plates
- Even over small distances, marks can be moving at different velocities. Cannot assume a static Earth
- Includes a deformation model which can be used to calculate epoch 2000.0 coordinates from observations at other epochs

Precise Point Positioning

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- Produces a coordinate directly from the GNSS satellites (no base station required)
- Similar to Single Point Positioning (handheld GNSS), but survey-accurate
- Relies on very accurate orbits and atmospheric models

Precise Point Positioning

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- Currently requires hours of data, but this is changing
- Produces a coordinate in terms of the satellite orbits (latest version of the International Terrestrial Reference Frame – ITRF)
- Produces a coordinate in the current epoch (eg 2012.85)
- In 10 years, may be the most common method of survey

ITRF and NZGD2000

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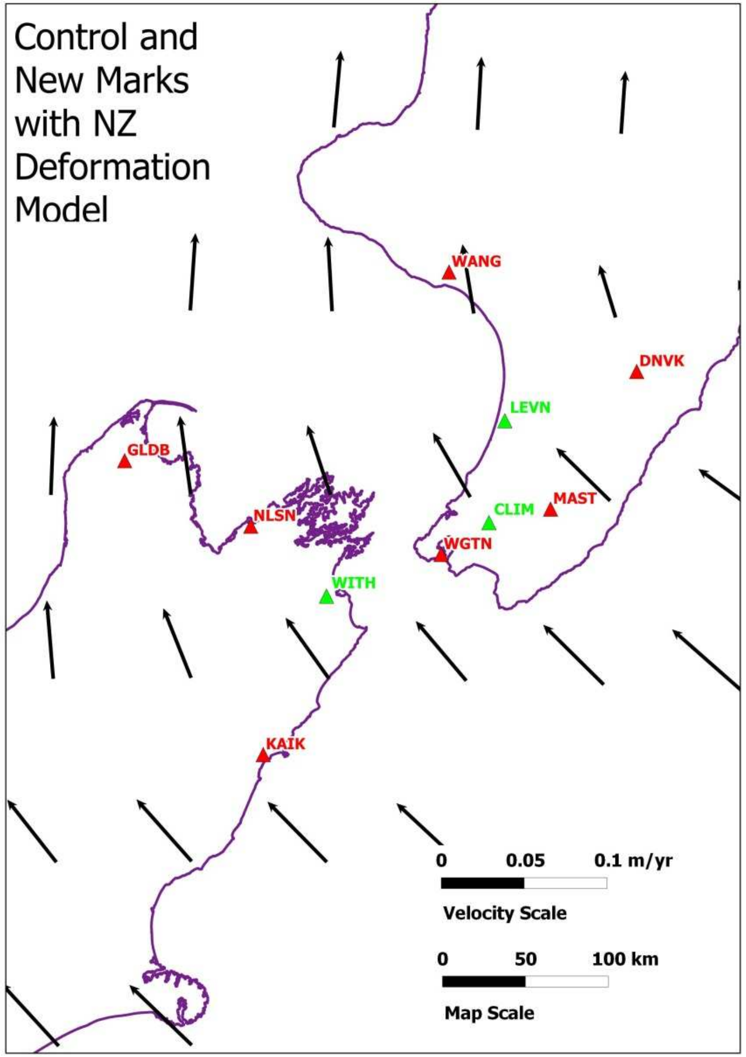
- Getting precise coordinates in the latest ITRF realisation has been greatly simplified through the provision of online GNSS processing services. Many of these provide absolute positions
- So we need to be able to transform coordinates from ITRF to the local datum
- We could always just make relative connections to control provided by the national geodetic agency, but this is not always the most efficient method
- Both coordinates may be required: ITRF for maximum precision and global consistency and NZGD2000 to meet regulatory requirements and ensure consistency with local datasets

Case Study

- Our project area is about 300km x 300km
- Station velocities vary significantly over this area
- PPP Processing method produces a coordinate in the current epoch (eg 2012.85)



GNSS Data Processing

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- We do all our processing in the more accurate reference frame, and then transform to any other desired reference frame and epoch
- Choose to use an online processing service (in this case JPL precise point positioning)
- This will give us ITRF2008 coordinates, in terms of the reference frame used by the IGS orbital products (IGS08).
- Process 24-hour sessions
- We end up with IGS08 coordinates at observation epoch, which is 2012 Julian Day 60 (2012.16)

