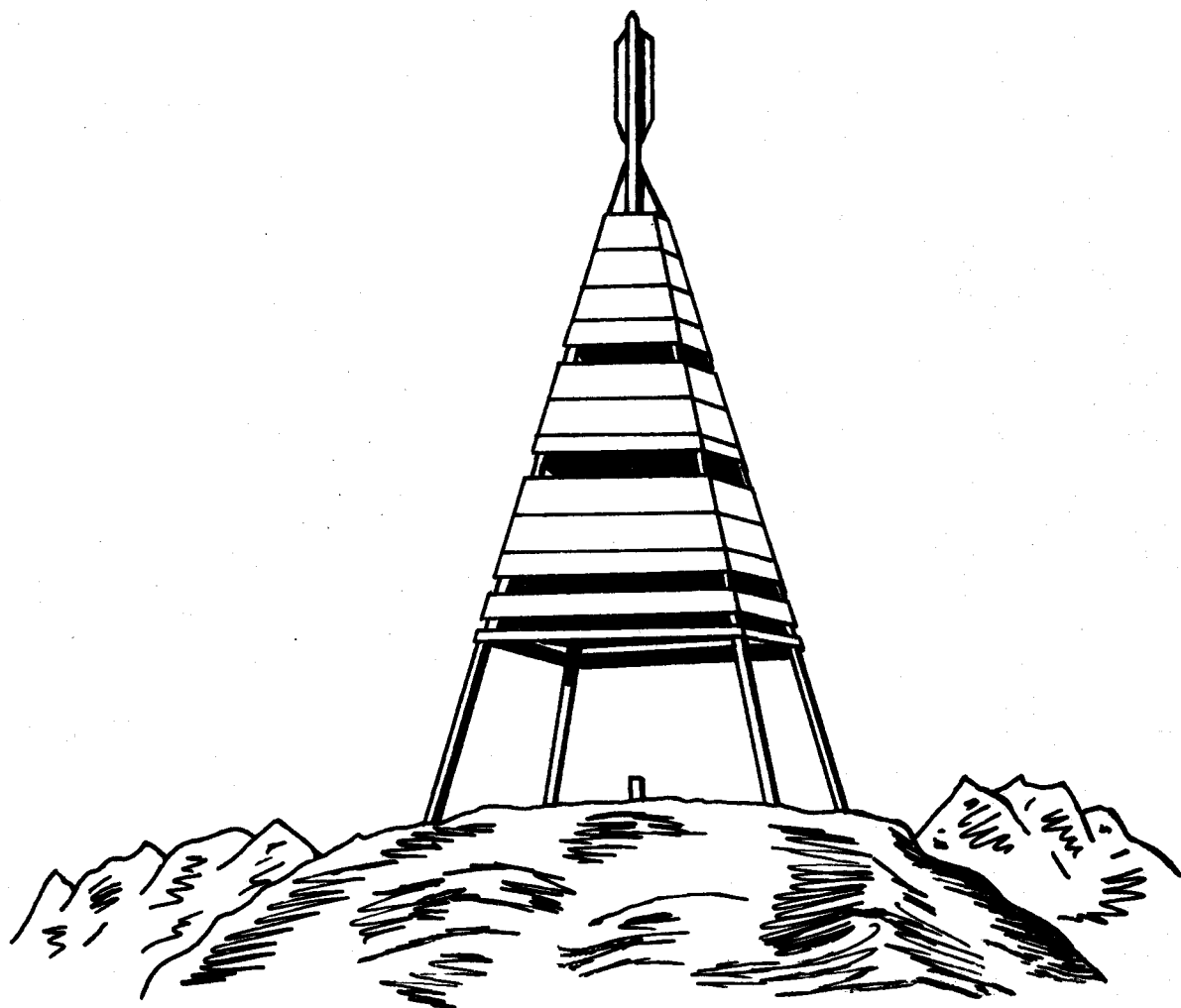


**First-order  
Geodetic Triangulation of New Zealand  
1909-49 and 1973-74**



**First-order  
Geodetic Triangulation of New Zealand  
1909-49 1973-74  
by L.P. Lee**

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## **Foreword**

**These notes, originally written in 1959 by direction of the then Surveyor-General, R. G. Dick, and revised in 1975, have been compiled from published annual reports and from periodical reports of survey parties on file in the Department of Lands and Survey. Some other publications from which information was obtained are listed in the bibliography at the end. The original notes were written by L. P. Lee, a member of the Head Office Computing Branch.**

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## PART I. HISTORY

### I. Historical Outline

#### Prologue

If the operations of a land survey could be performed in the order logically demanded, a general triangulation of the country would come first. It comprises the accurate measurement of at least one base-line and of the angles of a network of relatively large triangles, providing sufficient information for calculating the sides of the triangles and the positions of the vertices. In this way, a series of fixed reference points (trigonometric stations or simply "trigs") is spread over as wide an area of the country as possible. Connected to and depending on this primary triangulation would come lesser networks of smaller triangles, giving increased density of reference points, and usually carried out only in those areas where they are specially needed. Next would come standard traverses, lines of measured bearings and distances carried from trig to trig along the routes giving access to settlement surveys. And last of all would come the settlement surveys themselves, each fixed rigidly in position and orientation by connections to the standard reference points provided.

In New Zealand, as indeed in many a similar young country, history has determined otherwise, and the settlement survey proceeded well in advance of the triangulation which should have controlled it. Even the triangulations were done in reverse order, the smaller networks being observed first, the larger and more accurate networks later. Thus the procedure has been one of successive approximations, and each stage has led to a revision or recalculation of the work done earlier.

The geodetic triangulation of New Zealand is considered to have commenced with the measurement of the Wairarapa base in 1909. But the Wairarapa base was not, at that time, intended to be part of a general geodetic network, and to place the event in its proper historical setting it is necessary to survey events many years earlier. The geodetic triangulation is in fact the culmination of developments which reach back to the beginning of land surveys in the colony.

#### Earliest settlement surveys

Land surveys in New Zealand began with the earliest settlements and, like the settlements themselves, were confined to small local regions scattered widely over the country. No more was done than was required to meet the demands of the moment, the production of some sort of plan to accompany the sale of land to settlers, and neither scientific method nor great accuracy was claimed.

The first use of triangulation to control such local surveys was by Felton Mathew, first Surveyor-General, in 1840-41; the area covered was a limited one near Auckland. In 1849 another small triangulation was begun near Christchurch. Mention of these two triangulations is solely because of their historic interest; the observations were of inferior standard, and they were superseded by more accurate surveys later. Triangulation was the exception rather than the rule in the early surveys, and much of the work done reveals a sorry state of muddle, mismanagement, and neglect. Many of the surveys were inaccurate and misleading, and little or no attempt was made to place them in their correct positions relative to one another or even to orient them correctly. Bearings, where they were used, were magnetic; positions were often scaled from nautical charts; and there was no comprehensive control survey. Litigation over land boundaries, due to gaps or overlaps in the surveys, was frequent.

**Surveys in  
the Provincial  
period,  
1853-76**

In 1852 the administration of New Zealand was entrusted to six Provincial Governments, later increased to nine, each of which had its own Survey Department, while surveys of Maori land were conducted by a special Department of the General Government. The Provincial Survey Departments were faced with the task of introducing some semblance of order to the survey records, sufficient at least to give security of tenure to the purchasers of land. Some of the Provinces were unable to cope with the problem, and allowed the unsatisfactory surveys to continue, but others made an attempt, always under great difficulty, to provide a reliable control framework for the settlement surveys. There are several instances of triangulation being used in this period, but they were either surveys of indifferent accuracy or of too limited an extent.

A successful attempt to bring order into Provincial surveys was that of Henry Jackson in Wellington Province, beginning in 1865, which was based entirely on triangulation. There were three principal sections, in Wellington, Wairarapa, and Rangitikei, covering the three districts where settlements were located, and each section was erected upon its own base-line, with several check bases included for verification. Angles were measured at least four times, using either an 8 in. or a 6 in. (200 mm or 150 mm) circle. The sides of triangles were generally 3 to 5 km in length, but the three nets were linked by a fourth net of larger triangles with sides from 10 to 20 km long. True meridian was determined by astronomical observation at an observation point in the Hutt Valley for the orientation of the Wellington and Rangitikei sections, and at Opaki, near Masterton, for the orientation of the Wairarapa net. Initial latitude and longitude were also observed for the correct geographical positioning of all the stations, and rectangular coordinates referred to one initial station in each of the three sections were also computed. By 1875, 15 500 km<sup>2</sup>, involving 1200 permanently marked stations, had been triangulated in this way.

Another extensive triangulation scheme, carried out on behalf of the General Government for the control of surveys of Maori land, and located in the Provinces of Auckland, Hawke's Bay, and Wellington, was begun in 1868 by Theophilus Heale. In spite of interruption by war, it was carried on until it covered 40 000 km<sup>2</sup>, the stations being permanently marked. Begun in several independent sections which were gradually linked, it came to consist of two principal networks, a northern section resting on a base-line near the Kaipara harbour, and a southern section resting on a base-line at Maraekakaho near the present town of Hastings. The Kaipara base was measured three times with standard chains in 1870, the Maraekakaho base was measured three times in 1871 with a 66 ft (20 m) steel tape and straining apparatus.\* The sides of triangles averaged about 12 km in length, and angles were measured at least three times using instruments with 10 in. or 12 in. (250 mm or 300 mm) circles. Astronomical observations for true meridian and for latitude were made at three points, and geographical positions of all stations were computed; rectangular coordinates, referred to Mt. Eden, Auckland, for the northern section, were also computed.

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*\*This seems to be the first recorded use of the metal tape for base-line measurement. Experiments with such tapes had been made on the Thames goldfields in 1869 by Edwin Fairburn, who urged their use upon Heale. The measurement of the Maraekakaho base in 1871 was made by S. Percy Smith, who later became Surveyor-General. The superiority of the metal tape to the surveyor's chain soon led to its adoption as standard equipment. In regulations issued by J.T. Thomson, Surveyor-General, in 1876, it is specified that base lines be measured with a "standard chain"; the second edition of the same regulations, in 1879, refers to "standard chain or steel band"; the third edition, by James McKerrow in 1886, specifies "standard steel band" alone. The introduction of the metal tape is often ascribed to the experiments with steel wires made by Jaderin, of Sweden, about 1887, or the trials with steel tapes made by the U.S. Coast Survey in 1891, both of these being later than the regulations just mentioned. Although measurement with a steel tape became standard in New Zealand in the 1880s, surveyors have continued to call the tape a "chain", and to describe measuring with a tape as "chaining".*

In other Provinces – except for a very small area triangulated in Taranaki by Thomas Humphries in 1868 – triangulation was either avoided altogether or, when it was done, was carried out so inefficiently as to make it quite unreliable. But all the early triangulations, even those so competently done by Jackson and Heale, suffered from uncertainty as to the standard of length. Standards had been laid down and permanently marked in each of the Provincial centres. These generally took the form of stone or concrete blocks, firmly set 100 links (20 m) apart, with the terminals marked on brass plates cemented or leaded in on top of the blocks. These ground standards had been measured in various ways, usually either by beam compasses set with a standard scale or by direct measurement with a survey chain supposed to be correct imperial standard. As the standards were all independently laid down, and the terminal blocks could be affected by relative settling and shifting, the standard in each district differed more or less from the others – perhaps not sufficiently to make any great want of accuracy with the early methods of survey, but which became noticeable when the more refined measurements with the narrow steel tape came into general use.

#### **Meridional circuit system of Otago, 1856**

In the Province of Otago a control net of triangulation was not used, but a uniform system to govern the orientation of surveys was introduced by J.T. Thomson in 1856. The Province was divided into four large districts called “meridional circuits”. Within each circuit an initial station was selected and true meridian was determined by astronomical observation. Bearings, but not distances, were carried outwards by traverse from the initial station to the boundaries of the circuit, following the chief valleys suitable for settlement. Points on these traverses were called “geodesic stations”, and were usually from 15 to 25 km apart, providing a series of reference points by which any survey within the meridional circuit could be oriented in terms of the meridian of the initial station. Any further control was merely local, being based upon a small triangulation net extending only over the region where it was immediately required, so that a large number of independent triangulations came to be distributed throughout the settled area of the Province. Each such triangulation was regarded as the control for an area around it called a “survey district”, and each meridional circuit was eventually divided into many such districts, often irregular in shape, although the later additions tended to be bounded by lines parallel to and perpendicular to the meridian of the initial station. Local surveys could be coordinated with reference to the geodesic station within a survey district.

The system was not highly scientific or scrupulously exact – field observations being within an accuracy of 1' of arc – but it was simple and practical, and not likely to introduce inordinate errors or distortions. As surveys progressed, many of the smaller triangulations came to be joined, affording a check on their relative accuracy, but no adjustment of any discrepancies was attempted, and no controlling triangulation net was provided. The system satisfied the needs of the moment, but did not provide for the future.

#### **Proposals for reform**

The unsatisfactory state of most Provincial surveys was the cause of much concern. Heale, in particular, repeatedly urged that a general triangulation of the whole colony be carried out by the General Government, and always executed his own triangulations with a view to their eventual inclusion in such a general scheme, and the Secretary for Crown Lands, W.S. Moorhouse, recommended in 1872 that all surveys be placed under the control of a Surveyor-General and that a general triangulation be carried out. In 1873 a conference of the Provincial Chief Surveyors, presided over by Heale, recommended that a careful system of major triangulation, the sides averaging between 12 and 22 km, should be undertaken by the General Government over the whole area of the colony except where the rugged or forested nature of the country rendered it impracticable, and that minor triangulation or standard traverses, connected to this major triangulation, should be carried out by the Provincial Governments. Thomson was not present at the conference, but added his opinion to its report, advocating chains of minor triangulation as being more important than any major network. Parliament agreed in principle to the recommendation of the conference, and voted a sum of £5000 for the execution of a general triangulation.

In 1874, Major H.S. Palmer, of the Ordnance Survey of Great Britain, visited New Zealand to observe a transit of Venus, and when this assignment was completed he was asked by the General Government to report upon the state of the surveys. Palmer visited all but one of the Provincial survey offices, and in his report, presented to Parliament in 1875, the whole unsatisfactory state of affairs was quite bluntly revealed. Palmer insisted that, "whatever be the means introduced for systematising and carrying on future detail surveys and revising old ones, the basis of all such reform must be a general triangulation of the colony. In support of this view there could, perhaps, be no more convincing proof than this – that nearly all of the really good work hitherto done is that which has been founded on triangulation. That nothing short of trigonometrical survey will produce accurate estate maps of extensive areas is an axiom familiar to every educated surveyor; and in New Zealand accuracy is of special importance, from the responsibilities incurred in granting land, from the preponderance of undefined section boundaries, and from the scattered nature of the surveys."

In detail, Palmer recommended that a primary triangulation, with sides of from 24 or 32 km up to 80 or 100 km, and with angles observed with instruments of 12 or 18 in. (300 mm or 450 mm) circles, should be carried out over the whole country. A base-line of 10 or 12 km should be measured in each island, and the triangulation should be carried across Cook Strait to provide a connection between these bases and a verification of the observations. Latitude, longitude, and azimuth should be observed at one station of the survey and these values used in computing latitudes and longitudes of the remainder, azimuths of verification being observed at other distant points. Depending on this primary framework should be secondary triangulation with sides of from 12 to 30 km and observed with 8 or 10 in. (200 mm or 250 mm) instruments, and tertiary triangulation with sides of from 3 km to 5 or 6 km and observed with 6 in. (150 mm) instruments. The secondary triangulation would be extended only over those areas already occupied or likely to be occupied in the near future, and the tertiary triangulation could make use of those reliable networks already observed. All the triangulations should, as far as possible, incorporate existing stations. The work should be done by a special branch under the control of a Surveyor-General, and this branch could also be responsible for a levelling survey and a time service.

Some of these recommendations could be described merely as being standard practice. Nevertheless, reading the Palmer report today, one is struck by how closely the geodetic triangulation, when it did come, accorded with Palmer's recommendations, although some items – for example, his suggestion that stations be coordinated on a systematic projection with one central meridian in each island – had to wait 60 years for realisation. Palmer estimated that the whole operation could be finished in ten years, at a cost of £100,000 (\$200,000); his estimate of the cost was remarkably close to the truth, but in his estimate of the time he was much further from actuality.

At that time, the abolition of the Provincial Governments was already contemplated, and Palmer had in fact supported it in his report. The Abolition of the Provinces Act was passed in 1875, to come into force the following year, and as Palmer had had some praise for Thomson's system of survey in Otago, the General Government appointed Thomson as Surveyor-General, with the task of placing all the surveys of the colony on a uniform system. The Chief Surveyors of the Provincial Districts remained in local control of their surveys, but were now subject to the direction of the Surveyor-General.\*

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*\*In 1891 the new department adopted its present name of Lands and Survey, and the name Land Districts (provided by the Land Act 1877) was used instead of Provincial Districts which had been used until then. Since that time the boundaries of the Land Districts have been considerably altered from those of the Provincial Districts.*

**Adoption of  
the meridional  
circuit system,  
1877**

Thomson's report upon his visits to Provincial centres presented much the same picture of the prevailing chaos as Palmer's report had done, but his solution to the problem was a different one. At that time there were heavy arrears of land purchases awaiting demarcation of boundaries, and there was urgent need for a system that would rapidly bring all surveys under a reasonably correct system of control and record, so that settlers might be placed in secure possession of their land and the Government be safe to issue titles on reliable plans and descriptions. For this purpose, Thomson rejected the general triangulation as recommended by Palmer, and introduced, for all districts of the colony, the meridional circuit system which he had earlier used in Otago.

He was severely criticised in his own time for this decision, but it must be admitted that his reasons – the physical difficulty of triangulation in rugged country, the time and expense demanded, and the urgent necessity of correlating the settlement surveys already made and those waiting to be done – were cogent arguments at that date. Indeed, the difficulties later experienced in the geodetic triangulation, which, with interruptions, extended over considerably more than the ten years estimated by Palmer, completely vindicated Thomson's decision. Moreover, he never regarded the meridional circuit system as anything but a temporary expedient, a simple and rapid solution to the more pressing needs of the problem with which he was at that time faced, and he fully realised that a general triangulation must ultimately follow. But he considered that such a triangulation was remote. "Though the standard operations of a colonial settlement survey", he wrote, "cannot be called highly scientific, that is, microscopically exact, this in no way militates against their usefulness, nor prevents their stations ultimately being embodied in the more abstruse and refined analyses which the future wealth and leisure of the country may many years hence support and undertake."

Including those in Otago, 28 meridional circuits were established, 9 in the North Island and 19 in the South Island. The boundaries of the Provincial Districts, and the existence of stations which were already used as origins of coordinates in the Provincial surveys, were respected in determining the meridional circuit boundaries. At the initial station of each circuit, if it had not already been done, the astronomical meridian was determined by observations of circumpolar stars. Any existing triangulation was oriented on the true meridian of the appropriate circuit, and in those districts where triangulation had not been carried out, standard bearings were carried outwards by bearing traverses. Within three years, sufficient work had been done to enable all surveys to be correctly oriented within their meridional circuits.

Each meridional circuit was divided into survey districts, but on a much more regular system than had been used in Otago. In most cases, the survey districts are regular squares, bounded by lines at 1000 chain (approx. 20 km) intervals parallel to and perpendicular to the meridian of the initial station. Each survey district was named after a prominent topographical feature within it, and was further subdivided into sixteen square blocks, designated by roman numbers, by lines at 250 chain (approx. 5 km) intervals. In some districts there are departures from this regularity. The original intention had been that all survey points should be coordinated in terms of the initial station of the meridional circuit, and this practice was adopted in those districts which already had an extensive triangulation. In other districts, coordinates were referred to a local origin within each survey district, leaving the coordinates from the circuit initial to be determined at a later date.