Transforming Coordinates

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- Throughout, we are working in Cartesian (geocentric) coordinates. Any other transformations, such as to a mapping projection, are made at the end
- Step 1: Identify stations at which coordinates are available in both the desired reference frames
- Step 2: Use velocities at each station to obtain coordinates at a common epoch in the two reference frames
- Step 3: Calculate *appropriate* transformation parameters, using least squares. This will usually be three translation/rotation parameters, or three translation/rotation parameters plus one scale parameter over small portions of the Earth’s surface
- Step 4: Use the transformation parameters to convert coordinates between reference frames

Bilinear Interpolation

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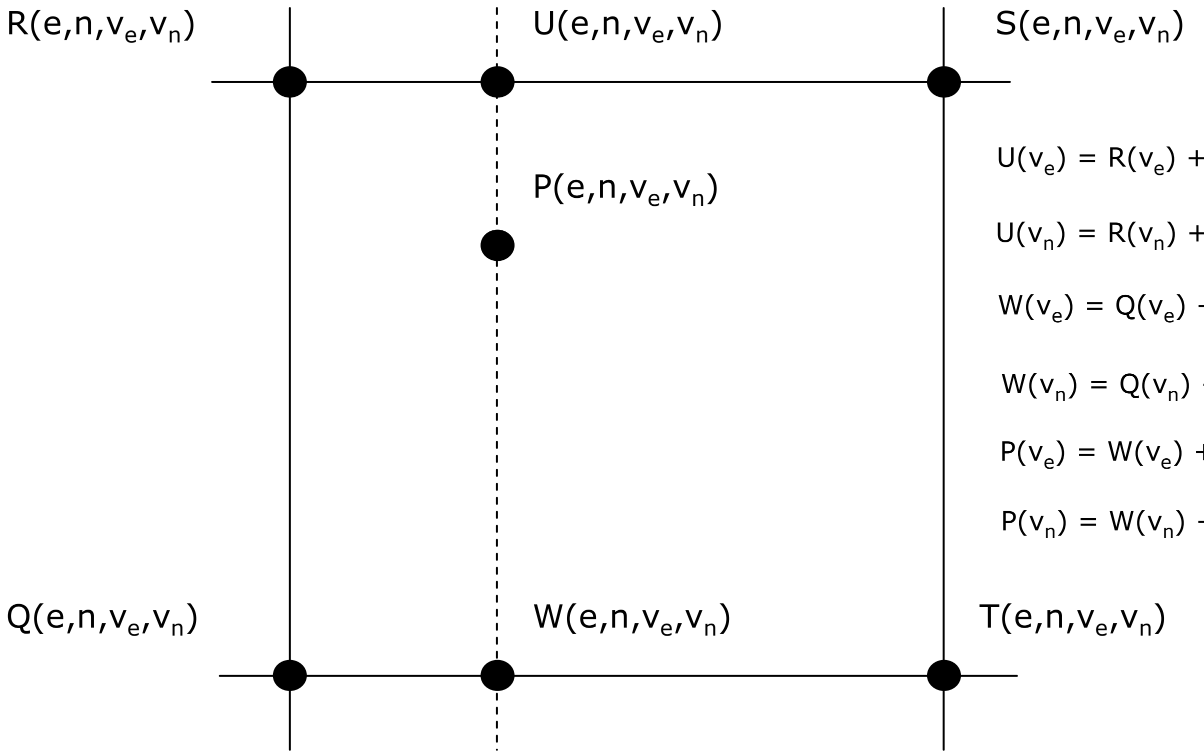


$$U(v_e) = R(v_e) + [(U(e) - R(e))/(S(e) - R(e))][S(v_e) - R(v_e)]$$

$$U(v_n) = R(v_n) + [(U(e) - R(e))/(S(e) - R(e))][S(v_n) - R(v_n)]$$

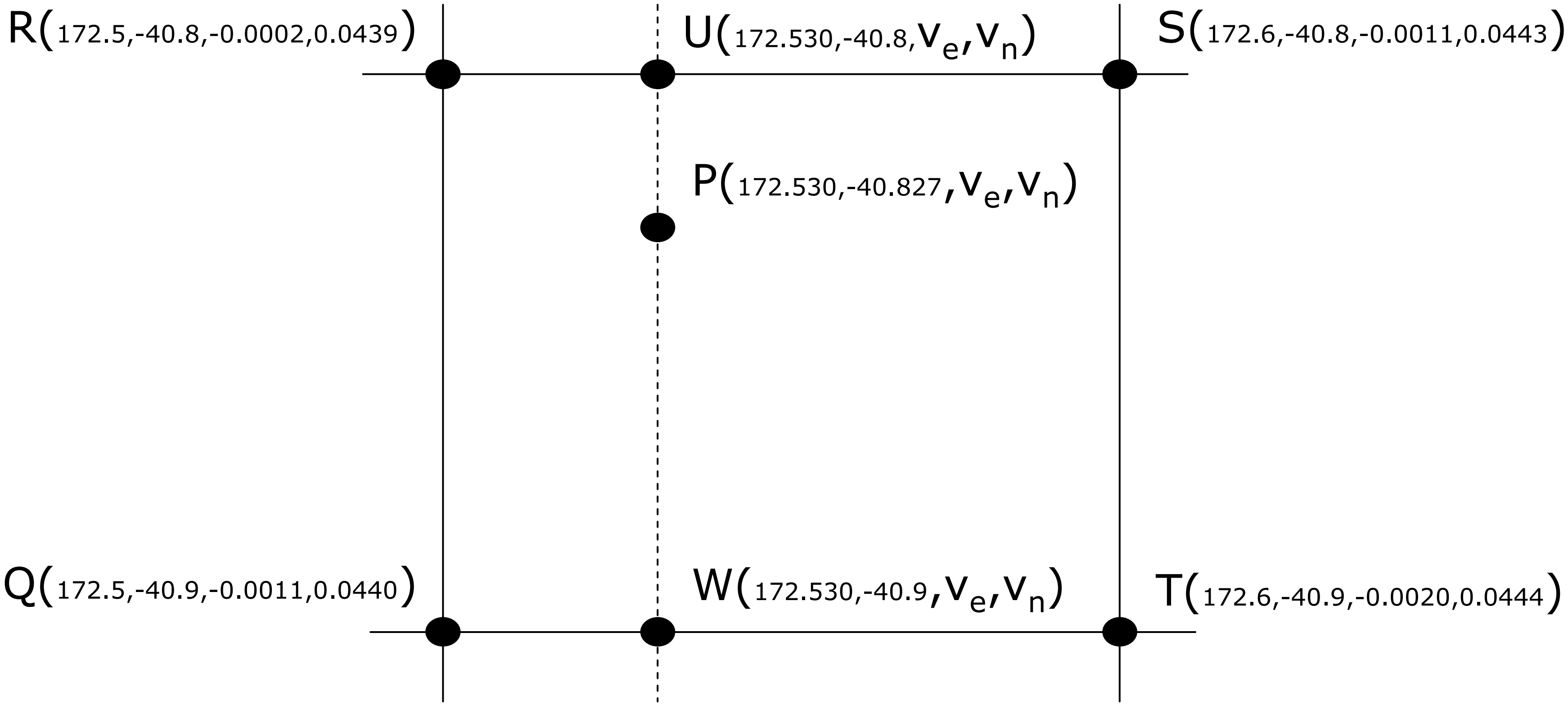
$$W(v_e) = Q(v_e) + [(W(e) - Q(e))/(T(e) - Q(e))][T(v_e) - Q(v_e)]$$

$$W(v_n) = Q(v_n) + [(W(e) - Q(e))/(T(e) - Q(e))][T(v_n) - Q(v_n)]$$

$$P(v_e) = W(v_e) + [(P(n) - W(n))/(U(n) - W(n))][U(v_e) - W(v_e)]$$

$$P(v_n) = W(v_n) + [(P(n) - W(n))/(U(n) - W(n))][U(v_n) - W(v_n)]$$

Calculating Velocity – Station GLDB



Calculating Velocity – Station GLDB

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$U(v_e)$ = -0.0002 + [(172.530 – 172.5)/(172.6 – 172.5)][-0.0011 – -0.0002] = -0.0005

$U(v_n)$ = 0.0439 + [(172.530 – 172.5)/(172.6 – 172.5)][0.0443 – 0.0439] = 0.0440