It was Thomson's intention that blocks and survey districts should also be used as a system of land appellation, sections for settlement being numbered within a block. Most of the Provinces had devised their own systems for this purpose, and such systems continued to be used, but the block and survey district system was adopted as lands were opened in new localities. Block and survey district boundaries were deviated where necessary to keep a section wholly within one block. The block and survey district system has also been extensively used as an indication of locality.

**Meridional  
circuit  
triangulation,  
1877-1900**

The meridional circuit bearings were sufficient for the orientation of any survey but alone could not fix its position; for this purpose, triangulation was necessary. In the twenty years following the general adoption of the meridional circuit system, minor triangulations were gradually extended over all the areas where settlement surveys had been made or were proposed. The Surveyor-General for the greater part of the period was James McKerrow, but the triangulations were the concern of the Chief Surveyors of the several Provincial Districts and do not seem to have been subjected to any specific control by the Surveyor-General except in the matter of general regulations for their conduct.

Mostly, they were chains or nets of polygons, without cross rays, and with sides averaging about 5 km in length. This type of figure is intrinsically weak, and can be greatly strengthened by the introduction of a few cross rays, but these, even where they were easily observable, seem to have been studiously avoided.\* In many districts, no systematic adjustment of the observations was made; quite often, the bearing or distance adopted for a particular line would be simply the mean of two or more values obtained from different routes of computation. Each meridional circuit was considered small enough to be regarded as a plane, and no attempt was made to apply any corrections for spherical or spheroidal shape.

When the most urgent needs of minor triangulation had been met, larger nets of major triangulation were observed, gradually linking together the various meridional circuits and providing checks by the comparison of bearings and distances of common lines where circuits met. By the mid-1880s a continuous chain of triangles extended for 1500 km from North Cape to Stewart Island, although there were many areas on the flanks of this chain awaiting the extension of the triangulation.

Many independent observations of latitude had also been made at the initial stations of most of the meridional circuits, and there were determinations of longitude (mostly by the method of moon culminations) at four stations. In 1882, a Survey Observatory was built at Mt. Cook, a small rise in the city of Wellington. Here in 1883 C.W. Adams made observations for latitude by Talcott's method with a zenith telescope, using 99 pairs of stars, while the longitude was determined by the exchange of time signals with Sydney Observatory by submarine cable. The observatory was connected to Mt. Cook trig station, less than two chains (50 m) distant, so that latitudes and longitudes could thence be carried through the triangulation nets. As the determinations at Mt. Cook were considered the most accurate that had been made, geographic positions on the Mt. Cook datum were computed for the initial stations of the various meridional circuits. (In 1902 a redetermination of the longitude of Sydney led to a revision of the New Zealand longitudes). Having served its purpose, the observatory was dismantled in 1885. Mt. Cook itself is no longer usable as a trig station, but its position is marked in the floor of the lobby of the National Museum, and its role as the initial station of the New Zealand surveys is commemorated in a tablet on the wall nearby.

The "geodetic" positions on the Mt. Cook datum differed in many cases from the observed astronomic positions, and the difference was of course put down to deflection of the vertical, although part of it was no doubt due to different degrees of accuracy in the determinations. No special study of this matter was made, although the influence of Mt. Egmont in causing such deflections was mentioned.

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*\*Field book records show that in some cases cross rays were observed but they were not shown on plans.*

In 1890 the Surveyor-General, S. Percy Smith, who himself had carried out much of the triangulation in the preceding years, was able to report: "The triangulation of the colony – either as major or minor work – is now well advanced, and its continuance in the future will be in somewhat small and detached portions where necessary to cover the settlement surveys as they advance; the cost, therefore, of this class of departmental work will be a gradually-decreasing one. The time will come when a great triangulation of the colony will be undertaken in the interests of science, and when this period arrives the work already done will facilitate its operations enormously, by furnishing the known positions of points to form the apexes of a series of triangles of any size required. So far as the settlement operations of the survey are concerned, no such governing triangulation is necessary, for the discrepancies on the closing lines of the work as it exists are far within the limits which could affect the boundaries of properties, or cause any displacement in the general maps of the colony."

Nevertheless, discrepancies were known, and, in spite of the optimistic note in the latter part of Smith's statement, were in some cases too large for comfort. Furthermore, with improvements in surveying instruments, later cadastral traverses were often more reliable than the triangulation to which they were connected. The main defects in the meridional circuit triangulations were the use of many different and uncertain standards of length, the poor quality of some observations with relatively large internal closing errors, the lack of systematic elimination of observational errors in the computation of the work, and the treatment of the triangulations as though the observations had been made on a plane. These have resulted in serious discrepancies along the boundaries of the meridional circuits. The framework provided by the triangulation of the late 19th century proved to be adequate for most of the needs of cadastral survey within any one meridional circuit, but has proved troublesome where any survey crossed the boundary of a circuit, and has also introduced difficulties in mapping on a national rather than a local scale. By the turn of the century, the necessity for a general triangulation of the whole country, to serve as the primary control for surveys of all kinds, was no longer a matter for argument. The need was realised, but it was still regarded as remote, to be done whenever the time, the staff, and the finance could be made available from the more immediate needs of cadastral survey. It was realised, too, that a geodetic triangulation could serve scientific purposes other than the purely utilitarian needs of cadastral and topographical survey, and in 1901 the Surveyor-General's report made mention of the possible measurement of an arc of the Earth's surface to determine the figure of the Earth.

#### Secondary triangulation, 1901-15

In 1901 a new secondary triangulation was commenced to bring into harmony all the different nets of minor triangles which had spread inland from the coastal chain in advance of settlement surveys in the Wellington and Taranaki districts. A chain of polygons, with sides varying from 10 to 40 km and averaging about 20 km, was carried from the southern part of the Wellington district for about 300 km into Taranaki. The angular observations were made by H.J. Lowe, using a 10 inch (250 mm) theodolite, the best in the Department at that time, reading by verniers to 10". In 1902 this work was suspended because of the urgency of settlement surveys, and it was not resumed until 1909. It was continued, by Lowe and H.E. Girdlestone, at intervals during the next three years until the chain had been completed.

At the time when this work was begun, it was also decided to obtain new standards of length to replace the untrustworthy chain standards which had been laid down in the Provincial centres. Accordingly twelve steel tapes, each  $\frac{1}{4}$  in. (6 mm) wide and slightly over 100 ft (30 m) in length, were procured in 1903 from Chesterman and Co., Birmingham, and these were compared with the imperial standard by the Standards Branch of the Board of Trade, London, which supplied certificates giving the particulars of the comparison. These tapes are referred to as Imperial standard bands Nos. 1 to 12. Secondary standard bands, of similar  $\frac{1}{4}$  in. tape, were also obtained and are referred to as Official standard bands Nos. 1 to 12. Each Chief Surveyor was supplied with an Imperial standard band and an Official standard band, with instructions to use the official standard band for hack comparisons of surveyors' working tapes, and reserve the imperial standard band for special cases when the highest possible accuracy was desired.

For the secondary triangulation two base-lines were selected: one, the Wairarapa base, near the southern end of the chain, and another, the Eltham base, at the northern end. These two bases were measured with extreme care by J. Langmuir in 1909-10, using invar tapes which were compared with the imperial standard bands at frequent intervals. The standard in terms of which each base was to be measured was to be the imperial band in the charge of the Chief Surveyor of the district in which the base-line was laid down. Thus the Wairarapa base was to be in terms of No. 1 (in Head Office, Wellington) and the Eltham base in terms of No. 4 (in New Plymouth).

During the next few years, three more base-lines, Waitemata, Matamata, and Kaingaroa, were measured in the Auckland district to serve for a similar secondary triangulation which it was proposed to carry out there, the Waitemata and Matamata bases being in terms of Imperial band No. 3 (in Auckland) and the Kaingaroa base in terms of No. 5 (in Napier). During the course of these measurements, as it was seen that a grave risk was being run by having the length of each base depend upon a different band, a series of intercomparisons between all available bands was made by Langmuir in 1912. As his comparisons were at variance with the original ones made by the Board of Trade, two of the imperial standard bands were forwarded to the National Physical Laboratory for re-testing in 1915. Both bands were found to have shortened. The shortening was not satisfactorily explained, but it was assumed that it had occurred before the measurement of the base-lines had commenced in 1909, and the lengths of the bases were computed on this assumption.\*

In this period also, a systematic investigation of mean sea level was undertaken at six ports (Auckland, Wellington, Lyttelton, Port Chalmers, Nelson, and Westport) where tide-gauges were in operation. The values actually derived, and published in 1909, were of what is now called "half tide level", a mean of mean high water and mean low water, and not mean sea level as it is usually defined, the mean of the tidal heights at certain intervals of time (usually one hour) throughout the period. As a datum for the heights of trig stations, the difference is unimportant.

Reconnaissance for the secondary triangulation in the Auckland district was carried out by Langmuir who had selected and beacons many stations up to the outbreak of the First World War. With heavy enlistments by members of the Department's staff, and with the need for economy in war-time, the work was suspended in 1915.

#### Proposals for primary triangulation, 1913-20

The secondary triangulation was never regarded as final. From the beginning, it was intended that it should at some future date be superseded by a primary net covering the whole country, and great care was exercised in the measurement of the base-lines so that they would be of sufficient accuracy to serve also as bases for such a primary net. It was soon realised that the base-lines had in fact been measured to such a high degree of accuracy that the angular observations were quite out of keeping with them. The shortcomings of the secondary triangulation were commented on even before the base measurement had been completed, and proposals for a primary net, observed with improved instruments, were again made. There was much support for the contention that the "some future date" could be in the very near future.

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*\*The custody of the official standard of length was formally given to the Surveyor-General in 1921 (Sec. 16, Land Laws Amendment Act 1921). In 1945 it was transferred to the Minister in charge of the Department of Scientific and Industrial Research (Sec. 6, Scientific and Industrial Research Amendment Act 1945). Effective custody is now with the Physics and Engineering Laboratory of that Department.*



In 1913, James Mackenzie, Surveyor-General, reported that new 8 in. (200 mm) transit micrometer theodolites had been obtained to replace the 10 in. (250 mm) Everest for future triangulation. He also commented: "The smoke from bush-fires during the summer, and the haze caused by the sun on the moisture-laden country during the winter, prevented the signals showing out distinctly even on comparatively short lines. Numbers of trigs. were situated on the high backbone ranges, which were very rarely clear from mist. In many cases over a month elapsed before the observations at a station were completed. When the primary work is undertaken, the observations will have to be taken at night to get over the vagaries of the atmosphere. Signals 25 miles distant can very rarely be seen during the day in New Zealand." In the same year, C.E. Adams, Chief Computer, also recommended that night observations should be given a systematic trial.

For a few years after the war, field work on triangulation of any kind ceased almost entirely, but the primary triangulation was not lost sight of. In 1920, W.T. Neill, Chief Inspector of Surveys, again drew attention to the need: "Notwithstanding the claims made in the early days, and frequently repeated, in favour of the original minor triangulation of the Dominion as being sufficiently accurate to control the section surveys on which titles are issued, it is now known by every surveyor who has made traverses connecting the trig. stations that the system has failed in regard to precision, and that the co-ordinate values of most of the triangulation stations of the original survey are not sufficiently accurate to check the values obtained by an ordinary traverse executed by an average 5 in. theodolite and the long band used by staff surveyors." He urged that a new triangulation should be executed on correct and scientific principles, making use of the increased precision of more recent instruments.

#### North Island geodetic triangulation, 1921-38

In 1921, when Neill had become Surveyor-General, he began the work which had been so often postponed. The project was now called "geodetic triangulation" for the first time, and the scheme envisaged was no longer a secondary network, but a new control network of the kind recommended by Heale and Palmer 50 years earlier, but incorporating the base-lines already measured by Langmuir for the secondary triangulation. Because of the extensive settlement surveys then commencing in the Urewera country, it was decided that observations should begin at the Kaingaroa base and be carried to the coast northward and eastward. A scheme of triangulation covering this area was prepared by Neill, although he allowed the observers to modify it at their discretion where field reconnaissance showed it to be desirable. Basically, the scheme was a network of polygons, but with several cross rays observed so as to introduce some quadrilaterals. The sides of triangles averaged about 30 km in length. In the years 1921-23 much reconnaissance was carried out by H.M. Kensington, T. Cagney, and A.C. Haase, and many beacons were erected. Existing stations were used as far as possible, but it was found convenient to establish several new ones.

Finally, in 1923, the field observations of the new triangulation began. Observations near the Kaingaroa base in the first year were made by A.C. Haase, but for the next eight years they were made by H.M. Ross, who was also responsible for extending the reconnaissance into the area ahead of the observations.

Ross carried the work northward to Tauranga, then eastward to East Cape, and southward through Poverty Bay and Hawke's Bay, completing observations in the latter district by 1927. Up to this time, angular observations were made in daylight, using a Troughton and Simms 8 in. (200 mm) transit micrometer theodolite reading to 1". Progress was relatively slow, each station requiring from 20 to 30 days for occupation, as several of the stations occupied were on the high axial ranges and among the most difficult of access of any in the North Island, sometimes involving a two-day tramp from the nearest road. Fog was prevalent, and observation was restricted to short periods in the early morning and late afternoon. Beacons were wooden quadripods, usually about 5 m in height. Heliostats were used at low stations where the beacons had poor background contrast. As the work progressed, Ross found that larger triangles were convenient, and he eventually adopted 50 km as the optimum length of sight. It was originally the aim to provide one astronomical station in every three

geodetic stations, and latitude observations were made at many stations in the Bay of Plenty region. Later it seems to have been realised that the proportion was unnecessarily high, and fewer stations were selected for latitude determination in other regions. Azimuth observations were not made during the first few years of the survey.

Some of the errors of triangle closure in the part of the network observed in the first years were uncomfortably large, occasionally reaching 5", although the majority were less than 1". In 1929, when the work had reached the Wairarapa base, Wild T3 geodetic theodolites, reading to 0".2, replaced the 8 in. transit. In 1930, observations were made to luminous signals instead of to the opaque beacon structures formerly used. As a result of the improved triangle closures obtained, the method of observing to beacon lamps at night was thereafter adopted. The new theodolites and the night observations made for more rapid progress as well as for increased accuracy. Triangle closures thereafter rarely exceeded 2".

In 1929 a small subsidiary network was observed to connect the main triangulation with the Dominion Observatory, Wellington, where international longitude observations had been made in 1926. From Wellington, the work was then carried northwards into Taranaki and towards Lake Taupo, and in this period public curiosity was aroused by reports of the mysterious nightly appearance of lights on distant hills. The Matamata base was reached in 1931, and the coverage of the North Island was about three-quarters complete. In these years, azimuth observations had been made at several stations, and in the central portion of the network some larger triangles, with sides of about 80 km, were used.

Progress was halted in 1931. In February of that year a disastrous earthquake occurred in the Hawke's Bay district, and a fire which followed caused the complete destruction of survey records in the Napier office. All available staff was transferred to the task of establishing new survey control in this area. As part of this work, the geodetic triangulation in the district was reobserved, and comparison of the new results with the earlier ones revealed rather large lateral shifts – up to 2 links (0.4 m) – in the positions of stations on the northern shore of Hawke's Bay. These were thought to be due to a lesser earthquake in that area in September 1932.

Much triangulation of a lesser order was carried out over the whole of the Hawke's Bay Land District, and it was in this period that the terms, "first-order", "second-order", and "third-order", were used to indicate the relative importance of the networks. Basically, these terms relate to the order of accuracy with which the observations are made, but they coincide also with the successive stages in the "breakdown" of the large triangles of the primary net into the smaller triangles of the lesser nets. The observations were made by Ross and J.P. Arthurs, and in 1933 Arthurs also participated in a new programme of international longitude determination at the Dominion Observatory.

A preliminary computation of the first-order net enclosing the three bases, Kaingaroa, Wairarapa, and Eltham, was used to establish the orientation and scale of the Hawke's Bay second-order and third-order nets. Coordinates of all the Hawke's Bay stations were then computed on the transverse Mercator projection, using the old initial station of the Hawke's Bay meridional circuit as origin. This was the first occasion on which a systematic projection was used for survey coordinates to replace the old "plane" coordinates. The projection was selected by H.E. Walshe, who was Surveyor-General at the time.

When the urgent work in Hawke's Bay had been completed, the geodetic triangulation was suspended for financial reasons during the depression of the early thirties. In 1935 a deputation from the N.Z. Institute of Surveyors waited upon the Minister of Lands, and supported the Surveyor-General in urging the completion of the triangulation, as well as the inauguration of standard traverse and precise levelling surveys. In 1936 the first-order triangulation was resumed under the field direction of T.W. Preston, with Preston and Arthurs as observers. Observations were commenced at

the Waitemata base and carried southwards to link up with the earlier work at the Matamata base, this area having been reconnoitred by Ross. In this district, some second-order triangulation was carried out in association with the first-order work. In the following year, observations were carried up the North Auckland peninsula, completing the network in the North Island. Several of the stations occupied were on islands, and some delays were incurred in awaiting favourable weather for landing. The original scheme of secondary triangulation had supposed a further base-line in the north, but it was now considered that there was no need for this. A small subsidiary network was observed to provide a connection to the point at Cable Bay where transpacific longitudes had been observed by Dr Otto Klotz in 1903.

**South Island  
geodetic  
triangulation,  
1938-42**

In 1938, observations in a quadrilateral across Cook Strait were completed, the longest line – 120 km from Papatahi to Attempt Hill – occurring in this figure.\* In that year and the following one, a network was spread over the high mountainous country in Marlborough and Nelson and extending to a site selected for a base-line at Culverden in north Canterbury. This work was under the direction of T.W. Preston, with J.P. Arthurs and R.P. Gough as observers.

During the reconnaissance an endeavour was made to avoid the highest ranges and to select stations at a lower altitude, but this was found to be impracticable without considerable loss of strength of figure, and eventually a scheme was adopted to give fairly complete coverage over the whole of the northern part of the South Island. This involved occupying a number of stations over 1500 m in height, including three over 2100 m. Reconnaissance in this area was made by aircraft, a three-seater single-engine Waco being hired from the Marlborough Aero Club, and three hours in the air saved weeks on the ground. Though fine weather was essential, it was found to be quite easy to check on the intervisibility of stations, and by cruising around a peak to decide on the feasibility of the station for access and occupation.

All members of the field party became expert mountaineers. Generally, the high country stations were too dangerous for lightkeepers to attend at night, and lamps were left on time-switch, the lightkeepers visiting the tops in good weather to service and de-ice the lamps, especially after snowstorms. Owing to the difficulty of using normal communications, when frequently both observers and lightkeepers were encamped at distances of one or two days travel from the nearest telephone, radio communication was introduced in 1938, so that the observer could issue instructions to lightkeepers. The transmitter, licensed as station ZLEN, was operated by the observer from his camp, the lightkeepers being equipped with receivers, and the saving in time more than paid for the equipment in one season. The sets were designed for a range of 80 km, but transmissions were often picked up by radio "hams" outside this range, in one case by an amateur in Manurewa, 800 km away. A geodetic Tavistock theodolite was used for some observations, but as its weight in the carrying case was nearly double that of the Wild, it was kept for the more accessible stations.

In 1939, W.G. Nelson assumed control of the field party and carried on the reconnaissance, with Arthurs and Gough still the principal observers. The work was carried south of the Culverden base towards a second base that had been selected on the flood plain of the Waitaki River. In this area the mountainous country was avoided as far as possible, and a chain of quadrilaterals was used, stations on the western side of this chain being on relatively high country, stations on the eastern side being on the low country of the Canterbury plains. Here two stations were established on the tops of water-towers, and at other stations it was necessary to elevate observers and instruments above the level of the plains by the use of temporary towers. Steel Bilby towers, erected to a maximum height of 36 m, were used. They were comparatively light, and could be rapidly erected, dismantled, and re-erected on various sites. It was necessary, however, to wait for calm weather before observing at tower stations. In other places, 9 to 12 m wooden towers were frequently used to clear plantations or other obstructions.

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*\*Longer rays to intersected stations have been used in some figures of lesser order, including a ray of 165 km to Mt. Aspiring.*

The outbreak of the Second World War did little to impede the progress of the survey. Unlike the conditions prevailing during the First World War, New Zealand now faced a threat of invasion, and topographical maps for military use were urgently required. Practically nothing suitable for the purpose existed, and the Department of Lands and Survey embarked upon the hurried production of a series of "provisional" topographical maps. Here, perhaps for the first time, the value of the geodetic triangulation was strikingly demonstrated. The provisional solution of the North Island network made it possible to harmonize survey data from different districts and of different degrees of reliability so that they could be reduced to a common datum. Accordingly, the observations of the South Island triangulation continued without interruption except by the weather.

Some incidents, however, were directly attributable to war-time conditions. The licence to operate station ZLEN was withdrawn, and most of the Canterbury triangulation was carried on without the assistance of radio communication. In 1940, when the field party moved into the mountainous country of Otago, it was almost impossible to operate without radio, and the reissue of the licence was applied for. This was granted, but subject to strict limitations; the station could be used for not more than 15 minutes at 9 a.m. and 9 p.m. each day. The appearance of lights and morse signals on isolated hilltops again attracted public attention, and instances are on record of police parties being organised to investigate the plight of persons supposed lost or in distress or, in at least one case, suspected Nazi spies. To allay public alarm and to avoid unpleasant incidents, it was necessary to publicise the activities of the survey party in local newspapers.

From 1940 to 1942 the network was spread over Otago and Southland and across Foveaux Strait to Stewart Island. A third base-line was selected at Riversdale, and inquiries were made about the loan of base-measuring equipment from the empire pool of surveying instruments. It proved difficult to obtain this during the war, and measurement of the three South Island bases was postponed until peace-time conditions had returned. In the meantime, the Riversdale base was measured provisionally, and, in conjunction with lengths carried over from the North Island, was used in a provisional solution of the South Island net to assist in the production of topographical maps for military use.

**Final  
field work,  
1947-49**

Base-measuring equipment was obtained on loan from Tanganyika and standardised at the National Physical Laboratory, Teddington, near London, before use in the field. The three South Island bases were measured in 1947 by a party under the charge of J.P. Arthurs. As the tapes used in the measurement of the North Island bases had not been compared with the imperial standards immediately after use, two of the North Island bases were now remeasured with the new equipment, and the tapes were returned to the National Physical Laboratory for restandardisation after use.

In 1948 twelve Laplace stations\* were selected, for ease of access as well as for their positions in the network, and longitude was determined by D.R. Brenchley at each of these, using time signals transmitted from the Dominion Observatory specially for this purpose. Latitude and azimuth observations were also made at these stations if they had not been made earlier. This work was finished early in 1949, and marked the completion of the field work.

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*\*A Laplace station is one where azimuth and longitude are both observed, and a Laplace equation can be formed, as explained under "Adjustment of the observations".*

## Office work, 1949

All computation of the triangulation was done by the staff of the Department's Head Office in Wellington. Preliminary adjustment of the geodetic network had been done piecemeal under successive Chief Computers E.J. Williams and F.H. Jennings, and reduction of the astronomical observations was carried on whenever there was a lull in other computing work during the war. The final adjustment was done in 1949 under the direction of the Chief Computer, R.J. Owen, as soon as the final base lengths and the Laplace observations were available.

When the adjustment was completed, it was possible to establish the "Geodetic Datum 1949", and to express all the triangulation in terms of a homogeneous system over the whole country. The stations were coordinated on the National Grids, each island being on an independent transverse Mercator projection, which had been selected by H.E. Walshe just before the war.

Several benefits from the new triangulation were immediately obvious. In the first place, the topographical maps could be issued in final instead of provisional form, with no possibility of gaps or overlaps where surveys from different districts met. Again, the development of civil aviation required the computation of air routes from homogeneous data, ignoring all merely local systems, and the Geodetic Datum made this possible.

## Costs

The cost of the field work from year to year has been published in the annual reports of the Department of Lands and Survey, but the cost of office computation has not been given. With some items estimated, the cost of the survey over the 40-year period has been as follows:

	£	\$
Base-lines, 1909-13	3208	6416
Field observations, North Island, 1921-37	52243	104486
Field observations, South Island, 1938-42	16975	33950
Base-lines, 1947	4455	8910
Laplace observations, 1948-49	1770	3540
Purchase of special equipment (approximate)	3000	6000
Computation, 1910-49 (estimated)	15000	30000
	<hr/> 96651	<hr/> 193302

## Extension of first-order control, 1973-74

After the completion of the first-order triangulation in 1949, second-order and third-order nets were observed in many parts of the country, but no further first-order work was contemplated for about 25 years – apart from the establishment of one new station in 1958 to replace one that had been destroyed by erosion. The west coast of the South Island was not covered by the original scheme. Proposals for using the beech forests and for investigation of hydroelectric power schemes showed the need for topographical mapping which would require a control survey as a preliminary, and in 1973 a scheme of extension of the first-order net in the northwestern South Island was drawn up.

Theodolites had changed little in the intervening period, but electromagnetic distance-measuring equipment had been developed and was by this time standard equipment in control surveys. The extension therefore was not pure triangulation but involved distances measured by AGA Geodimeter Model 8. Provisional topographical maps of the region were available, and these were used for drawing up the scheme, without check by reconnaissance on the ground. Even after six hours flying over the region in a Cessna 172, few lines of sight had been verified because of cloud.

The geodetic survey section attached to Head Office had long since been disbanded, and the work of beaconing was done by staff from the Nelson and Westland district offices, under the direction of L.S. Ricketts. Second-order stations and photo control points were beaconed as well as the first-order stations. The country was very

rugged, and helicopter transport to stations was considered essential. A hired helicopter was engaged for about 250 hours in transporting beaconing and observing parties. Two-way radio communication was also used, the equipment being supplied by the New Zealand Forest Service. Beaconing was carried out during three weeks in February, 1974, and the final verification of the scheme was done at the same time. Weather was the main problem, as stations were frequently under cloud. Many of the landings and pick-ups were done in marginal conditions, when the risk of being stranded was great. As it happened, all three of the beaconing parties were stranded on the same day, one party walking out that night, another on the third day, and the last being picked up on the fourth day.

Observation and measurement of the first-order rays were done during three weeks in March, 1974. Five two-man observing parties and two light parties were used, comprised of surveyors and survey assistants from several districts, the surveyors being L.S. Ricketts, C.W. Thompson, J.D. Collett, J.R. Busby, and A.A. Radcliffe. Under optimum conditions the work would have taken four nights. The first week was lost, with rain followed by snow, but weather was comparatively good during the next two weeks. Cloud obscuring a line and blown light bulbs were the only hold-ups which necessitated extra nights on a particular hill, but one station had to be reoccupied because the first attempt at observation had been defeated by the weather.

The average triangle misclosure was just outside the specified limit for first-order observation, but, because of the strengthening of the figure by measured lines, this was accepted. Geodimeter measurement of lines previously fixed by triangulation agreed remarkably well with the earlier values.

Electronic computers had become standard equipment in the period since the computation of the original first-order net, and adjustment and computation of the new work was done using the Elliott 503 computer owned by the Applied Mathematics Division of the Department of Scientific and Industrial Research. Staff who had worked on the computation in 1949 were still employed, but by this time were unwilling to use any method other than the electronic computer.

#### **Final touches**

Extensive second-order and third-order networks have already been observed over large areas of both the North and the South Islands. These networks have utilised existing trig stations wherever possible and have provided the means of coordinating a great many old stations in terms of the geodetic network. Old triangulation has also been recomputed to fit the newer control, but this is not regarded as final; it is styled "fourth-order" and is superseded whenever new field work can be done.

Several proposals have been made for replacing the old meridional circuits by new coordinate regions. These have included one origin centrally placed within each Land District, and belts bounded by defined meridians. Investigations have also been made as to whether one projection could be made to cover the whole country, but it was not found possible to devise a projection with a range of scale sufficiently small to make it suitable for all purposes. When metrication of surveys and maps was begun in 1973, a one-projection coordinate system (the N.Z. Map Grid) was adopted for topographical maps, but for cadastral survey it was decided to retain the meridional circuits with only minor amendment of boundaries in some places. The old "plane" coordinates, however, are superseded by transverse Mercator coordinates referred to the old origins. Uniform coordination was, of course, the main purpose of the geodetic triangulation.

## PART II. FIELD WORK

### II. Reconnaissance and General

#### Nature of country

New Zealand consists of two main islands, the North Island of 114 700 km<sup>2</sup> and the South Island (including Stewart Island) of 150 500 km<sup>2</sup>, separated by Cook Strait which is about 22 km wide at its narrowest part. Both islands are disposed along an axis extending for about 1300 km in a northeasterly direction, with, in the North Island, a secondary axis extending for about 500 km in a northwesterly direction. The country in general is mountainous, the main mountain chain lying along the axis of both islands.

In the North Island, the main chain, extending from Wellington to East Cape, consists of the Taranaki, Ruahine, Huiarau, and Raukumara Ranges, which are mostly above 1000 m in height but do not quite attain 1800 m; these, with the lesser ranges on their flanks, are heavily forested up to about the 1200 m level. The secondary axis, reaching into the North Auckland peninsula, is composed of lower ranges reaching 1200 m at the southern end. Near the junction of the two axes and extending northwards in the angle between them is the volcanic plateau from which rise the intermittently active volcanoes in the middle of the island where the highest peak is over 2700 m. About 130 km to the west, the great bulk of Mt. Egmont, a recent volcano, causes a protruberance in the coastline. Much of the surface of the island, even where it does not attain very great heights, is rugged, with several large rivers flowing in deep gorges. Expanses of level plain are few.

In the South Island, the main mountain chain, extending almost the whole length of the island, is the dominating feature. In the central part of the island it is known as the Southern Alps and lies parallel to the west coast which is about 30 km distant. Many of the peaks exceed 3000 m in height, the highest exceeding 3600 m. On the west coast, the descent to sea level is steep, with many short glaciers and rapidly-flowing rivers, and the lower country is heavily forested. On the eastern side, the distance to the sea is about 130 km, with an extensive piedmont plain, the Canterbury Plain, along the coastal strip. The plain is crossed by several rivers with very wide beds. The volcanic hills of Banks Peninsula form a protruberance in an otherwise simple coastline. In the northern part of the island, the main mountain axis is more centrally placed and somewhat lower, but many subsidiary ranges contribute to the generally mountainous nature of the region, with many peaks above 2000 m. The southwestern part of the island is excessively mountainous, the coastline being penetrated by a number of deep fiords.

In the North Island the greatest density of population is in the farming area extending from Auckland southwards to the Waikato district; in other regions, the population is coastal or is spread along the valleys of the principal rivers. Transport is chiefly by road, as there is an extensive network of main highways; the principal railway is the main trunk between Auckland (the largest city) and Wellington (the administrative capital). In the South Island the population is concentrated mainly along the eastern coastal strip, which also carries the principal railway, that between the chief cities of Christchurch and Dunedin. The main roads generally follow the valleys between the mountain ranges. The main road and railway route between the east and west coasts crosses the Southern Alps through Arthur's Pass at a height of 900 m.

#### Climate and weather

New Zealand lies within the temperate zone, and has a temperate rainy climate with warm summers but no marked dry season. The prevailing winds are westerly, and the main axial ranges rising in the path of these winds make sharper climatic contrasts from west to east than from north to south. Cook Strait, the only large break in the main ranges, is a particularly windy locality. The greatest rainfall is in the ranges and to the west of them, and there are some especially dry areas to the east of the

ranges. The sunshine experienced is about 50% of the possible sunshine, with but little seasonal change.

The seasons did not unduly affect the triangulation. Observations were made in all months of the year, with a preference, in the North Island, for observations during the winter. Weather conditions preventing observation are possible at any time of the year, and the atmosphere is often clearer in winter than in summer. In the South Island, however, the higher peaks were not occupied in winter, although work continued at the lower stations. In the North Island, long delays were occasioned by the almost continuous fog on the high points of the main ranges.

## Reconnaissance

The type of triangulation figure aimed at from the very beginning was a network of polygons and quadrilaterals covering as wide an area of the country as possible. The attainment of this aim has led to the occupation of some very difficult stations, and the scheme has not passed without some criticism. Ross, for example, maintained that chains of quadrilaterals on either side of the main axial ranges of the North Island could have been observed more quickly and cheaply than the network which involved occupying the higher stations in the axial ranges. There can be no doubt, however, that the wide network is a stronger control figure than chains of quadrilaterals would be, and so it was attempted except in the central part of the South Island where it was physically and financially impracticable.

A tentative scheme was drawn up from information available on existing maps, and including old stations wherever possible. The surveyor responsible for reconnaissance then visited each station in the scheme in order to prove the rays or to modify the scheme when proposed rays were found to be not observable. At each station, sketches and photographs of the horizon were made, and an endeavour was also made to recognise as many hills as possible that might be suitable for an alternative scheme. Any amendment to the original scheme could then be made without a return visit to the station. When all the requisite information was available in any particular region, the selection of rays to give a sound figure could be made, having due regard to the cost of occupying the stations. The scheme was then submitted to the Surveyor-General for approval and, after any amendments which he considered necessary had been made, was finalised.

During the reconnaissance, full notes were made as to access and means of transport and communication, camp sites, and means of keeping up supplies to lightkeepers. The programme of operation could then be drawn up, and a chart was made showing the movements of each observer and the lamps requiring to be shown to each station as occupied. Each member of the party was then supplied with a schedule of movements and all the necessary particulars regarding the stations he was required to visit.

## Station sites

Sites for trigonometrical stations range from the "easy" sites where a vehicle can be driven right to the station to the more remote sites which involve one or two days' journey from the nearest road.

To obtain a network covering the North Island it was necessary to occupy many stations in the main mountain chain, four of these being over 1500 m in height and, except for Tongariro on the central volcanic plateau, being the highest stations in the island. These are above the limit of permanent forest, but on many other stations it was necessary to clear the forest from the station site and along some of the rays to be observed. Several of the North Island stations are on isolated volcanic cones, others are merely peaks in more extensive mountain ranges. To obtain a strong net in the southern part of the North Auckland peninsula, several stations were established on islands of the Hauraki Gulf.

In the central part of the South Island, the Southern Alps were avoided altogether, but stations in the main axial ranges were occupied in the northern and southern districts where the net is wider. In Nelson and Marlborough, there are fourteen stations over 1500 m in height, five of them being over 2100 m. In Otago and South-



land, there are eleven stations over 1500 m, although none reaches 2100 m. Access to these stations involved some hard foot-slogging, although packhorses were hired from local settlers wherever possible. On the eastern side of the chain on the Canterbury Plains, the stations were, by contrast, of very easy access, two of them being located on water-towers in towns.

#### Legal access and protection

Stations have been established on Crown land, Maori land, and private land indiscriminately, although, as the main work was done in the 1870s, the greater number of the stations then established would be on Crown and Maori lands. With subsequent alienation of the land, many such stations came to be on private land. The land on which any trig station is situated, notwithstanding any such alienation, is legally Crown land, and the title to land containing a trig station is subject to a right-of-way to and from the trig.\* In a few instances in the early days, the sale plans of Crown land expressly reserved an area of 10 ft (3 m) radius around each trig station, but in the majority of cases Crown grants and titles do not mention trig stations or any right of access to them, and many property owners are unaware of the legal provision. Surveyors have also the legal right of entry upon any land whatever in the execution of their duties.\* This right is enforced only as a last resort; in the normal course of events, the surveyor carrying out reconnaissance is responsible for interviewing property owners and obtaining their consent to the entry of the survey party upon their lands.

In cases where it was desired to establish a new station on private land, the consent of the owners was obtained in writing. This applied also to land belonging to or administered by a local authority, and to Crown land reserved for a specific purpose, such as the wild life sanctuary of the Littler Barrier Island where station Hauturu was established and the railway reserve on which the Culverden base was laid down. In only one case was the location of a station affected by local opposition. At Maungapohatu, the highest point of the range is tapu because of a burial ground and the Maoris would not permit a station to be established there. It was therefore necessary to establish a station some distance to the north of the highest point. An earlier station had been sited to the south of the highest point for the same reason.

Any person who destroys or displaces a trig station (or any other survey mark) is guilty of an offence for which the penalty is a fine or imprisonment.\* The majority of stations have been preserved, not so much on account of legislation as to the fact that they have been placed on hilltops where cultivation or road-making activities would not reach them. In a number of cases, however, it was found that, although old stations were undisturbed, fences had been erected over them or buildings or plantations obstructed the sights, and it was necessary to establish new stations. In settled districts, or on hilltops frequented by tramping parties, it is inevitable that some stations will be lost, mostly inadvertently, occasionally by vandalism. Even loss from natural causes must be reckoned with; in 1958 it was found that station Muarangi had been destroyed by erosion, and a new station was established in a safer position.

#### Station Names

Practice in the naming of trig stations has varied considerably in the past, and stations have been designated by numbers, by letters, and by proper names indiscriminately. Wherever an old station has been reoccupied for new observations, the old

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\*Sec. 9, *Land Act 1885*, re-enacted in later Acts.

\**Trigonometrical Stations and Survey Marks Act 1868*. The right is restricted to the daytime.

\**Trigonometrical Stations and Survey Marks Act 1868*. The current penalty is a fine not exceeding \$400 or imprisonment for a period not exceeding one year.

designation has been adhered to. About 1944, it was decided that all first-order stations should be known by proper names, and names were provided for some stations which had not previously possessed any. A station name is usually the name of the hill on which the station is established, but in some cases the name of a nearby locality has been used. The new names (which may be additional to earlier numbers or letters) were approved by the New Zealand Geographic Board, and the Board also decided several cases of doubtful spelling of Maori names. In some cases, the name now attached to a first-order station is not the name under which it has appeared in earlier reports.

The southern terminal of the Matamata base, in the Mangawhero Settlement, was originally called Mangawhero. There are, however, two other stations of the same name in the district, and in 1948 the southern terminal was renamed Langmuir in memory of the surveyor whose careful measurement of the North Island base-lines marked the beginning of precise geodetic survey in New Zealand. Station B in the Ruahine Range was called Matanganui by Ross, but he appears to have confused it with another station (Matanginui) about 6 km to the northeast. In 1966 station B was renamed Ross.

Where a new station is established very close to the site of an old one, either because the old station is inconveniently sited or because the old ground mark cannot be found, the old designation followed by the description "No. 2" is used, to draw attention to the fact that the new station is not in the same position as the earlier station of the same name.

#### Station marks

In New Zealand, the commonest station mark is a galvanised iron tube, about 760 mm in length and 50 mm in diameter, with a flange at the bottom. The tube is inserted in the ground with the top approximately at ground level or a few centimetres above it, and when the site is readily accessible, the base of the tube is set in concrete. Sometimes a centred brass or stainless steel bolt set in concrete or cemented into rock may be used as a station mark. The instructions for triangulation in New Zealand now provide also for the insertion of a buried mark, such as a bottle, below the surface mark. This may not have been done in the case of some old stations. A surveyor occupying a station now is instructed to search for a buried mark and to insert one if an earlier one cannot be found. In populated areas, where a station runs a risk of being disturbed, witness marks are inserted nearby, so that a disturbed station can be reinstated.

#### Organisation of party

During the period 1923-31, only one surveyor was engaged upon the work, and he was responsible for the reconnaissance and the planning of the network as well as for making the observations. In the first few years, when observations were made to permanent beacons, the only assistants required were those employed in beacon building and station clearing. When night observations were introduced in 1930, the strength of the field party was necessarily increased because beacon lamps had to be attended at several stations simultaneously, even when some of them could be left on time switch. In a report made when he relinquished control of the field work, Ross recommended that the party should consist of two surveyors, one recorder, and six lightkeepers. The senior surveyor would be in charge of the whole operation and would be responsible for the reconnaissance, while the junior surveyor would be principally an observer. If two instruments were available, it would often prove economical if one surveyor could make any necessary astronomical observations while the other carried on with the terrestrial observations; where only one instrument was available, it was not economical to keep a large party waiting while the observer endeavoured to make astronomical observations.