$W(v_e)$ = -0.0011 + [(172.530 – 172.5)/(172.6 – 172.5)][-0.0020 – -0.0011] = -0.0013

$W(v_n)$ = 0.0440 + [(172.530 – 172.5)/(172.6 – 172.5)][0.0444 – 0.0440] = 0.0441

$P(v_e)$ = -0.0013 + [(-40.827– -40.9)/(-40.8 – -40.9)][-0.0005 – -0.0013] = -0.0007

$P(v_n)$ = 0.0441 + [(-40.827– -40.9)/(-40.8 – -40.9)][0.0440 – 0.0441] = 0.0441

Transforming Velocities to Cartesian Reference Frame

- Recall that we are always working in Cartesian (XYZ) coordinates, so need XYZ velocities. Call this column vector $\mathbf{v}_{XYZ}$
- But the velocity model is topocentric (ENU). Call this column vector $\mathbf{v}_{ENU}$
- We can convert between the two using the geocentric to topocentric rotation matrix, $\mathbf{R}_{GT}$, for the point's latitude (ϕ) and longitude (λ)
- $\mathbf{v}_{ENU} = \mathbf{R}_{gt}\mathbf{v}_{XYZ}$
- $\mathbf{v}_{XYZ} = \mathbf{R}_{gt}^{T}\mathbf{v}_{ENU}$

$$R_{gt} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}$$

Transforming Velocities to Cartesian Reference Frame – Station GLDB

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- $\mathbf{v}_{XYZ} = \mathbf{R}_{gt}^{T}\mathbf{v}_{ENU}$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -0.130 & -0.992 & 0 \\ -0.648 & 0.085 & 0.757 \\ -0.750 & 0.098 & -0.654 \end{bmatrix}^{-1} \begin{bmatrix} -0.0007 \\ 0.0441 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.0285 \\ 0.0045 \\ 0.0333 \end{bmatrix}$$

Calculating NZGD2000 Epoch 2012.16 Coordinates – Station GLDB

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- $\mathbf{x}_{\text{NZGD Epoch 2012.16}} = \mathbf{x}_{\text{NZGD2000 Epoch 2000.0}} + 12.16\mathbf{v}_{\text{XYZ}}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{2012.16} = \begin{bmatrix} -4792405.831 \\ 628416.781 \\ -4148068.669 \end{bmatrix} + 12.16 \begin{bmatrix} -0.0285 \\ 0.0045 \\ 0.0333 \end{bmatrix} = \begin{bmatrix} -4792406.177 \\ 628416.835 \\ -4148068.263 \end{bmatrix}$$

Calculating Transformation Parameters

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- Use least squares to obtain the best solution, as we have more observations than parameters
- Functional model: **At** = **b**, where **A** is the design matrix, **b** = Calculated (IT96) minus observed (IGS08) and **t** is the matrix of unknown transformation parameters
- Stochastic model: **W** = **I**, in this case we choose to weight all coordinates equally
- So $\mathbf{t} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$, the standard least squares solution
- And $\mathrm{Cov}(\mathbf{t}) = \sigma_0^2(\mathbf{A}^T\mathbf{A})^{-1}$
- The Aposteriori Standard Error of Unit Weight is $\sigma_0^2 = (\mathbf{A}^T\mathbf{t}-\mathbf{b})^T(\mathbf{A}^T\mathbf{t}-\mathbf{b})$/(degrees of freedom)
- This is a linear problem, so no need to iterate
- Note: if you wish to weight your coordinates: $\mathbf{t} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{b}$

Calculating Transformation Parameters

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$$
\begin{array}{ll}
GLDB & x \\
 & y \\
 & z \\
NLSN & x \\
 & y \\
 & z \\
KAIK & x \\
 & y \\
 & z \\
WGTN & x \\
 & y \\
 & z \\
MAST & x \\
 & y \\
 & z \\
DNVK & x \\
 & y \\
 & z \\
WANG & x \\
 & y \\
 & z
\end{array}
\quad
A=\begin{bmatrix}
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1
\end{bmatrix}
\quad
b=\begin{bmatrix}
-4792406.177\\
628416.835\\
-4148068.263\\
-4775888.435\\
549740.211\\
-4177981.109\\
-4685479.521\\
531055.197\\
-4280819.034\\
-4777269.652\\
434270.387\\
-4189484.267\\
-4801933.943\\
370789.222\\
-4167752.305\\
-4860760.939\\
325692.752\\
-4103646.312\\
-4888073.52\\
443004.771\\
-4060015.325
\end{bmatrix}
-
\begin{bmatrix}
-4792406.117\\
628416.851\\
-4148068.23\\
-4775888.398\\
549740.2\\
-4177981.061\\
-4685479.471\\
531055.245\\
-4280819.009\\
-4777269.602\\
434270.406\\
-4189484.221\\
-4801933.888\\
370789.24\\
-4167752.257\\
-4860760.892\\
325692.771\\
-4103646.255\\
-4888073.493\\
443004.775\\
-4060015.31
\end{bmatrix}
=
\begin{bmatrix}
-0.06\\
-0.016\\
-0.033\\
-0.037\\
0.011\\
-0.048\\
-0.05\\
-0.048\\
-0.025\\
-0.05\\
-0.019\\
-0.046\\
-0.055\\
-0.018\\
-0.048\\
-0.047\\
-0.019\\
-0.057\\
-0.027\\
-0.004\\
-0.015
\end{bmatrix}
$$

$$
Cov(X)=\begin{bmatrix}
3.22\times10^{-5} & 0 & 0\\
0 & 3.22\times10^{-5} & 0\\
0 & 0 & 3.22\times10^{-5}
\end{bmatrix}
$$

Three Parameter Transformation Results

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- SEUW = 0.015 m
- t_x = -0.046 ± 0.006 m
- t_y = -0.016 ± 0.006 m
- t_z = -0.039 ± 0.006 m
- Note: In this case least squares simply gives us the average of the coordinate differences, so we could have avoided the matrix algebra, but would not get the precision information so easily

Four Parameter Transformation Results

- SEUW = 0.015 m
- t_x = -0.103 ± 0.211 m
- t_y = -0.011 ± 0.021 m
- t_z = -0.088 ± 0.183 m
- s = $-1.19 \times 10^{-8} \pm 4.40 \times 10^{-8}$
- None of the parameters is significant, so this is not the best transformation

Calculate IT96 Epoch 2012.16 for CLIM

- $\mathbf{x}_{\text{NZGD Epoch 2012.16}} = \mathbf{x}_{\text{IGS08 Epoch 2012.16}} + \mathbf{t}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NZGD\,2000\quad 2012.16} = \begin{bmatrix} -4793404.120 \\ 407108.010 \\ -4175081.520 \end{bmatrix} + \begin{bmatrix} -0.046 \\ -0.016 \\ -0.039 \end{bmatrix} = \begin{bmatrix} -4793404.167 \\ 407107.994 \\ -4175081.559 \end{bmatrix}$$

Calculate IT96 Epoch 2000 for CLIM

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- $\mathbf{x}_{NZGD\ Epoch\ 2000} = \mathbf{x}_{NZGD2000\ Epoch\ 2012.16} - 12.16\mathbf{v}_{xyz}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NZGD\ 2000} = \begin{bmatrix} -4793404.167 \\ 407107.994 \\ -4175081.559 \end{bmatrix} - 12.16 \begin{bmatrix} -0.0196 \\ 0.0277 \\ 0.0250 \end{bmatrix} = \begin{bmatrix} -4793403.928 \\ 407107.657 \\ -4175081.864 \end{bmatrix}$$