When the triangulation was resumed in 1936, two surveyors were engaged upon the work, one being occupied with observations for most of the time while the other made observations whenever possible in a programme that also included reconnaissance and general supervision. In the South Island, the number of surveyors was increased to three, two being occupied with observations, leaving the senior surveyor free to

devote all his time to reconnaissance and general supervision, and making observations only when necessary to relieve one of the other two. The party included two recorders, a head chainman who also performed clerical and storekeeping duties, and five or six lightkeepers.

#### **Camp sites**

It was found convenient to establish a central headquarters for the geodetic party in the region in which the observers were operating, the location of this headquarters being changed from time to time as the work progressed. For much of the time the headquarters was simply a temporary camp, but in closely settled country it was often possible to rent an old house so that valuable equipment and records could be locked up when the camp was left unoccupied. When working at a particular station, the observer or lightkeeper would camp as close to the station as possible, and this often meant pitching a tent at quite a high altitude.

#### **Transport**

Details of transport in the early days of the survey are not recorded on the file, and the party did not acquire a motor truck until 1927. Until then, it is assumed that the surveyor would travel by railway or service car to the vicinity of the station and would then hire a packhorse or proceed on foot.

In the later stages of the field work the party had three motor vehicles, two light trucks and one heavy truck. The cabs were wide enough to give moderately comfortable seating for two men and a driver. The heavy truck was in the charge of the head lightkeeper, and carried the tools and equipment necessary for station clearing and beacon building. It was found preferable to have all the equipment likely to be necessary moving forward with the trucks instead of being stored in one centre. A special trailer was made to carry the parts of the steel observing towers.

Other transport was arranged as it was required. Packhorses could usually be hired from local settlers. For visiting islands, a launch could be hired from local fishermen. In North Auckland, where several stations are on islands, a scow was chartered. In the 1973 extension of the first-order net, helicopter transport to station sites had become the accepted means of travel – a far cry from the arduous foot-slogging of the 1920s.

#### **General duties of lightkeeper**

Lightkeepers were obtained through advertisements in the press. As a lightkeeper was often required to be absent from headquarters, working without supervision, as long as three months, and as one man's failure to show his light to the observer when required could delay the whole party days or even weeks, it was essential that trustworthy men, who were faithful in the performance of their duties, should be employed. A high standard of physical fitness was also required. Suitable men were obtained without difficulty, and some of them remained with the geodetic party for several years. It was desirable also that a lightkeeper should hold a driver's licence, and the qualified drivers were asked to give instruction and practice to the unqualified men as opportunity offered. Possession of a driver's licence is essential for promotion to the position of head chainman.

On being assigned to a particular station, the lightkeeper was required to cut a track where necessary and to mark it for night travelling. In open country white paint marks on rocks and white calico on poles made good night marks. A top camp was pitched as near to the station as practicable. Inquiries and searches were made for water near the top, and if water could not be found it was carried up from the nearest supply in 9-litre tins. The lightkeeper was responsible for keeping the camp stocked with food and fuel.

Each lightkeeper was supplied with a map or diagram of the triangulation net, a prismatic compass, and a small telescope (which proved superior to binoculars). The lines of sight to other stations were found by visual observation using the map as

far as possible, and compass bearings were used only when conditions of visibility rendered other methods unsuitable. At a station in bush or scrub country all lines to be observed had to be cleared 20 ft (6 m) wide and all growth cut down to a level 10 ft (3 m) below the line of sight. At a station where there was a permanent beacon, back lines were cut so that the beacon would show on the skyline to all other stations.

Additional men could be engaged by the lightkeeper when delay in the progress of the observer would be caused by the lightkeeper's working single-handed at bush-felling or carrying equipment to the station. On difficult or dangerous stations men were required to work in pairs, and if two permanent lightkeepers had not been assigned to such a station the lightkeeper could engage another man locally to accompany him. Dangerous areas, such as precipitous bluffs, in the close vicinity of a station, were required to be fenced off by a suitable timber or rope barricade.

## Field books

The field books used in the geodetic triangulation (except for some used in the period 1923-24 when some of the work was experimental) consist each of 52 pages measuring 203 mm × 140 mm and bound along the short side in clothboard covers. This size proved convenient for field use, and the hard covers gave adequate support for writing. A pocket for loose papers is provided inside the front cover, and a cloth tunnel for holding a pencil is attached to the top edge of the cover. An elastic band is attached to the cover for keeping the whole or part of the book closed.

Four types of field book came to be used, each with a distinctive colour for the cover cloth: base measurement, green; horizontal angles, dark blue; vertical angles and astronomical observations, red; precise levelling, maroon. The use of these books, except those for base measurement, has continued in the second-order and third-order triangulation still being carried out and in the precise levelling which was commenced after the first-order triangulation had been completed. The field books were numbered consecutively as they were issued from Head Office, one series of numbers including all the types of book. When filled, they were returned to Head Office for storing in a fireproof safe. Under later policy, the field books were sent to the appropriate district offices for permanent storage.

Instructions for the making of field book entries do not seem to have been issued during the progress of the first-order triangulation, although some are contained in the technical instructions for second-order and third-order triangulation which have been issued since. The general principles governing survey records would apply. The field book is the original, and the most important, record of the observations; the practice of recording observations in a scrapbook and later copying them into the field book is not permitted by the Department of Lands and Survey. Figures must be clearly and legibly made with a hard pencil, with sufficient pressure to leave a slight indentation in the paper which gives permanency to the record. Erasures are not permitted; where an error has been made, the incorrect figure should be crossed out and the correct figure written in a clear space. In addition to the observations, the field book should contain also a record of all factors which may affect later computation, such as weather conditions and target visibility. Such information may be important when the question of rejecting some observations or of weighting rays unequally may arise, and its recording at the time of observation is especially necessary as the later computation is done by someone who was not a member of the field party.

## Summary sheets

When the observations at one station are completed, a summary of the observations is made on a specially designed form. These were not introduced until the resumption of field work in the North Island in 1936, but they were used after that date for all the observations made earlier, and their use has continued in the second-order and third-order triangulation. The computer then works from the summaries instead of from the field books which are stored away. Four kinds of summary sheet were designed: base measurement (this was a computation form as well as a summary of measurements),

horizontal directions, vertical angles, and astronomical observations.\* Except for the base measurement form (which it is not expected will be required again), each form has three parts. The upper part provides for the particulars of the station occupied, the observer's name, the instrument used, the date of the observations, and the field book reference. The lower part provides space for diagrams (e.g. of eccentric stations) or any relevant remarks. The central and largest part contains the summary of the observations, this part being necessarily different on each type of form.

### Classification of triangulation

According to the degree of accuracy with which the observations are made, triangulation is classified into first-, second-, or third-order. The terms coincide also with the steps in the breakdown of the large triangles of the primary net into the smaller triangles of the secondary and tertiary networks. The following specifications have been adopted in New Zealand:

	Triangle Closure		Length Closure
	Average	Maximum	
First-order	1 "	3 "	1 in 25 000
Second-order	3 "	5 "	1 in 10 000
Third-order	5 "	8 "	1 in 5 000

The limits are narrower than those recognised internationally, the specifications laid down by the International Association of Geodesy (originally due to the U.S. Federal Board of Surveys and Maps) allowing a maximum triangle closure of 8" for second-order and 12" for third-order. In practice, the specified limits are rarely approached. In the earliest geodetic observations, made in daylight with an 8 in. (200 mm) transit micrometer theodolite in the area from the Kaingaroa base to East Cape and south to Hawke's Bay, some of the work lay outside the limit for first-order work, triangle closures occasionally exceeding 5". Being part of the primary network, however, these observations are still regarded as first-order. Later work, done with Wild theodolites at night, was well within the specified first-order limits.

In New Zealand, the sides of first-order triangles average about 50 km, of second-order triangles about 15 km, and of third-order triangles about 5 km. With Wild theodolites, angles are read to 0".1, but the mean of a number of sets is carried to one or two decimal places more than those in the separate observations. The degrees of accuracy adopted for horizontal angles are:

	"
First-order observations, 12 sets	0.001
Second- and third-order observations, 6 or 3 sets	0.01
Less accurate work	0.1

Old triangulation readjusted to fit newer geodetic control is regarded as fourth-order.

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\*More recently, one summary form for horizontal directions, vertical angles, and measured distances, at one station, has been proposed.

### III. Base Measurement

#### Base measuring equipment, 1909-13

Methods of measuring geodetic bases had not been standardised in the earlier 1900s, and each field party could use its own ingenuity in the design of equipment. The equipment used by Langmuir in 1909-13 was made to his own design.

#### Tapes

The principal equipment consisted of three 5-chain (100 m) invar tapes, 0.5 mm thick, two of these being 3 mm wide and the third 6 mm wide. These tapes were not standardised in an official laboratory before or after use; instead they were compared, at frequent intervals during the course of base measurement, with a temporary one-chain comparator laid out with one or other of the imperial standard steel bands in the custody of the Chief Surveyors, these bands having been certified by the Standards Department of the Board of Trade, London, in 1903. During field measurement, the tapes were supported at 50-link (10 m) intervals and subjected to a tension of 15 lb (6.8 kg) at one end. Values of the elastic extension and the horizontal sag on a 50 link length were found experimentally; values of the modulus of elasticity and the coefficient of expansion were supplied by the makers.

The tapes were numbered 01, 02, and 03. Each tape was graduated in chains, the chain length at each end being further graduated in links. All the tapes were graduated by fine engraved lines drawn right across on the material of the tape. These lines in many cases were not precisely at right angles to the length of the tape, so that the graduations on one edge were not in accord with those on the other; each edge therefore presented the features of a separate tape, and was treated as such, and standardised accordingly. The ends of each tape were marked, one A and the other B, and the two edges of each tape were also distinguished by the letters A and B, all engraved on brass sleeves soldered to the tape.

#### Measuring heads

When the tapes were used in base measurement, the greatest number of measurements were of approximate 5-chain (100 m) lengths, these having been marked out with temporary ground marks in a preliminary operation. At one end of a 5-chain bay, a tripod was erected over the ground mark; this carried a sliding adjustment gunmetal top, with a further fine centring movement obtained by two pairs of opposing screws. A capstan screw in the centre of this top supported the tape when in use, and an inset of lead provided a surface for making a zero mark with which a chain graduation of the tape could be aligned.

At the other end of the bay, a 5 inch (130 mm) Troughton and Simms theodolite was erected over the ground mark. This was equipped with a brass scale, 1 link (25.4 mm) in length and divided to 0.02 of a link, attached to the end of the horizontal axis of the telescope, and with a central zero graduation coinciding with that axis. The brass scale carried a sliding steel scale, 0.1 link in length and divided to 0.001 of a link, which could be made to coincide with any of the 0.1 link spaces on the brass scale.

As a chain graduation on the tape was aligned with the mark on the head of the tripod at one end of the bay, a reading was taken only at the theodolite at the other end, where the small amount greater or less than the full chain (or any other graduation if the bay were not of 5-chain length) was measured on the scale attached to the theodolite. A strong magnifying glass was always used, and the 4th decimal of a link was estimated on the sliding steel scale. With the tape coming from the left as the observer faced the instrument, the link reading was always that of the mark on the tape immediately to the left of the central zero on the brass scale. If the tape

graduation which fell within the length of the brass scale was to the left of the zero, the full reading would be, say, 60.2855, the decimals of a link read from the left-hand portion of the brass scale being added to the tape graduation. If the tape graduation was to the right of the zero, then the link value would be that of the graduation next to the left, outside the range of the brass scale, and the decimals of a link to be added to it were read from the right-hand portion of the brass scale, the full reading being, say, 59.7145. If the tape came from the right, the method of reading was relatively the same, although the scale figures would be upside down.

#### Tension apparatus

Measurements were always made with the tape under a tension of 15 lb (6.8 kg) applied at the theodolite end of a bay, the tape being clamped to a stand at the other end. The tension was applied by means of the straining apparatus shown in the

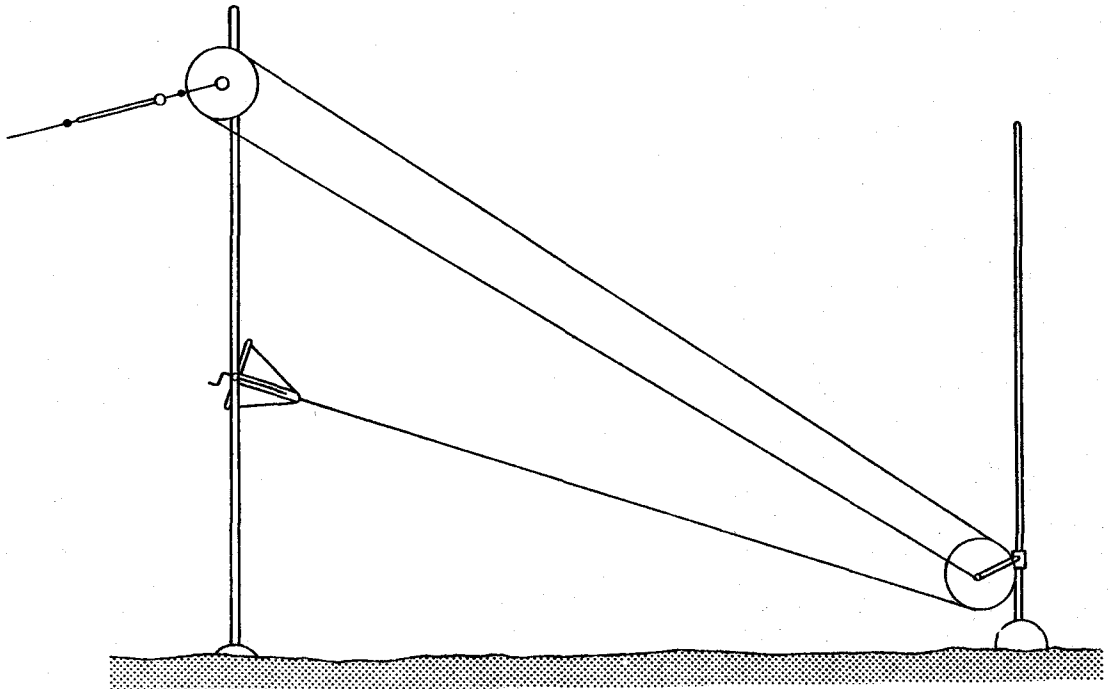


diagram. One end of the tension wire was attached to the frame of a fixed pulley near ground level and on the line of the base; the wire was then passed under and over a second pulley, also on the line of the base but at a higher level and about 2 m nearer to the theodolite. From the second pulley the wire was carried back to the first pulley, which it passed over and under, and was then clamped to the slow-motion screw of a tension frame which moved on the shaft carrying the upper pulley. A spring balance was attached to a rider on the shaft carrying the upper pulley, and the measuring tape was clamped to this balance. Tension was increased by raising the tension frame, until the spring balance registered the desired tension. The balances used were tested frequently, and when evidence of a weakening of the spring became apparent, a corresponding correction was applied to the measured length of the line.

#### Measuring procedure

Six men were needed for the measurement of a bay, comprising one contact-observer at the tripod end, one tape and tension adjuster and one scale reader at the theodolite end, and three men for lining in supports, placing tapes properly on the supports, and assisting to carry tapes forward.

At the commencement of measurement for the day, a tripod was centred by plummet over the terminal ground mark, and a theodolite was set up about 3 m from it on a line through the terminal mark and at right angles to the line of the base. The terminal

mark was then intersected by the wires of the theodolite and a vertical line was ranged up to the tripod-head where a fine needle-point mark was made on the lead insertion to serve as the zero mark for the first bay. The theodolite was then centred over the ground mark at the forward end of the bay, and readings of the vertical angle to the contact-mark on the rear tripod were taken.

When the vertical angles had been recorded, the support-stands, 50 links (10 m) apart, were ranged into line, and the spring supports were run up or down the support-rods so as to be placed on the grade between the horizontal axis of the theodolite and the contact mark on the rear tripod. Both the theodolite and the rear tripod were centred on the base-line itself, but the actual measure made with the tape was that of a line from the rear contact mark to the outer end of the horizontal axis of the theodolite, which was 0.372 link (75 mm) from the base line. The support-stands were therefore placed on a line parallel to the measure being made and about 40 mm from it, so that the stand at the midpoint of the bay was on the base-line. The corrections for eccentricity of the line measured were obtained by inspection from a table computed for the purpose.

The first tape was then lifted and placed on the supports, being carefully watched to see that it lay flat, without twists, throughout. The observer at the rear tripod then clamped the end of the tape to a slow-motion screw on a support-stand to the rear of the tripod. On a signal by whistle from the rear observer, the tension-adjuster at the forward end then pulled the tape taut and clamped it to the balance on the tension gear. He then adjusted the tension, first rapidly by sliding the tension-frame upwards on its support, and then accurately by the slow-motion handle. The rear observer, on receipt of a signal, then made the final adjustments to bring the tape graduation into coincidence with the zero mark on the measuring-head. The reading of the length of the bay was then made by the observer abreast of the theodolite. The tape number, the edge and the end used, were all noted in the field book. At least four readings were taken, the tape being lifted from the supports and lowered again before each new reading. The last reading of the series was made by the tension-adjuster as a check. The whole operation was then repeated with the second tape.

While the measurement of the first bay was being made, one man erected a tripod at the forward end of the second bay, so as to be ready for the measurement of the second length. As soon as the first bay was completed, the two tapes were lifted by four men and carried forward to the next bay. The theodolite remained in position, serving now as the rear point of the bay. In this case, if the A end of the tape had been used in readings at the scale for the first bay, the B end would be used in readings for the second bay, so that the A and B ends were read alternately as base measurement proceeded.

At the conclusion of the day's measurement, if the final mark was not a permanent ground mark, the mark on the finishing tripod was transferred to the ground, being marked on a strip of lead tacked to the top of a stout peg driven about 600 mm into the ground and with the top 50 or 75 mm below the surface. All such pegs were protected from injury by carefully covering them until the measurement of the section had been completed.

Each base was divided into sections of convenient length for measurement, the section terminals being marked by permanent ground marks which were intended to serve as standard reference points for any future surveys in the locality. Each section was measured on two separate occasions, using each of two tapes; on the second occasion, a different edge of the tapes would be used, so that the procedure was equivalent to four measurements. On certain difficult sections, this was increased to six measurements, or sometimes eight.

#### **Wairarapa base**

The base was selected by H.J. Lowe in December, 1908, as being suitable for the first base near the southern end of the Wellington-Taranaki triangulation. The length of the base is 13.0 km, at an average height of 52 m above sea level, on the Wairarapa Plain. The southern terminal, Bidwill, was an existing station of a very early triangulation scheme; the northern terminal, Woodside, was newly established



by Lowe. The line was divided into nine sections by the placing of eight intermediate marks, iron tubes set in concrete, with brass plugs having fine centre holes set in the tops of the tubes. The terminal marks were similar, but in addition sub-surface marks were provided, brass plugs set in bluestone, the whole set in concrete.

From Woodside, alongside the Wellington-Napier railway, the line runs on an azimuth of  $177^\circ$  and for the most part slopes fairly gently downwards. In parts it intersects closely occupied country, and 41 barbed-wire fences had to be crossed during the course of the measurement. The northernmost 4 km, Sections 1, 2, and 3, are of a very shingly nature, there being in many places little or no soil overlying the loose stones, which increased the difficulties of setting up both theodolite and tripod. Sections 4 and 5 are mostly free from shingle, and are good measuring country. Section 6 is a surface-dry swamp, but the subsoil to a depth of 2 m in places consists of soft peat, through which heavy stakes of sawn timber,  $125 \times 100$  mm, were driven down to a firm foundation to provide stable supports for both theodolite and tripod legs. Sections 7 and 8 are good measuring ground, but Section 9, the southernmost mile, is for the greater part irregular, with steep rising and falling grades, in some cases approaching  $15^\circ$  in inclination. A satisfactory base could have been obtained without this last section, but it was considered desirable to extend it to Bidwill, which was one of the main stations in the secondary triangulation scheme as it was then envisaged.

Measurement of the base occupied 47 days, from March to May, 1909. The Wairarapa district is among the most windy in New Zealand, and, although the measurement was carried out in what was considered the most favourable season of the year, much time was lost in waiting for suitable weather. On Section 9, owing to the prevalence of wind, it was found necessary to follow the undulations of the country closely, so that many bays were shorter than the usual 5-chain lengths. Great care was taken in measuring the vertical angles; where these were large, eight readings were taken with the instrument in direct and reversed positions, particular attention being paid to the level bubble readings. As an additional check on the measurement of Section 9, an auxiliary base of about 2.2 km was measured on the plain, this auxiliary base forming one side of a triangle which included, as a second side, all but one chain of Section 9. The length of Section 9, as computed from this triangle, differed from the direct measurement by 0.0061 link (1.22 mm), or 1 part in 170 000, which was satisfactory verification.

**Eltham  
base**  
(in earlier  
reports  
sometimes  
called Okaiawa  
base).

This base was first suggested by H.M. Skeet in 1897 when he was carrying out triangulation of the Taranaki district. As no better site had since presented itself and as a base somewhere in this region was desirable, it was in 1909 approved as a base-line of the secondary triangulation. The length of the base is 16.0 km, at an average height of 180 m. The terminals were existing stations, Eltham in the north and station B, since named Karimoi, in the south. The ground marks at these stations were replaced by new marks similar to those used at the Wairarapa base, and intermediate marks were iron tubes set in concrete. The base was divided into ten sections, but in addition to the iron tubes marking the ends of these sections, tubes were also placed on public roads crossed by the base-line.

From station Eltham, locally known as Birk's Hill, about 2 km west of the town of Eltham, the line runs on an azimuth of  $206^\circ$  and with a general slope downwards. In detail, however, the country is exceptionally rough for base measurement, with many streams and gullies on the line. In measurement across the gullies, additional height had to be provided for support-stands, some of these being up to 5 m above ground level. In addition to the natural roughness of the line, artificial obstacles such as barbed-wire fences and large boxthorn hedges were frequent, 120 fences of all descriptions having to be crossed in the course of the measurement. During the preparatory work, stiles were built over 40 hedges with ditches and banks, and 400 m of plank footways were required to cross streams, ponds, and swamps. Measurement of the base occupied 46 days, from April to June, 1910.

### Matamata base

The site was selected by R.T. Goulding after examination of many suggested sites including those along the railway from Hamilton to Morrinsville, where there proved to be too much swamp land to allow of proper measurements being made. The base is 11.0 km in length, at an average height of 64 m above sea level, and for the most part on level ground. Both terminals were newly established stations, the northern one, near Waharoa railway station, being called Waharoa, and the southern one since being renamed Langmuir. At Waharoa, the station mark was similar to those inserted on the Wairarapa and Eltham bases; at Langmuir, the ground mark consisted of a brass centre plug in a stone block, set in concrete, and a concrete pedestal, 2 m high, was erected over this, with another brass plug set in the centre of the top of this pedestal. Langmuir explained the erection of this pedestal as follows: "Owing to curvature and the configuration of the ground along the line I had some trouble in finally deciding on the position to be adopted, as from either end only a portion of the 27-ft signals, as now erected, at the other end can be seen under the best conditions. From the top of the 7 ft high concrete mound at the southern end perhaps five-eighths of the signal at the northern end can be seen in the early morning for from one to two hours after daybreak. From the ground at the northern end about one-third of the signal at the southern end can be seen during the same early hours, disappearing altogether before 8 a.m., as refraction lessens and the air becomes unsteady owing to the rising heat. The position of the line and the height of the necessary mound at the south end were finally decided upon after experimenting with sun-flashes from a point about 15 ft above the ground at the northern end, viewed from a temporary staging 9 ft high at the southern end."

The line runs on an azimuth of 163°, passing through the Matamata Settlement which, at the time of the original measurement, was mostly in grass or under crop. The country was closely subdivided, and 64 fences and hedges had to be crossed. The base was divided into eight sections, and only on the southernmost section are there any steep grades, the greatest inclination being 23°. Measurement of the base was made in 22 days, during December, 1910, and January, 1911.

### Waitemata base

The site was selected by Langmuir in 1911, as it was considered desirable that a base should be measured somewhere in the locality where the triangulations from the north and the south converged to meet on the Auckland isthmus. The base is 8.4 km in length, being the shortest of the geodetic bases, and is at an average height of 37 m above sea level, about 16 km west of Auckland city. The western terminal, Kumeu, was an existing station; the eastern terminal, Waitemata, was newly established by Langmuir. From Kumeu, the line runs on an azimuth of 103°, and the country along the line is very irregular, covering a range of 82 m in height, the steepest grade being 11°. The main north road and railway line are both crossed by the base-line. The line was divided into five sections for measurement, the terminal and the intermediate marks being similar to those used on earlier bases. Measurement occupied 21 days, from June to August, 1911.

### Kaingaroa base

The base was selected by Langmuir because of its central position in controlling triangulation in five meridional circuits. It is the longest of the geodetic bases, being 18.3 km in length, and at an average height of 550 m above sea level, on the Kaingaroa Plains. The area is now covered by a vast plantation of pines, but at the time of measurement was open country in fern and scrub. Both terminals were existing stations, Ahiwhakamura, near the present Kaingaroa forestry camp, in the northeast, and Kaingaroa in the southwest. The line runs on an azimuth of 38°, about 11 km of the central portion being in great part nearly level, and fair to good chaining country. Both ends of the line are exceptionally rough for base-line work, with steep grades up to 36° in inclination, Langmuir noting that Section 8 was the most difficult in all the five bases measured. The base was divided into nine sections, the terminal marks and the intermediate marks being iron tubes set in concrete.

Measurement of the base occupied 49 days, in January, 1912, and February to April, 1913. A marked difficulty of the Kaingaroa base, and source of considerable expense and delay, was the great scarcity of water, the whole supply for the camp, including

horses, having to be carted about 6 km. As an additional check on the measurement of Sections 8 and 9 at the southwest end of the base, an auxiliary base of 2.4 km was measured as one side of a triangle of which Sections 8 and 9 formed a second side. The difference between the computed and the measured lengths of the sections was 0.0539 link (10.8 mm), or 1 part in 267 000, but Langmuir stated that the results of the direct measurement were much more reliable.

#### Base-measuring equipment, 1947

The equipment used for the measurement of the South Island bases and the remeasurement of two North Island bases was the "Macca" base measurement equipment designed by G.T. McCaw in association with Cooke, Troughton and Simms, Ltd. It was obtained from Tanganyika on loan from the empire pool of surveying instruments. A detailed description of this equipment has been published by Hotine,\* and the "drill" adopted in New Zealand was largely based on Hotine's recommendations, although the allocation of duties was somewhat different owing to the absence, in New Zealand, of the seemingly inexhaustible supply of cheap native labour that had been used in Africa.

#### Tapes

Invar tapes, compared with the standards of the National Physical Laboratory, Teddington, England, both before and after their use in the field, were used. They were of 3 mm × 0.5 mm cross-section, four being 100-ft (30 m) tapes and one a 20-ft (6 m) tape. Two of the 100-ft tapes were kept as field standard tapes, the other two being the working tapes. Comparisons between the field standard tapes and the working tapes were made three times at each base, before the first measurement, between the first and second measurements, and after the second measurement.

The greater part of each 100-ft tape, a length of 99.6 ft, was ungraduated. At each end, a length of 0.4 ft was graduated in 200 divisions, an estimated tenth of a division being 0.0002 ft. Every tenth graduation was numbered, the numbers in each case reading from 0 to 20 outwards towards the end of the tape, so that a measured length between two marks approximately 100 ft apart was 99.6 ft plus the sum of the measurements on the two end scales. This system of numbering is designed to avoid booking plus and minus readings, with consequent risk of errors in reading, booking, or summation. The 20 ft tape was graduated throughout its length in tenths and hundredths of a foot.

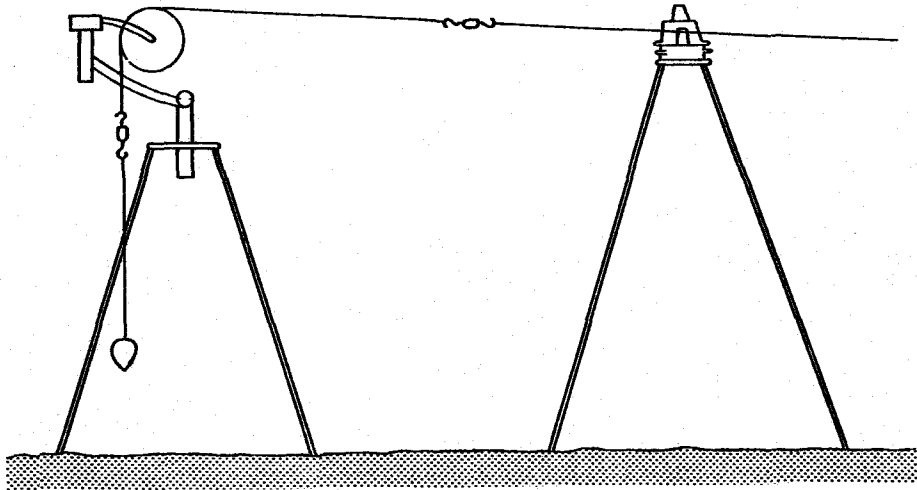
#### Measuring heads

At each end of the bay to be measured a tripod carrying a special measuring head was erected. The mark on this head which defined the end of the bay, and against which tape readings were made, was engraved on a flat polished brass boss, one half of which had been cut away to allow the tape to lie at the same level as the mark. The boss was provided with levelling, centring, and rotational adjustments so that it could be brought accurately onto the line of the base, and the edge of the flat part of the boss aligned with the direction of the base. A lens attached to the measuring head could be swung out of the way while the tape was being brought into position, and returned to its working position for reading the scale on the tape.

#### Tension apparatus

At each end of the bay, in front of the leading measuring head and to the rear of the rear one, a trestle carrying a pulley was erected. The end of the tape was attached, by means of a double swivel-hook, to a cord which passed over the pulley, a 20 lb (9 kg) weight being attached to the other end of the cord by another swivel-hook. During measurement the tape was allowed to hang free under the tension exerted by the 20 lb weight at each end. The supports of the pulley were free to turn about horizontal and vertical axes so that the pull of the tape would bring the pulley into the vertical plane of the base. Screws were provided for raising or lowering the pulley

\*M. Hotine: *The East African arc. IV. Base measurement.* Empire Survey Review, 3 (1935):203-218.



and for moving it horizontally at right angles to the base, so that the tape could be adjusted to touch the mark on the measuring head. The position of the pulley trestles was arranged so that about one-quarter of the length of the cord hung between the pulley and the straining weight and about three-quarters lay between the pulley and the tape. The swivel-hooks prevented any torsion in the cord from causing the tape to twist.

#### Preparation for measurement

The base-line was first divided into approximate 100 ft lengths by temporary marks, usually nails driven through pieces of white cloth. A 100 ft setting-out wire was used for this purpose, and alignment was done by eye, since the positioning of the marks was not required to be precise. Bays shorter than 100 ft, to be measured with the 20 ft tape, were put in to close onto a terminal mark or a permanent intermediate mark where the section length was not exactly divisible into 100 ft lengths, or where a 100 ft bay would be awkwardly placed because of obstacles or infirm ground.

A tripod carrying a measuring head was then erected over each ground mark, about six or eight tripods being in position at any one time. The measuring heads were accurately aligned on the base, with the aid of a small telescope placed over the rear base terminal or on the last measuring head which had been correctly aligned, and sighted on the next forward permanent mark or on the beacon at the forward base terminal. The difference of height between adjacent measuring heads was determined by levelling as each tripod was set up, readings being taken on two staves, one graduated in feet and the other in links, the difference in links, converted to feet, being used as a check on the difference in feet.

#### Measuring procedure

The field party consisted of two surveyors, Arthurs and Gough, six survey cadets, and three chainmen. In addition, a computer and another survey cadet attended to office work in camp, but, when it was desired to take full advantage of fine weather in speeding up measurement, they were often called upon to assist in the field work.

Seven men were required for the measurement of a bay: two observers (survey cadets), two men to place the straining trestles (chainmen), two to carry tapes (survey cadets), and the booker (one of the surveyors) who supervised the operation from near the midpoint of the bay. In addition, three men were required for placing tripods and levelling ahead of the measurement, and for carrying rear tripods forward as the measuring party advanced. When not in use for measurement, the tapes were hung from spring-hooks attached to poles, and as the tape-holders moved forward from one bay to the next, the booker would hold the tapes near their middle. The trestle men carried their trestles forward, and the observers carried the canvas buckets containing the straining weights, cords, and swivel-hooks.

On arrival at the new bay, the tape-holders stationed themselves abreast of the measuring heads, and the trestle men placed and adjusted their trestles. The first alignment was made by sighting over the pulley and, with a little experience, it was found that the height at which a pulley should be placed so that the tape should lie at the level of the measuring head could be judged fairly well, leaving only minor adjustments to be made when the tape was attached to the cord. When the trestles were firmly in position, the trestle men reported that they were ready, and the buckets containing the weights were placed on the ground beneath the pulleys.

The rear observer then unhooked his end of the tape, the upper of the two tapes on the pole, and called "Unhooked", whereupon the forward observer unhooked his end of the same tape and called "Unhooked". The observers then took the tape to the bay, the booker standing near the middle of the bay assisting with this. The trestle men had meanwhile lifted the weights and passed the cords over the pulleys, and the tape was then hooked onto the cords. The trestle men remained holding the weights until the rear observer called "Weights", whereupon the weights were lowered gradually and let go. The observers retained a grasp on the swivel-hooks until they felt that the tape had taken the strain, when they let go carefully. The trestle men then made any necessary adjustments to the positions of the pulleys to bring the tape against the marks on the measuring heads. When the observers were satisfied that everything was in order, they swung the lenses on the measuring heads into position for reading, and called "Ready". The booker helped to steady the tape and also checked that it was lying without twists or kinks.

Then, having heard both observers call "Ready", the booker called "Measure", and each observer estimated the tenth of a scale division in line with the mark on the measuring head. The booker then called "Read", and the rear observer called his reading. The estimated tenth of a division was called first, as being the only value which, owing to subsequent creep of the tape, could not be verified by inspection, and as being the value on which the observer had concentrated his attention while waiting for the order to read; the unnumbered graduation was called next, and the numbered graduation last, so that the whole reading was called, and booked, backwards. When the rear observer had finished calling, the forward observer then called his reading, which had also been made on the command "Read". The booker entered the two readings and their sum.

The forward observer then gave the tape a gentle pull forward, and called "Ready" when it had steadied. A second pair of readings was then made as before, and, after another pull on the tape, a third pair. The booker checked that the three sums did not differ among themselves by more than three units in the last figure; if they did, he would require additional readings to be taken. When the booker was satisfied, the tape was then returned to its spring-hooks by a reversal of the former procedure. A thermometer was carried on each pole, and the observers then called the temperature readings. The booker also carried a thermometer, and the mean of the three readings was taken as the temperature of both tapes for that bay. The measurement procedure was then repeated with the second tape.

For field book record, bays were numbered continuously from one end of the base to the other in the direction of measurement, so that, if any bay required remeasurement, the second set of readings would be recorded under the same bay number. Each base was measured in two directions, but the bays for the second measurement were not identical with those of the first measurement (as the ends of the bays were defined by the marks on the measuring heads, not by the ground marks) and they were numbered independently. The observers changed ends after ten bays, to eliminate any constant errors due to personal bias. The forward observer thus became the rear observer, but he continued to tug the tape after the first and second readings, so that over the whole base the tape was pushed and pulled an equal number of times. Duties among the field party were rotated after the completion of a section between permanent ground marks.

As the measuring party advanced, tripods at the rear were carried forward and re-erected by the aligning and levelling party. Until the measurement of any bay was completed, the tripods defining the previous bay were left in position, lest any accident to the rear tripod of the bay under measurement should necessitate remeasurement of the earlier bay. If it was necessary to cease work before a permanent

ground mark had been reached, a peg was inserted beneath the last tripod and the position of the mark on the measuring head was transferred vertically to a tack in the peg by means of theodolite observation.

The speed of measurement averaged about four minutes per bay, and the average daily distance measured was about 1.2 km. The maximum number of bays measured in one day was 82 (2.6 km) on the Waitaki base. Wind was troublesome, and it was not possible to obtain satisfactory results in any wind stronger than a very light zephyr. Measurement began at daybreak, to take advantage of the usually calm conditions in the early morning, and work was often stopped by wind in the early afternoon. On some occasions, it was possible to use a 2 m high canvas screen, 33 m long, rigged between two trucks, to shelter the tapes from light cross winds, but because of fences and roads to be crossed, it was not possible to use this screen consistently.

#### **Field computation**

A computer was attached to the field party, so that computations could keep pace with measurement. A special computation form had been printed, and the necessary field book data were entered on this, and the entries checked by a surveyor or a survey cadet, before the field books were forwarded to Wellington for safe custody. The computations of the length of each section of the base, with all the corrections except that for restandardisation of the tapes, were then made by the computer and checked by a survey cadet. Many of the corrections could be read from tables, in critical form wherever convenient, constructed for the purpose. A calculating machine, adaptable to hand or electric operation, was available, and could be operated electrically at those bases where a vacant farmhouse was used as a survey camp.

#### **Riversdale base**

The site was selected by W.G. Nelson in 1940, on the Waimea Plain near the village of Riversdale. The base is 11.4 km in length and lies at an azimuth of  $123^\circ$  on fairly level ground at an average height of 140 m. The terminals were newly established stations, called West Base and East Base, marked by iron tubes set in concrete; intermediate marks, dividing the base into 50 chain (1000 m) sections, were also iron tubes in concrete. The line lay across cultivated land, and many post-and-wire fences had to be crossed, as well as several roads and a branch railway. Measurement occupied 19 working days in February and March, 1947, the period including an interruption of a week while several survey cadets sat for examinations. The weather was mostly fine and much of the measurement was made in hot sunshine.

#### **Waitaki base**

The site was selected by W.G. Nelson in 1939, on the wide floodplain of the Waitaki River, near its mouth. The base is 12.1 km in length, and lies at an azimuth of  $100^\circ$  at an average height of 70 m. The land is fairly level, but the soil is stony and occasionally gave difficulty in setting up tripods. The terminals, Awamoko and Waitaki, were newly established stations; they and the intermediate marks at 50-chain (1000 m) intervals were marked by iron tubes set in concrete. The line crossed many post-and-wire fences, but was free from other obstructions. Measurement occupied 19 working days in March and April, 1947, the weather during this period being mostly fine, often beginning with cold, misty conditions in the early mornings but becoming sunny and hot later in the day. Work was often stopped by wind in the afternoon.

#### **Culverden base**

The site was selected by T.W. Preston in 1938, alongside the Christchurch-Culverden railway, from near the Hurunui River to near the Pahau River, and 25 links (5 m) to the southeast of the centre-line of the rails. The southwestern portion of the base for rather more than half of its length was adjoined on both sides by a pine plantation. The base is 11.1 km in length, at an azimuth of  $61^\circ$  and at an average height of

220 m; the line slopes fairly gently downwards towards the northeast. Part of the line is over rough shingle country which caused some difficulty in obtaining rigid tripod set-ups. Both terminals were newly established stations, Balmoral and Pahau, and were marked by brass rods set in concrete. The intermediate marks, iron tubes in concrete, were placed at 10-chain (200 m) intervals, but only the marks at 50-chain intervals were fixed by measurement with the 100 ft tapes. Measurement occupied 17 working days in April and May, 1947, the weather being mostly calm during the first measurement, although strong winds delayed the second measurement.