Calculate IT96 Epoch 2000 for CLIM, LEVN, WITH

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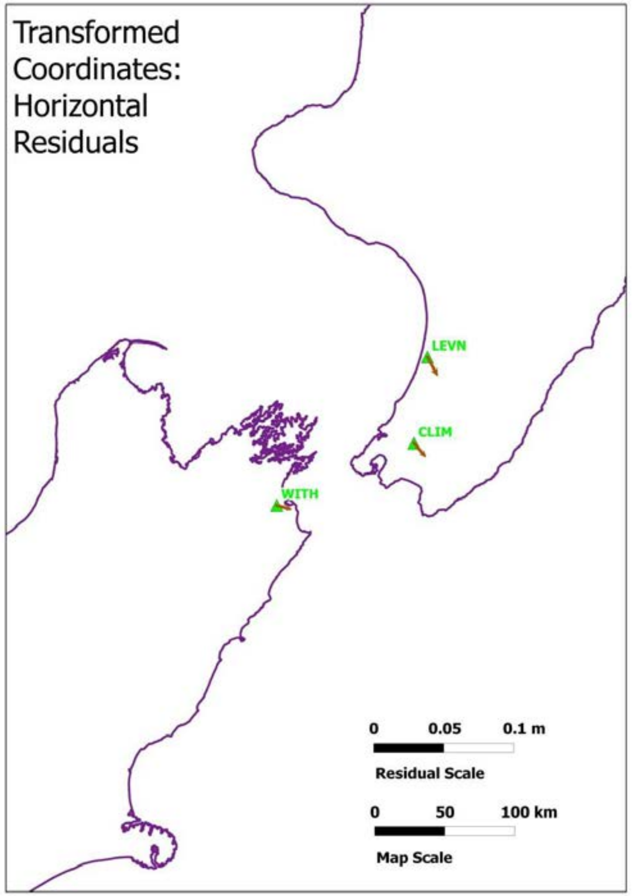
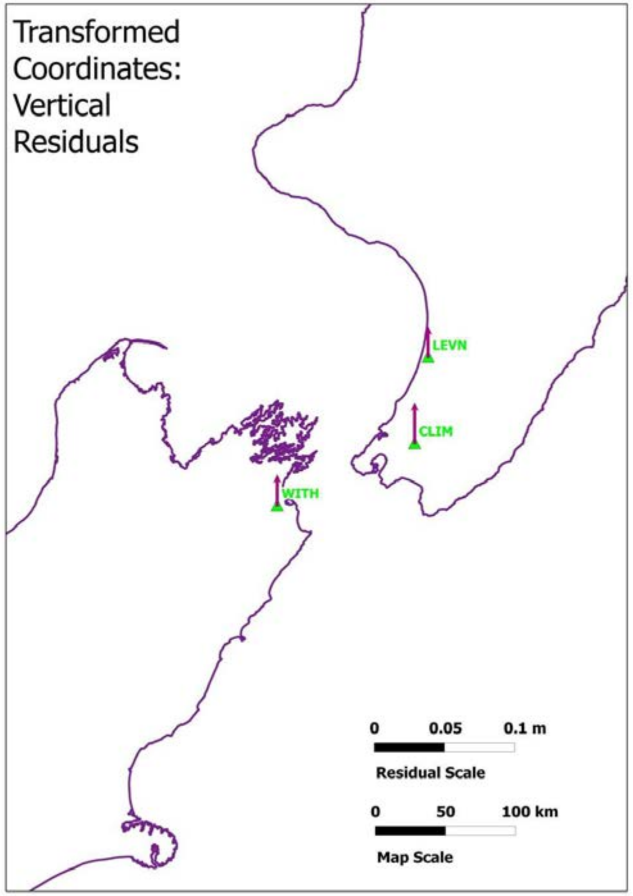
Station	IGS08 Epoch 2012.16 (XYZ)	Velocity (ENU)	NZGD2000 Epoch 2000.0 (observed)	NZGD2000 Epoch 2000.0 (GDB)	Difference (ENU)
CLIM	-4793404.120	-0.026	-4793403.928	-4793403.914	0.007
	407108.010	0.0333	407107.657	407107.663	-0.008
	-4175081.5204	0	-4175081.864	-4175081.841	0.025
LEVN	-4833775.0621	-0.0164	-4833774.861	-4833774.854	0.006
	402451.2374	0.0335	402451	402451.006	-0.011
	-4127913.8068	0	-4127914.155	-4127914.134	0.018
WITH	-4753506.3677	-0.0195	-4753506.156	-4753506.143	0.009
	500939.4145	0.0352	500939.133	500939.14	-0.002
	-4209496.456	0	-4209496.815	-4209496.801	0.018

Residuals Plots





Summary

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- Absolute positioning is readily available, and its use will increase
- These positions are in terms of the satellite orbit reference frame (latest IGS realization of current ITRF)
- Software to convert to a local reference frame may not exist, or may need to be tested
- This conversion can be done by the surveyor using a spreadsheet and the procedure outlined in this presentation