#### **Remeasurement of Matamata and Waitemata bases**

Because of a doubt as to the standardisation of the tapes used in the original measurement of the North Island bases — a comparison with the imperial standard being made several years after the field work — it was decided to remeasure two of these bases with the "Macca" equipment before the tapes were sent to the National Physical Laboratory for restandardisation.

The Matamata base was remeasured in 25 working days in June and July, 1947, the work being interrupted by rain and wind on several occasions. Housing development near the town of Matamata had encroached on the line since it had been first laid out, and it was necessary to deviate from the old line in three places to avoid obstructions. This was done by erecting a right-angled triangle upon each section of the base which could not be directly measured, and measuring the hypotenuse and the offset from the base. It was found that in some places the steep slope of the base-line approached the limit for the equipment, and difficulty was experienced through the tendency of the tape to run downhill.

The Waitemata base was remeasured in 21 working days in July and August, 1947, the work being interrupted fairly often by wind and rain. Parts of the line had been obstructed by buildings, though to a lesser extent than on the Matamata base, and it was necessary to make two small deviations from the old line. In other cases, it was decided to overcome the obstructions rather than to deviate from the line; a pig-sty was moved bodily out of the way, and the tapes were threaded through a hole made in the wall of a hen-house.

## IV. Horizontal and Vertical Angles

### Theodolites

As the period during which angular observations were made extended from 1923 to 1942, a period during which considerable advances were made in the design and construction of theodolites, it is not surprising that different types of instrument were used, with a consequent variation in the standard of accuracy. From 1923 to 1928, an ordinary transit theodolite was used, with exposed metal circles each read by two independent micrometers at opposite ends of a diameter. From 1929 to 1942, modern double-reading theodolites were used, with enclosed glass circles internally illuminated, the optical system permitting images of the graduations at opposite ends of a diameter to be seen simultaneously and a mean reading of the two to be obtained from one micrometer reading. This system has since become familiar to surveyors, even on cadastral survey work, and the older style of theodolite would no longer be used on geodetic triangulation.

### Troughton and Simms 8-inch transit theodolite

Two of these instruments were ordered in 1911 and were made to the Surveyor-General's specification which included "to be made with all the latest improvements." They were numbered 218 and 219, and were examined at the National Physical Laboratory, Kew Observatory, in 1912 before delivery to New Zealand. No. 218 was used on triangulation observation from 1923 to 1928.

In this instrument, the telescope was of focal length 400 mm and aperture 50 mm. Two inverting eyepieces were supplied, of magnification  $\times 20$  and  $\times 30$ , and also a diagonal eyepiece of magnification  $\times 30$  for the observation of stars. The telescope could be transited at the eye end only, so that the standards could be made as low as possible. Two diaphragms were made for hairs, the first with one horizontal hair and two vertical hairs 20" apart, the second with one horizontal hair and two oblique hairs. Another diaphragm was made with lines cut on glass, one horizontal line and five vertical lines arranged for star observations for time.

The horizontal and vertical circles were each of 200 mm diameter and divided to 5'. Each circle could be read by two micrometers to 1" or by estimation to 0".1. The levels fitted to the base-plate and to the micrometers, and the striding level, were each divided to 8".

### Wild T3 theodolites

Two of these instruments, numbered 77 and 84, were purchased in 1928, and replaced the Troughton and Simms transit theodolite on triangulation in 1929. In each instrument, the telescope was of 75 mm aperture, and the eyepiece magnifications were  $\times 24$ ,  $\times 30$  and  $\times 40$ . The diaphragm, engraved on glass, consisted of one horizontal line with a single vertical line above this and two vertical lines below it. This arrangement proved satisfactory for a luminous signal, but hid too much of an opaque beacon because of the closeness of the two vertical lines. A new diaphragm, made by the manufacturer to Ross's specification, was fitted to No. 84 and proved satisfactory. This consisted of one horizontal line with two short vertical lines 20" in length and 20" apart, the horizontal collimation being midway between these. At a distance of 60" from these lines was a 30" gap in the horizontal line, this gap being used for pointings in altitude.

The measurement circles were of glass, the horizontal circle being of 135 mm diameter and graduated to 4', the vertical circle of 90 mm and graduated to 8'. Each circle could be read by micrometer to 0".2 or by estimation to 0".1. The levels were divided to about 6" or 7".



### **Tavistock theodolite**

A geodetic Tavistock, numbered 38116, made by Cooke, Troughton and Simms Ltd., was used for observations at some South Island stations from 1938. The telescope was of aperture 50 mm and magnifications  $\times 20$  and  $\times 30$ . The circles were of glass, the horizontal circle being of 125 mm diameter, the vertical circle of 70 mm diameter. Both circles were divided to  $10'$  and read by micrometer to  $0''.5$  or by estimation to  $0''.1$ . The peculiarity of the Tavistock is that the optical system permits two lines, one at each end of a diameter of the circle, to be brought into focus in the same field of view and moved by turning the micrometer head until they are equidistant from a fixed mark between them. The reading obtained in this position is the mean reading required.

### **Tests of theodolites**

No special tests of the accuracy of the theodolites were undertaken in New Zealand, and the practice of obtaining National Physical Laboratory certificates was abandoned, as the standards set by the laboratory for this class of work were too low to be of value. The results of tests made overseas, especially the work of Rannie and Dennis on the performance of Wild theodolites in Canada,\* were studied and observing procedure was planned in the light of such tests.

At the Culverden base, when three surveyors and both Wild and Tavistock instruments were in camp together, the opportunity was taken of comparing the two instruments in the field, and each surveyor observed 12 sets of directions with each instrument at Pahau. The results showed that the two instruments were of comparable accuracy, with no significant difference in the probable errors obtained. However, as the weight of the Tavistock in its carrying case (26.5 kg) was more than double that of the Wild (11.5 kg), it was kept for the more easily accessible stations.

### **Instrument and lamp stand and observer's platform**

The tripod supplied with the theodolite was not used after the first few years of the survey. It was found more satisfactory to build an instrument stand over the station mark, this instrument stand serving also as a lamp stand when required. The legs of the stand were of 100 mm square timber, about 2 m long and sunk about 1 m in the ground. Even on rock sites, a crowbar was used or holes drilled so that the legs could be sunk as far as possible. The inside of each leg at ground level was about 250 mm from the station mark. The sides of the stand were well braced with 100 mm  $\times$  50 mm timber, and the top was a level 305 mm  $\times$  305 mm  $\times$  25 mm board, with a central aperture placed vertically over the station mark.

A triangular platform, made from 300 mm  $\times$  25 mm planks was placed around the instrument stand with at least 25 mm clearance of the legs. This platform was supported, at about 150 mm above the ground, on beams of 75 or 100 mm square timber which were themselves supported on blocks 1 m or more away from the legs of the stand. The observer's weight, as he moved around on this platform, therefore did not disturb the instrument stand. Additional timber to increase the height of the stand for a tall observer was kept ready, as it was essential that the observer stand erect while observing. Any stooping during observations over a long period would inevitably produce a crick in the observer's back which could not be conducive to the best results towards the end of the evening's programme.

### **Instrument shelter**

The instrument in use was always protected by a canvas observing shelter. At a station where a permanent beacon had been built, the canvas was fitted around the sides of the beacon. At a station without a beacon, it was erected on a frame of poles or of tubular steel. This frame occupied a square of about 2 m side at ground level, and narrowed to a square of about 1.5 m at roof level. The canvas sides of the shelter terminated above, at instrument level, in a number of overlapping flaps, any number of which could be opened for observations. The roof was pyramidal, and

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\*J.L. Rannie and W.M. Dennis: *Improving the performance of Wild precision theodolites*. Empire Survey Review, 3 (1935): 2-4; and other papers there referenced.

each of its four sections was provided with a flap which could be opened for star observations.

## Observing towers

At stations in level country or in settled districts among buildings and shelter plantations, it was frequently necessary to elevate the observer and his instrument, and also the target to be sighted, above the level of the surrounding obstructions. In order to avoid aberrations due to local variations in atmospheric refraction it is advisable to have the line of sight clear all obstacles by at least 3 m. Removal of the obstructions was in most cases impracticable – a surveyor was apt to receive a frosty welcome on approaching a farmer for permission to cut or top some of his much prized trees – and the most economical method of dealing with the problem was the erection of towers.

For heights up to 12 m, timber towers were used. They comprised an inner tripod on which the theodolite was supported, and an outer quadripod to carry the observer's platform and shelter. When reconnaissance was carried out in the South Island, it was realised that higher towers would be needed on the Canterbury Plains, and two steel towers, constructed to the specifications of the "Bilby towers" of the U.S. Coast and Geodetic Survey, were made by Andersons Limited, Lyttelton, under the supervision of the Public Works Department. These provided for heights from 12 m up to 36 m.

The Bilby tower was constructed of steel parts bolted together, and was easily erected and just as easily dismantled for further use elsewhere. It comprised two units, an inner tripod on which the theodolite was placed, and an outer tripod which carried the observer's platform. The inner tripod was provided with an adjustable top by means of which the height of the theodolite could be varied to suit the observer. A feature of the outer tripod was the C-shaped joint on each upright at theodolite level; should any line of sight be obscured by an upright the joint could be turned away clear of the line. A superstructure on the outer tripod enabled the lamp or heliograph to be set and plumbed over the instrument 3 m above the top of the inner tripod. Observations could therefore be made simultaneously by observers at opposite ends of a ray.

The tower was erected vertically in sections, each section adding 4 m to the height of the structure, except the uppermost section of the inner tripod which was a welded unit 3 m in height. Every piece of steel was clearly marked with the number of the section to which it belonged, and parts of inner and outer tripods were distinguished by bands of red and blue paint respectively. To give a range of possible heights, erection could be commenced at any of the first six sections, and increased height could be gained with a 3 m extension inserted below the observer's platform. The diagonal ties of the first four sections and all the upright and horizontal members were of angle-steel, ranging in size from 70 mm × 70 mm × 6 mm at the lowest section to 25 mm × 25 mm × 3 mm on the superstructure. Above the fourth section twelve diagonal ties of 10 mm rods were used per section instead of six as in the lower sections. The weight of the complete tower, including timber for platforms, was about 3 tonnes.

The foundation of every tower, whether of wooden or steel construction, was a most important contribution, not only to the stability of the tower, but to the accuracy of the observations. It was designed so that slight movements or vibrations set up in the outer tower as the observer moved around his instrument would not be conducted to the inner tower supporting the theodolite. Bilby's foundation comprised six angle-iron anchor posts 1.5 m in length riveted to flanges which in turn were bolted to wooden sills, 1 m × 200 mm × 75 mm. Holes 1.5 m deep and 1 m square were dug and the anchors bedded firmly in position, one post for every leg of each tripod. The sills were covered to a depth of a few centimetres with soil well tamped, and two plates of 1 m × 250 mm × 50 mm were placed crosswise on the sills, on each side of a pair of anchor posts. The holes were then half-filled pending erection of the first two sections of the tower, after which the filling was completed using a heavy rammer. Care was taken to allow no stones to become embedded between sills or plates as these might conduct vibrations from outer to inner tripod. In Canterbury,

as it was necessary to provide against heavy gales, the sizes of the foundations were increased by fastening the anchor posts to jarrah sills, 1.5 m × 200 mm × 125 mm. Two timber plates, 1.2 m × 300 mm × 50 mm, were placed on each side of the anchor posts.

The uprights of the first sections of the inner and outer tripods were then bolted to the anchor posts, and then the centre horizontal braces were bolted in position. The diagonals were bolted at their lower ends to the uprights and to the horizontals. Steps were provided on one of the uprights of each tripod, and temporary triangular platforms were placed on the horizontals. The upper horizontals and upper ends of diagonal members were then bolted in position. The platforms were then moved up to the next level, and the next set of uprights and mid-horizontals bolted in position as before. A departure was made from the American method of assembly for this and the succeeding two sections, the upper horizontals and the upper ends of the diagonals being bolted in position next. The diagonals, being tie-rods, frequently required the application of considerable tension to align the holes sufficiently for the insertion of the bolts without damage to the rods. The inner and outer tripods were erected together for the first six sections, after which the outer tripod was completed, including platform and superstructure.

To complete the inner tower a tackle was fastened to the top of the outer tower and lowered through the centre of the platform. The seventh section of the inner tripod was assembled on the ground, stood up over the station mark in the centre of the tower, and the welded uppermost section was bolted to it. The two sections were then hauled up and bolted in position. If the extension, which for the inner tripod was also a welded unit, was used, the three sections were bolted together before being sent aloft.

The time required to dig holes and set foundations depended on the type of subsoil encountered. Sufficient experience in working at heights to which they were not accustomed was gained by four chainmen during the erection of the first tower. It was found that, given favourable weather, these men could erect a tower subsequently in 2½ working days. The towers had to be painted orange and black in accordance with the requirements of the Civil Aviation Branch of the Air Department.

No guy ropes were used on the towers. Towers erected over two stations were subjected to a severe test in November, 1939, when exceptionally strong gales were experienced for a week. After the gales had subsided, one of the towers was found to be still plumb. A slight settlement in the outer tripod of the other had produced a displacement of 25 mm in the top of the tower; the inner tower had not moved perceptibly. The towers functioned very well in Canterbury, although it was necessary to wait for practically calm weather before observations conforming to geodetic specifications were obtained.

After the completion of the first-order observations, the towers were sold to an Australian survey department.

## Beacons

A beacon was erected over every first-order station during the period up to 1930 when observations were made in daylight; after the adoption of night observations to lamps, the beacons were no longer necessary and were erected only over stations which were to be used also for second-order or third-order observations. The beacon is usually a wooden quadripod, with a centre mast or target-board projecting above the apex of the pyramid. For a first-order station, the height of the beacon is about 5.5 m to 6 m to the apex. More recently, standardised beacons, 3 m in height for a second-order station and 1.5 m for a third-order station, have been adopted, and in some cases one of these has been erected over a first-order station. The 5.5 m and 3 m beacons are large enough to permit a theodolite to be set up under them, but a 1.5 m beacon must be removed when the station is occupied, and so is designed to be lifted bodily when the screws securing the rafters to the corner posts are removed. A beacon, whatever its height, was removed when the station was occupied for astronomical observations.

For the height in feet of a beacon from ground to apex, Ross adopted the working formula  $\frac{7}{3}s + 6$ , where  $s$  is the length of a ray in miles. This gives a 5.5 m beacon

at 60 km and a 6 m beacon at 80 km, but eventually the 5.5 m beacon was adopted as standard. The base of the beacon was a 3.5 m square, centred on the trig tube, with four corner posts of 200 mm × 200 mm timber about 1 m long driven into the ground, leaving only enough protruding for attaching the anchor irons which secured the rafters. The square was so oriented that the rafters would not obstruct any of the rays to be observed, although occasionally it was found that at a station where a beacon is already in existence this condition was not satisfied, and the beacon had to be shifted or an eccentric station established. The rafters were of 125 mm × 75 mm timber about 6 m long, meeting at the apex vertically above the trig tube. A mast of 75 mm × 75 mm timber projected about 1 m above the apex, and target-boards were usually attached to the mast. The walls of the pyramid were of 225 mm × 25 mm boards nailed to the rafters, and extending from the apex down to a height of about 2 m above the station mark. Where boards were used below this, an observing gap or look-out would be left at theodolite level. The upper portions of the beacon were usually painted black, the lower portions white, although on some high stations which had sky backgrounds when viewed from all the others, the beacons were painted completely black.

### Heliographs

When observations were made in daylight, heliographs were used as targets at low stations where the beacons had poor background contrast or at stations where permanent beacons had not been built. Several types of heliograph were used, but the military heliograph, provided with a key for morse signalling, proved the most satisfactory. When night observations were introduced, heliographs were still used in observing missing sets of directions or in observing vertical angles.

### Beacon lamps

When night observations were first introduced, imported beacon lamps were not available and lamps were assembled as required. It was found that a 3½ or 4½ volt torch bulb or cycle lamp bulb fitted into the reflector of a motorcar headlamp was a satisfactory beacon lamp. Fittings to carry the lamps and to permit directional adjustment were manufactured locally. Equipment of this kind was used for several years, but eventually factory-made beacon lamps, using 6-volt 3-candlepower bulbs, were obtained from Cook, Troughton and Simms Limited. Keys for morse signalling were also provided. Dry cells were used to run the lamps, arranged in series-parallel to give adequate brightness for the length of sight.

Time switches were used when lamps were left unattended. Originally, time switches were made from alarm clocks, but later on Sauter time switches were purchased. The switches were set to switch the lamp on at sunset and off at eight hours after sunset. With the switch in operation, a lamp could be left unattended for three weeks if necessary.

### Lightkeeping

The lightkeeper appointed to a station was required to build an instrument stand over the station mark and to identify the lines of sight to other stations. The beacon lamp was set up over the station mark, but if a heliograph were used it was set up over a stake on the line to the observer's station, in a position where it would be clear of shadows. Where such a set-up point was found to be eccentric, its use would be continued, and the observer would measure the eccentricity on his arrival at the station. The lightkeeper erected a brushwood or canvas shelter for his own comfort, and aligned his telescope on the observer's station, on forked sticks or other suitable mounting braced in position. During observations, the lightkeeper kept his lamp pointed to the observer's station and watched for signals from the observer.

At a difficult or dangerous station, the lightkeeper visited the top at night as little as possible. If he was completely satisfied that his lamp was showing a good light and was correctly aimed, he could, after all daylight work at the station was completed, leave the lamp on time switch and return to his main camp to await word from the observer. After bad weather he would make a visit to the top to ensure that the lamp was still showing and the lamp glass was clean.

When the observer was expected at the lightkeeper's station, the lightkeeper erected a frame for the observer's shelter and an eccentric lamp stand for signalling to other stations.

### Signalling with lights

For signalling between observer and lightkeeper when both stations were occupied, the morse code was used. The observer called the lightkeeper by a steady light until he was answered by rapid dots; he then extinguished his light. The lightkeeper then showed a steady light while the observer sent his message. The lightkeeper called the observer by slow dashes until he was answered by a steady light; he then sent his message.

Certain messages were abbreviated in the following code:

III	Increase your light.
MMM	Moderate your light.
DA	You may leave your station for the day; light not required.
W	Wait, I may finish tonight.
TS	Time switch.
FINI	I have finished to you only.
FINI ALL	I have finished to all stations from here and will occupy my next station immediately.
OK	Light satisfactory, or message received, or signals finished.
MS	Message following.
G	Go, or go to.
H	Helio.
HQ	Headquarters.
L	Lamp
S	Show, or show to.
R	Repeat.
Z	Observer, or observer's camp.
ZG	Observer gone to next station.
VP	A vehicle is being sent to pick you up.

When it was necessary to send numbers, they were spelt.

### Radio communication

In the South Island, radio equipment was used for the issuing of instructions to lightkeepers. The transmitter was operated by the observer from his camp, the lightkeepers being equipped with receivers. This one-way communication proved satisfactory. The equipment was made by Collier and Beale Limited, Wellington, and consisted of a 10-watt transmitter operating on 4060 kHz, and weighing about 18 kg. It could be set up ready for voice transmission in about 10 minutes. It was run mostly from a 6-volt battery, but often 1½-volt dry cells were used, the signal strength being then cut by about 25%. The set was easy to operate and the signal was very clear up to 160 km, being very strong at about 9 p.m., which was the usual time for communication. The lightkeepers were able, however, to receive the transmission at all times and they reported that the signal faded very little. The receivers were 3-valve regenerative units weighing about 11 kg. When the first-order field work was completed, this radio equipment was transferred to the Air Department.

### Observation of horizontal angles

All observing was done by the direction method, one of the stations to be observed being selected as the origin, and directions to other stations being measured in the clockwise sense from that origin. In the original instructions issued to observers in 1923, the circle was required to be set to the approximate azimuth of the station of origin, but this requirement was later dispensed with and the initial direction was taken as 0°. The telescope was always turned clockwise, care being taken not to overshoot any station, and anticlockwise movements were avoided except for the fine adjustment of the pointing. A "set" consisted of a face left and a face right

observation on the same circle setting.

The usual precautions in procedure to obtain the most accurate results possible with the instrument were followed, the observations being ordered to eliminate several possible instrumental errors:

- (1) To eliminate errors due to eccentricity of the circle, simultaneous readings were made of the micrometers at opposite ends of a diameter. With the Troughton and Simms instrument, this involved reading two micrometers on each occasion, but with the Wild and the Tavistock the mean of two such readings is obtained automatically through the optical system of the instrument.
- (2) To eliminate errors due to incorrect collimation, the line of sight not coinciding with the optical axis of the telescope, each round of readings was made with the telescope "face left" and repeated with the telescope "face right".
- (3) To eliminate errors due to imperfect graduation of the horizontal circle, several sets were observed, with different initial settings for each set, so that the same angles were measured in different regions of the circle. In first-order observations, 12 sets were observed, the initial setting moving through every 15° of the circle.
- (4) To eliminate errors due to imperfect graduation of the micrometer, the initial setting for any set was increased by a multiple of 5". Thus, for Set 1 the initial setting would be 0° 00' 00", for Set 2, 225° 00' 05", for Set 3, 90° 00' 10", and so on.
- (5) To eliminate errors, in single-axis instruments, due to warping of the theodolite axis, the observations were divided into three series, the directions of the three footscrews of the tribrach being altered by 120° for each new series, so that the whole instrument was rotated clockwise.

In accordance with the makers' recommendations, the telescope was given a complete clockwise rotation before readings were commenced. That is, for any set, the telescope was pointed to the station of origin and an approximate setting of the circle was made. The telescope was then rotated in a clockwise direction and brought back to the station of origin, when the first reading was made. Each other station around the horizon in a clockwise direction was then observed and a reading made, with a closing pointing and reading to the origin. Face was then changed and the procedure repeated.\*

If Set 1 was begun on face left and concluded on face right, then Set 2 would be begun on face right to save the time of resetting. Successive sets were therefore begun on face left and face right alternately. The circle was turned through 45° after each set in each of the three series, the series beginning at 0°, 15°, and 30° respectively, so that the initial settings for the series were as follows:

Face	Set	Initial Setting	Set	Initial Setting	Set	Initial Setting
		°   °		°   °		°   °
L - R	1	0 - 180	5	15 - 195	9	30 - 210
R - L	2	225 - 45	6	240 - 60	10	255 - 75
L - R	3	90 - 270	7	105 - 285	11	120 - 300
R - L	4	315 - 135	8	330 - 150	12	345 - 165

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\*It has been argued that this procedure gives undue weight to the origin, which is observed twice, the other stations only once, and that it also requires application of a correction to remove the difference between the initial and final readings of the origin. An alternative procedure, used on some second- and third-order work, is to observe the origin and every other station around the horizon in a clockwise direction, but without a closing pointing to the origin. Face is then changed, and the telescope is turned past the origin so that the first reading is made on the first

In the original instructions, the three series were to be observed on three different days. The observations could be made whenever the beacons were visible, and in the case of any station being lost sight of by mist, smoke, or haze, the observations of the visible stations were continued, leaving the missing station to be observed later as opportunity offered. On an overcast day, observations could be made at any time but in clear weather observations were usually restricted to the early morning and the late afternoon. When observations were made to beacon lamps at night, work could be commenced as soon as the lamps became easily visible, which was usually a short while before sunset.

The observation of 12 sets could be carried out by a good observer in four or five hours on one night, but, although the original instructions were later relaxed, results were not in general made to depend on one night's work. The usual experience, in fact, was that several nights were required for observations at each station. In an earlier report, Ross stated: "The conditions under which it is safe to observe will give much concern to the inexperienced observer. Nights on which the lights appear as steady star-like points are few, but records show little difference in the steadiness of the observations between those on nights when pointings are made with every confidence and those on nights when accurate pointings appear impossible. On clear, cold nights the lamps often appear as unsteady flaring lights subtending an angle which it appears impossible to bisect with a probable error of one second. Again on smoky or misty nights the lights may appear so faint as to be invisible with the usual telescope illumination. A procedure must then be adopted which to the uninitiated appears impossible. If the light cannot be seen with the naked eye, the instrument is set on the distant station by bearing and vertical angle. Search is then made for the light with the telescope illumination cut off. If the light can be seen at all an accurate pointing may be made by switching the illumination on and off several times, and, by alternately impressing on the retina of the eye the images of the cross wires and the light, coincidence may gradually be obtained."

#### Reduction of horizontal observations

The recorder reduced all the field book entries as the observing proceeded, the observer being responsible for ensuring that all reductions were independently checked and that each page of the field book was initialled by the checker.

For each set the mean of the face left and face right observations was required, and in the case of the Wild theodolite, the micrometer being graduated in double seconds, the mean was obtained by adding the two seconds values to the degrees and minutes reading. An additive correction was then applied to the reading of the origin to make it 00' 60".0, any difference between the corrections to the initial and return readings being distributed among the intervening directions. An example of one set of directions at a station is the following:

		At Hihitahi			Set 6		
					Mean	Corr.	
		°	'	"	°	'	"
Waipuna	R	240	00	20.3	00	40.8	19.2
	L	60	00	20.5			
Tongariro	R	308	32	43.9	33	26.1	19.4
	L	128	32	42.2			
Mangaweka	R	94	46	01.3	46	02.3	19.5
	L	274	46	01.0			
Waipuna	R	240	00	20.3	00	40.3	19.7
	L	60	00	20.0			

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*station clockwise from the origin, and the final reading is made on the origin. This gives one face left and one face right reading on every station, including the origin, and eliminates the need for a closure correction.*

The lay-out of the field books, both for horizontal and for vertical observations, was designed for the old-style theodolite, so that column headings are not applicable to double-reading optical-scale instruments, but it was used without change during the whole period of the first-order triangulation. Re-design of the field books, for modern theodolites, was not undertaken until 1969.

When a set was completed, the degree value of the origin was then subtracted from each reading, so that each set was converted into terms of an initial reading of  $0^{\circ} 00' 60''$ . The readings above would thus be:

	°   '   ''
Waipuna	0 00 60.0
Tongariro	68 33 45.5
Mangaweka	214 46 21.8

The final directions for all sets were summarised in a notebook and the mean of the 12 sets of readings to each station was found. Any observation for which the residual exceeded  $4''$  was repeated.

As soon as possible after the observations at a station were completed, the mean results of each set were entered on the summary of observations. The form provided ten columns, the names of the stations observed being entered at the heads of the columns. The degrees and minutes of the directions were entered below the station names, and the seconds of each set were entered down the columns. All observations were shown, any that were outside the allowable residual limit being crossed out, and where any set had been repeated without rejecting either, the mean of the two observations was shown as the value of that set. The mean of 12 sets was carried to  $0''.001$ , and corrections for eccentricity of instrument or beacon, to the same degree of accuracy, were applied where necessary.

If the observations at any station were sufficient to close one or more triangles, the closure of those triangles would be checked before the observer left the station, approximate sides and spherical excesses of triangles being computed as the work progressed. If a triangle did not close within the limits,  $3''$  in first-order work, check observations would be made to ensure that the observations at the station being occupied were correct. If they were, it would be necessary to re-occupy one of the earlier stations. Re-occupation of a station because of large errors of closure, although occasionally necessary, was in fact a rare event.

#### Observation of vertical angles

The instructions stipulated that vertical angles were not to be observed at the same time as horizontal angles. During the early period when all observations were made in daylight, vertical angles were observed as far as possible between midday and 2 p.m., the effect of refraction being least at about that time. Later on, when most observations were made at night, vertical angles were still observed near midday when occasion offered, heliographs being used as beacons. When midday observation was not feasible, vertical angles were observed at night, as near to midnight as possible.

Every ray observed in the course of measurement of horizontal angles was included also in the programme of vertical angles, except in one or two cases where weather conditions would have meant too long a wait for satisfactory beacon visibility. In some cases the depression of the sea horizon was also observed; such observations were never used in the computation of heights, but they proved useful in verifying the value of the coefficient of refraction.

The instructions did not lay down the number of times a vertical angle was to be observed. In general it would be observed four times, with face left and face right readings each time, but was never observed less than twice.

The form for the summary of vertical observations was divided into ten horizontal panels, one for each station observed, and each provided for the particulars of four sets. The information summarised was the station observed, the zenith distance, the



date and time of observation, the barometer and thermometer readings, the height of the point observed above the station mark, and any relevant remarks. The height of the instrument above the occupied station was recorded in the upper part of the form.

## V. Astronomical Observation

### Astronomical stations

Although the proportion varied at different times and in different parts of the net, stations where latitude has been determined by astronomical observations amount to one station in every three in the North Island and one station in every six in the South Island, or one station in every four in both islands together. The greatest density of latitude stations is in the Bay of Plenty region north of the Kaingaroa base, where the field work began. In later years, when the difficulties of observing at the high stations in the axial ranges had become apparent, the proportion of latitude stations was reduced, although the instructions still specified that latitude should be observed whenever convenient. Consequently, latitude was more often observed at stations of relatively easy access, such as those in Taranaki, than at the more difficult stations. Over the whole network, the proportion of latitude stations has been ample.

Azimuth observations were made at one station in every seven, the proportion being the same in both islands. The original instructions did not mention azimuth observations, and none were made in the first three years of the survey. It may have been the intention to retain the azimuths on which the meridional circuit triangulation had been oriented. From 1927 onwards, however, azimuth observations were included in the observer's programme, the new network being recognised as independent of all previous observations. In all cases except one, an azimuth station was also a latitude station.

When the other field work had been completed, seven Laplace stations were selected in the North Island and five in the South Island, a proportion of about one station in every twenty-five. They were well distributed throughout the network, including stations in the far north, near East Cape, and near the southern end of the net. They were chosen for ease of access as well as for their strategic positions in the net. Most of them were adjacent to roads, and, except in two cases, the truck carrying the instruments could be driven right to the site. Longitude was observed at all of these stations, and latitude and azimuth were also observed at those stations where such observations had not been made previously.

### Instruments used

In general astronomical observations were made with the theodolites in use at the time for horizontal and vertical angle measurement, but at Kelburn a 12-inch (300 mm) Troughton and Simms theodolite was used for time and latitude observations.

Several chronometers are listed in the field books, but only one of these, Mercer 12781, is now held. It is a marine type box chronometer, with half-second escapement and a 24h dial.

At Laplace stations, a portable transit instrument and a tape chronograph were used.

### Methods of reading chronometers

In most observations the telescope was pointed a little in advance of the position of the star, and the chronometer time was recorded when the star reached the thread, horizontal or vertical as the case may have been. One diaphragm, for the Troughton and Simms theodolite, had five vertical lines, so that the passage of a star across each of these could be observed and timed without moving the telescope. With other theodolites, the telescope had to be moved so that several observations of a star could be made within a short period.

Standard methods of reading the chronometer were used. If the observer had the assistance of a recorder, he would warn the recorder to be ready as the star approached the thread. The recorder would then count seconds by following the beat of the chronometer, so that, when the observer called "Read" as the star reached the thread, he could record the second and estimated tenth of a second at that instant. The time would be recorded in the reverse order of second, minute, hour.

If the observer worked without a recorder, he would use the "eye and ear" method, counting seconds from some instant a little before the star's transit, mentally noting the second and tenth of a second at the instant of transit, and continuing the count until the next 10<sup>s</sup> reading of the chronometer. The recorded time would be this last chronometer reading minus the seconds count at this reading plus the seconds count at the instant of transit. If a stopwatch was available, it was used to measure the interval from the star's transit to the following 10<sup>s</sup> reading of the chronometer.

#### **Observations for time**

The chronometer was set to keep approximate local sidereal time at each station, and observations to determine the chronometer error and rate were made at the beginning of each night's work, at the end of the night's scheduled programme, and, as a safeguard against interruption by cloud, at some convenient time during the programme.

The method used was by zenith distances of stars near the prime vertical, two or three observations of a star in the east being paired with two or three observations of a star in the west at about the same time. The stars observed were not selected in advance; the surveyor observed any star which happened to be in a suitable position. The approximate azimuth and magnitude were also noted to assist in identification. On some occasions a planet was observed; it proved easy to bisect the apparent disk, so that the correction for parallax was the only difference from observation of a star.

#### **Observations for azimuth**

The method of determining azimuth was always by observing the direction to a close circumpolar star at any hour angle, and the referring mark was always the lamp at another station of the net. Twelve sets of observations were taken, the initial circle settings on the referring mark being the same as those on the initial station in the measurement of horizontal angles, and using every 15° of the circle. The procedure in measurement involved first pointing the telescope to the referring mark and making an approximate setting of the horizontal circle. The telescope was then given a complete rotation in a clockwise direction, a fine pointing on the referring mark was made and the circle was read. The telescope was then swung in a clockwise direction to a position a little in advance of the star, and the chronometer time was noted at the instant that the star crossed the vertical thread in the diaphragm of the telescope. The horizontal circle was then read, and striding level readings were also taken. For each set, several time observations of the star were made, usually four but sometimes as many as eight. The telescope was then turned to the referring mark again and a closing reading of the circle was made.

In most cases the same star, usually  $\sigma$  Octantis, was used for all the sets at one station. Occasionally, however, a different star was used for each of the three series into which the twelve sets were grouped.

#### **Observations for latitude**

The standard method of latitude determination was that of circummeridian zenith distances, which was used from the earliest field work until the end of the project. At some stations, however, several other methods were tried, the results of the trials being in general to show the superiority of the method of circummeridian zenith distances for theodolite observations.

### **Circummeridian zenith distances**

These were observed at 65 of the 71 latitude stations. In the first year of the survey, the stars observed were not paired in equal zenith distances to the north and to the south of the zenith, about six stars being observed at each station. From 1925 onwards, the stars were paired, a star observed to the north of the zenith being balanced by a star of about the same zenith distance, within  $2^\circ$ , to the south of the zenith. This gave better results by reducing the uncertainties of the correction for refraction. The number of stars observed was also increased; for most stations six pairs of stars were usual, and in the later years ten pairs or more were often used.

For each station, a list of the transit times of stars at suitable zenith distances was prepared, and the stars observed were selected from this list on the night of the observations. Because of the ever-present possibility of cloud interrupting the work, it was inadvisable to select beforehand all the stars to be observed. Observations usually extended over about ten minutes on either side of meridian transit. For each observation, the telescope was set a little ahead of the star, and the chronometer reading was recorded when the star reached the horizontal thread. The vertical circle was then read. Barometer and thermometer readings were also noted, at some time during the series of observations on one star, as they were needed for the refraction correction. In the earliest observations, face was changed systematically throughout a series of observations, the index error of the vertical circle being deduced from the difference between the results of the face left and the face right observations. Later on, it was realised that change of face was unnecessary provided that both stars of a pair were observed on the same face, and it became usual to use the same face throughout a whole night's observations. Index error would be indicated by the results derived from north stars being consistently different from those derived from south stars, but the error was removed by taking the mean result for each pair. The avoiding of change of face also meant that more observations could be made in a given time, and the zenith distances of a star were observed at intervals of about three-quarters of a minute.

### **Exmeridian zenith distances of a circumpolar star**

For the south star to be paired with a north star in the method of circummeridian zenith distances, Ross occasionally chose a close circumpolar star and observed exmeridian instead of circummeridian zenith distances. The method was never used except in association with north circummeridian observations, and gave quite satisfactory results.

### **Meridian zenith distances**

At two stations in Taranaki, Ross used single meridian zenith distances of stars, about twelve pairs of stars being observed in each case. In several cases, three stars about the same meridian zenith distance were observed, but in such cases the mean zenith distance of the two stars on the same side of the zenith was paired with the zenith distance of the star on the other side of the zenith.

### **Almucantar observations on the colatitude circle**

From 1927 to 1929, experiments were made of a method of obtaining time and latitude simultaneously by observing the chronometer times of the passage of stars across the almucantar passing through the pole, advocated by the American astronomer Chandler in 1884. The zenith distance being equal to the colatitude of the station occupied, Chandler called this almucantar the "colatitude circle", and devised an instrument (which he inappropriately called the almucantar) for observation by this method. Chandler's instrument employed a telescope whose supports were floating in a bath of mercury so that the almucantar observed with the telescope was very accurately defined. In New Zealand, a theodolite was used for Chandler's method, the diaphragm having five horizontal threads and the passage of the star across each of these threads being timed. Except for a necessary correction to the central time because of unequal thread intervals, the method of computation followed that described by Chandler, the observation equations formed from all the observations on one night being solved by least squares to give corrections to the assumed latitude, clock correction, and zenith distance. The preparation of an observing programme for this

method entailed more labour than in the methods mentioned earlier, as it was necessary to compute the hour angle and azimuth of each star when on the almucantar, and to ensure that the stars observed were suitably distributed in azimuth. The striding level was read for each star.

The method, used at three stations, gave very disappointing results, and it seems that there were two reasons for this. A theodolite, even with striding level readings, cannot define the almucantar with sufficient constancy to take full advantage of the intrinsic precision of the method (a criticism which does not apply to the prismatic astrolabe or to a theodolite with an astrolabe attachment). In the observation equation for each star, the absolute term is the difference between the observed and computed times of passage across the almucantar; this difference is remarkably sensitive, and the method needs both a chronometer with a constant rate and very great care in reading the times. These conditions were not satisfied in the field, and the almucantar observations were eventually rejected.

At Kelburn, a station in the Dominion Observatory grounds, Ross observed latitude both by circummeridian zenith distances and by almucantar passages, with a disturbingly large difference between the results of the two methods. A chronograph wired to one of the Observatory's pendulum clocks was used for the almucantar times, so that the condition of reliable timekeeping was satisfied, and 70 observations were made on 41 stars. The results were better than those obtained at other stations without a chronograph, but the residuals were nevertheless disappointing, and quite different results were obtained from different groups of stars, the range of latitude values being 4", and the mean result of all the almucantar observations differing by nearly 3" from the result of the circummeridian observations.

During the International Geophysical Year of 1958, observations for time and latitude were made at Kelburn with a Danjon prismatic astrolabe, an instrument which defines the almucantar of 30° zenith distance by means of a 60° angle prism and a mercury horizon, and by which impulses are sent to the chronograph automatically as the observer follows the motion of a star. The results of these observations, the most accurate yet made in New Zealand, confirmed the essential correctness of Ross's circummeridian observations and demonstrated the inadequacy of a theodolite for almucantar observations.

#### Almucantar observations on the 60° circle

Towards the end of the field work in the South Island, the almucantar method was tried again at three stations, this time observing the passages of stars across the almucantar of 60° zenith distance. Stars were observed in groups of six, suitably distributed in azimuth, and the results of each six were obtained from solution of the observation equations by least squares. The striding level was not read. The method suffered from the same defects as the method of observation on the colatitude circle, and these observations too were eventually rejected.

#### Fundamental longitude

The fundamental longitude adopted for the triangulation was that observed at the Dominion Observatory, Kelburn, Wellington, by R.C. Hayes and I.L. Thomsen, of the Observatory staff, in October and November, 1933, as part of a world-wide programme inaugurated by the Bureau des Longitudes, Paris. These observations have been described elsewhere.\* Observations were made with the Observatory's transit instrument which was equipped with a hand-driven self-registering micrometer. At the same time, almucantar observations on the colatitude circle were made by J.P. Arthurs with a Wild theodolite at Kelburn first-order station. Both the meridian and the almucantar observations were recorded on the same chronograph in terms of the Observatory's sidereal clock, but as Arthurs' observations were recorded by hand key and were subject to personal error, they were not used in the final determination of longitude.

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\*R.C. Hayes and I.L. Thomsen: *International longitude determinations; 1933, October-November. Results of observations made at the Dominion Observatory, Wellington.* N.Z. Journ Sci. and Tech., 18 (1936):512-519.

Time signals from several radio stations were received and recorded, but the longitude adopted was based only on those broadcast from GBR Rugby at 10<sup>h</sup> G.M.T., which were received on 27 nights. The mean result gave the longitude of the transit pier east of Greenwich as 11<sup>h</sup> 39<sup>m</sup> 03<sup>s</sup>.97, from which the longitude of Kelburn first-order station was derived as 11<sup>h</sup> 39<sup>m</sup> 03<sup>s</sup>.95.

#### **Observations for longitude at Laplace stations**

As the longitudes at other Laplace stations were determined from chronograph readings made with a hand key and were therefore affected by the observer's personal equation, further observations were made at Kelburn to evaluate this equation, the previously adopted fundamental longitude being regarded as free from personal effects. Three series of such observations were made, before observing at other stations, after the completion of other field work, and once during the progress of that work. The point occupied at Kelburn was the No. 2 transit pier, in an outbuilding of the Observatory, its position relative to the transit pier and to the first-order geodetic station Kelburn being known.

#### **Portable transit instrument**

For longitude observations, a portable transit instrument made by Troughton and Simms was used. It had been purchased before 1874 but, in spite of its age, was in good condition and suitable for the purpose. The object glass was 2½ inches (60 mm) in diameter and the eyepiece used gave a magnification of ×55. The diaphragm consisted of nine vertical threads, and electric illumination was used. The vertical circle enabled zenith distances to be set to 1'. The striding level, tested by the Dominion Physical Laboratory, had a sensitivity of 0".92 per division. The telescope was reversed by hand by lifting it from its Y supports and changing the axis end for end. An azimuth adjustment was provided for bringing the line of collimation into the meridian. A transit micrometer was not fitted, observations being made by the hand key method.

#### **Chronometers and chronograph**

Times were recorded on a tape chronograph made by the Dominion Physical Laboratory. It used paper tape 25 mm wide, which was driven by clockwork at the rate of approximately 12 mm per second. A Mercer chronometer No. 12781, set to keep approximate Greenwich mean time, was connected to the chronograph so that every second was marked on the tape by a pen; minutes were indicated by the absence of every sixtieth second mark. A Blockley chronometer No. 31629 was used as a sidereal time clock.

#### **Radio receiver**

A portable radio receiver, made by the Dominion Physical Laboratory, was used for the reception of broadcast time signals. It used dry cells, and operated on a frequency range of from 300 kHz to 900 kHz. The radio masts were of spruce in sections which could be bolted together into two masts each 6 m in height. The time signals, as they were received, were recorded automatically on the chronograph tape. The radio was also the switch panel, all electrical connections from other instruments being to the set.

#### **Orientation of instrument**

The transit instrument was set up on a brick pillar about 1 m high, which had been erected on a concrete block enclosing the station mark. A canvas shelter on a portable spruce frame was erected around the instrument, allowing plenty of room for the recorder's table, and with an opening about 0.6 m wide in the top for observation. A built-up platform for the observer protected the instrument against shocks. The transit instrument was oriented approximately during daylight using triangulation information or by an observed sun azimuth.

The final orientation was carried out each night before beginning the observing. The error of the sidereal chronometer was found from the transit of a star near the zenith, where an azimuth error has little effect. Then a circumpolar star or one with a large zenith distance was followed with the centre thread until the instant of transit as indicated on the chronometer. The instrument at that instant was then correctly oriented. The whole operation was usually done twice.

### Observing programme

The method of observation with the transit instrument consisted essentially in recording the Greenwich mean time of transits of stars across the meridian, using a chronometer corrected by means of radio time signals. The Greenwich sidereal time could then be computed, while the local sidereal time was given by the right ascensions of the stars. The difference between local time and Greenwich time then gave the longitude of the station.

A normal night's work consisted of transits of 24 stars, divided into four half-sets of six stars each. The instrument was reversed between the first and second half-sets and between the third and fourth half-sets, giving two half-sets with clamp east and two with clamp west. When cloud intervened, three half-sets could be taken as one night's determination, one half-set being combined with each of the other two half-sets. In exceptional cases, a half-set could consist of four stars. The complete determination consisted of four nights' observations.

- Only stars with a zenith distance less than  $15^\circ$  were used. All stars in the ephemeris within this limit were listed, the final selection being made at the time of observation, after taking into account the time needed for striding level readings and the reception of time signals. The time of contact of a star with each of the nine threads was recorded on the chronograph tape by the pressing of a hand key, the recorder having started the chronograph a little before the first contact. The chronograph was left running long enough to record one minute mark with each group of star contacts. The striding level was read at least four times per half-set, once before commencing, once at the conclusion, and twice or more during the half-set. Actual observing time per night ranged from  $2\frac{1}{2}$  hours to  $3\frac{1}{2}$  hours.

During observing, the field book contained only the level readings and an observation summary of the stars used, all other information being recorded on the chronograph tape. The following day the tapes were read and the data entered in the field book in chronological order. The times of the nine contacts were meaned to give the observed time of transit. As a safeguard against gross error, the contacts were summed in symmetrical pairs (1+9, 2+8, etc.); if one thread was missed for any reason, the corresponding contact on the other side of the centre thread was discarded.

### Time signals

For the longitude programme, radio time signals were broadcast through the Post Office radio station ZLW every hour from 6<sup>h</sup> to 12<sup>h</sup> G.M.T. The signals originated at the Dominion Observatory, Kelburn, Wellington, the signal clock being a pendulum clock regulated to keep Greenwich mean time by daily comparison with overseas time signals, generally with those broadcast by station NSS Annapolis, Md., U.S.A. The signals consisted of six "pips" marking the last five seconds in each minute at the last three minutes in the hour. The signals were usually correct within 0<sup>s</sup>.1, and the error of any signal could be interpolated from the daily comparisons, no allowance being made for any error in the original transmissions from overseas. Corrections to the broadcast time signals were supplied weekly by the Observatory. The signals from station ZLW were usually well received; on the occasions when they were not, the identical signals could be obtained from the National Broadcasting Service station 2YA. At 11<sup>h</sup> G.M.T., station 2YA relayed the time signals broadcast from the B.B.C., and these could also be used after correction for travel time. The error of the chronometer at any instant could be obtained from the record of the time signals, so that all the data on the chronograph tape could be expressed in terms of Greenwich mean time (U.T.).

**Astronomical  
summaries**

The form used for preparing a summary of the astronomical observations in the field books was designed to cater for all the kinds of observations, for time, azimuth, latitude, and longitude. The purpose of each set of data was indicated in the heading to that set, and additional information or instructions (such as the other star with which a given star was to be paired in latitude observations) could be written below the star name. Not every column was needed for the record of any one type of observation. Columns were provided for: the date and time (N.Z. standard time or G.M.T. to the nearest minute); the name or catalogue number of the star observed; the set (in azimuth observations) or face, or whether a star was observed in the east or the west in time observations, or in the north or the south in latitude observations; the chronometer time, a vacant heading being also provided for stating whether this was local sidereal time or any other; the horizontal circle reading of the referring mark; the horizontal circle reading of the star; level readings; the zenith distance of the star; and barometer and thermometer readings. The form also provided for the record of the value of one division of the level, and the name of the station used as the referring mark, in addition to the usual particulars of the station occupied, the observer's name, the instrument used, and the field book reference.



## PART III. COMPUTATION

### VI. Base-line Computation

#### Computation of the North Island base-lines.

The original computation of the lengths of the North Island bases was done by the field party under Langmuir's direction as the measurement proceeded, and was checked by the Computing Branch at Head Office, so that the results were available for publication in the annual report which described the field work. Special computation books were printed and bound for the purpose. The information entered from the field book for each bay included the mean measured length by the particular tape used, the angle of elevation or depression to the nearest second, the observed temperature, and the field book page reference. The bays were not numbered, but were entered in their natural order, the leading end of a bay being denoted by a sketched symbol for a tripod or for a theodolite.

#### Corrections to base measurements

The measured lengths and the corrections were expressed to 0.0001 link (0.02 mm). Six corrections were applied to the measured length of a bay, as follows.

#### Correction for temperature

This was derived from the known coefficient of expansion of the tape and the difference from the temperature of standardisation. Details are not shown in the computation books, and, as the majority of the bays were 5-chain lengths, it was probably entered from a table.

#### Correction for standardisation

At the head of the page for the computation of each section, the differences from the standard length of the tape are shown, for each full chain and for parts of a chain. These differences were interpolated between the comparisons with the standard steel band preceding and following the measurement of a section. The majority of the bays being 5-chain lengths, the correction for standardisation was most often taken from this table; for odd measurements less than 5 chains, a proportional correction was applied.

#### Correction for alignment

The length measured with the tape was that of a line slightly inclined to the direction of the base, the offset at the theodolite end being 0.372 link. The correction required because of this inclination was taken from a table, copies of which have not survived. The correction to a measured length  $l$  is

$$-l + \sqrt{l^2 - 0.372^2},$$

which has its minimum numerical value of 0.0001 link when  $l$  is the full tape length of 500 links, and increases as  $l$  decreases.

#### Correction for sag

The tape was supported at 50-link intervals, and Langmuir stated that the amount of sag on a 50-link length was determined experimentally. A description of the experiment is not known.

#### Correction for tension

Tension was applied to the tape, as described earlier, until the reading of the spring balance was 15 lb. With progressive weakening of the spring, revealed by frequent tests, the actual tension applied gradually lessened, and a further correction for change of tension was necessary. Details of the computation are not shown, but the correction was always very small.

**Correction  
for slope**

The slope length of the bay, corrected by the amount of the previous five corrections, was then used to give the horizontal length by multiplying it by the cosine of the vertical angle. This was done using seven-figure logarithms.

**Correction  
for height  
above sea  
level**

The horizontal length of a section was next found by summing the lengths of the individual bays, and the correction for height above sea level was applied to the length of each section. The computation of the correction is not shown; presumably it was based on the mean height of all the set-up points in a section, and on a mean value for the radius of curvature of the spheroid in the direction of the base-line, or perhaps on a mean radius of the Earth in the region of the base. The corrections are sufficiently small, except in the case of the Kaingaroa base, for the precise value of the radius to be unimportant.

Some loose papers inserted in the computation books show heights computed from vertical angles compared with heights derived from levelling. No mention of levelling had been made in any available report.

**Corrected  
lengths of  
bases**

The results of the various measurements are summarised in the annual reports of the Department of Lands and Survey between 1910 and 1914. The individual measurements of each section are shown, with the tape number and edge used. Generally, there were four measurements, but sometimes the number had been increased to six or occasionally eight. In some cases, a weighted mean of the measurements was taken, the weights ranging from 1 to 5, but the reasons for weighting are not given.

**Computation  
of the South  
Island base-  
lines**

At the time of the measurement of the South Island bases and the remeasurement of two of the North Island bases, a computer from the Head Office Computing Branch was attached to the field party so that computation could proceed along with the measurement. A calculating machine was used. A form serving both as a summary of observations and as a computation form had been printed, and at the head of this form the relevant particulars were entered: the name of the base, the number of the measurement, the section (i.e. the names or numbers of the terminal marks), the tape numbers and thermometer numbers, and the field book reference. Columns were provided for: the date and time, the number of the bay, the number of the tape used, the sums of the rear and forward readings of the tape, the mean temperature, and the difference in height between the terminal marks of the bay (i.e. the measuring heads on the tripods) — all these being copied from the field book — and for the computed slope length and a number of corrections. For each bay, two horizontal panels were provided, one for each tape.

**Slope length  
of 100-ft bay**

The slope length of each bay was computed after the mean of each set of summations had been entered. As explained above, an estimated tenth of a scale division was 0.0002 ft, so that

$$\text{slope length} = 99.6 + (\text{mean sum of readings} \times 0.0002).$$

For computation, a decimal point was inserted (mentally) before the thousands digit of the sum of the readings, so that the formula used was

$$\text{slope length} = 99.6 + (\text{mean sum of readings} \times 2).$$

**Corrections**

The following corrections were applied to the slope length as so derived:

**(a) Corrections to each bay:**

- (1) Correction for inclination of scales,
- (2) Correction for deformation of catenary,
- (3) Correction for temperature,
- (4) Correction for slope,

**(b) Corrections to each section:**

- (5) Correction for error of scales,
- (6) Correction for N.P.L. standardisation,
- (7) Correction for field comparison with standards,

**(c) Corrections to whole base:**

- (8) Correction for variation of gravity,
- (9) Correction for difference of tension,
- (10) Correction for height above sea level,
- (11) Correction for N.P.L. re-standardisation.

All corrections were carried to 0.000 001 ft, the rounding-off to a lesser degree of accuracy and the conversion to links being left till the final step. In the case of corrections to each bay, the first three decimals (usually 0.000) were inserted as leading figures at the head of the computation columns, and only the last three decimals were shown as individual corrections. Corrections were derived from tables wherever possible, the tables being nearly always in critical form.

**Notation**

The symbols used in the following paragraphs are collected here:—

$l$	is the length of the tape	= 100 ft,
$w$	is the weight of the tape per unit length	= 0.00824 lb per ft,
$t$	is the tension	= 20 lb,
$a$	is the cross-section area of the tape	= 0.0025 sq in,
$E$	is Young's modulus of elasticity for invar	= $22.8 \times 10^6$ lb per sq in,
$h$	is the difference in height of the ends of the tape,	
$s$	is the sum of the rear and forward readings,	
$T$	is the temperature in degrees Fahrenheit,	
$\phi$	is the mid-latitude of the base,	
$R_A$	is the radius of curvature of the international spheroid in the azimuth of the base,	
$H$	is the mean height of the base above mean sea level.	

**Correction  
for inclination  
of scales**

When the tape is suspended in a single horizontal catenary, the graduated ends of the tape are inclined to the horizontal at an angle of  $\tan^{-1}(wl/2t)$ , or  $1^\circ 11'$ , and the readings are therefore made on a foreshortened scale. The length given in the certificate of standardisation for each of the 100-ft tapes refers to the horizontal distance between the mid-graduations at each end (i.e. 10 and 10). Therefore when the sum

of the scale readings is 20 (or 0.2 in the units adopted for computation), no error is introduced by the inclination of the scales, but when the sum of the readings is in excess or defect of 20, the effect must be taken into account and the readings reduced to the corresponding horizontal measurement. If the sum of the scale readings is  $s$ , the slope length is taken as

$$99.6 + 2s = 100.0 - 2(0.2 - s),$$

whereas the correct value is

$$100.0 - 2(0.2 - s) \cos 1^\circ 11'$$

The correction to be applied is therefore  $+ 2(0.2 - s)$  vers  $1^\circ 11'$ , or

$$+ (100 - \text{slope length}) \text{ vers } 1^\circ 11'.$$

This correction was tabulated against the slope length for the complete range of tape readings. As the maximum value is 0.000 085 ft, the correction was given in a critical table constructed from an inverse formula, and the tabular values were checked by the direct formula above.

This correction (which always reduces numerically the tape reading in excess or defect of 20) is applied so as to bring the slope length closer to 100, i.e. it is positive for lengths less than 100, and negative for lengths greater than 100.

The correction for inclination of scales could have been applied to the length of each section by working from the sum of the tape readings for all the bays of that section, but in this case it could not be so conveniently tabulated and would best be computed for each section.

#### Correction for deformation of catenary

When the ends of the tape are at different levels, the tape does not assume the horizontal catenary in which it was standardised, and a correction for deformation of catenary (or for difference of sag) is given by

$$+ \left(1 + \frac{2l}{aE}\right) \frac{w^2 l h^2}{24t^2}$$

The numerical value of the correction to a 100-ft length is

$$+ 0.000\ 000\ 70776\ h^2$$

For differences of height up to 12 ft, for which the correction is 0.000 101, this was given in a critical table; for differences of height greater than 12 ft, the correction was computed from the above formula.

#### Correction for temperature

The coefficient of linear thermal expansion of each tape had been determined at the National Physical Laboratory in 1937, and certificates of standardisation made later than this continued to quote the same coefficients. The tapes were standardised at 68°F, and the coefficients of expansion were given in equations containing terms in  $(T - 68)$  and  $(T - 68)^2$ .

The corrections for temperature to a 100-ft length of each of the four tapes were computed in a table covering rather more than the expected range of temperature. The table contained values for every degree, with differences and an auxiliary table of corrections for tenths of a degree.

The thermometers used in the field were tested at the Dominion Physical Laboratory, and corrections to the readings were given. Three thermometers were used at each bay, and it was found convenient to construct, for each selection of three thermometers in use, a critical table giving the corrected mean temperature corresponding to

the sum of the three readings. The temperature corrections to each of the two working tapes were also tabulated opposite the mean temperature.

The temperature corrections were computed for 100-ft lengths only. Numerical trials in extreme cases showed that use of the actual measured length made no significant difference to the correction.

#### Correction for slope

The slope length of each bay was obtained from the measured length by the application of the three corrections, for inclination of scales, for deformation of catenary, and for temperature. The horizontal length was then obtained from the formula,

$$\text{horizontal length} = \sqrt{(\text{slope length})^2 - (\text{difference in height})^2},$$

which was easily evaluated by calculating machine. The original computations were made by applying this formula independently to the two tape measurements of each bay.

In checking the computations, use was made of a table giving the corrections to 100-ft lengths for differences of height up to 4 ft, and a chart comprised of graphs of additional corrections for the actual slope length. The slope correction is the algebraic sum of the correction from the table (which is always negative) and the correction from the graph (which is negative for slope lengths less than 100 and positive for slope lengths greater than 100). The horizontal axis of the graphs represented slope length from 99.6 ft to 100.4 ft on a scale of 0.1 ft to 1 inch; the vertical axis represented the difference of height on a scale of 0.5 ft to 1 inch up to 4.0 ft. The correction curves, symmetrical about the middle ordinate (100.0 ft), were at a correction interval of 0.000 010 ft and ranged up to 0.000 310 ft.

#### Correction for error of scales

The standardisation certificates revealed that there was no error in the scales of tapes Nos. 1a and 2a, but that the scales on tapes Nos. 4 and 5 were slightly short. The error in the graduation of tape No. 5 (which was used only for measuring the 100-ft test bays) was not taken into account in field computation.

On tape No. 4, each scale was short by 0.0001 ft, i.e. the length of each scale was 0.3999 ft, and an estimated tenth of a scale division was therefore 0.0001 9995 ft instead of the assumed 0.0002 ft. There is no error, however, when the sum of the readings is 20 (i.e. 0.2 in the units adopted for computation), since the length of the tape under these conditions was given in the N.P.L. standardisation. An error enters when the sum of the readings is in excess or defect of 20.

If the sum of the scale reading is  $s$ , the slope length of a bay is taken as

$$99.6 + 2s = 100.0 - 2(0.2 - s).$$

The correct slope length is

$$100.0 - 1.9995(0.2 - s),$$

and the correction to be applied is therefore

$$+ 0.0005(0.2 - s).$$

If there are  $n$  bays in a section, the total correction to be applied is therefore

$$+ 0.0005(0.2n - \sum s),$$

where  $\sum s$  is the sum of the means of the tape readings.

### Correction for N.P.L. standardisation

The "standard length" of each tape was given in the certificate supplied by the National Physical Laboratory. The difference, standard length - 100, multiplied by the number of bays in the section (normally 33), was applied to the total length of each section as given by each tape.

### Correction for field comparison

A correction derived from the comparison of tapes in the field, on the assumption that the field standard tapes (Nos. 2a and 5) did not differ from their standard lengths, was applied to the total length of a section as given by each of the working tapes (Nos. 1a and 4), in a manner similar to that of the correction for N.P.L. standardisation.

A field comparison of tapes was made before the first measurement of each base, between the first and second measurements, and after the second measurement. The correction applied to the first measurement of a section was derived by linear interpolation between the values given by the first and second comparisons, while the correction applied to the second measurement of a section was derived by interpolation between the values from the second and third comparisons.

### Corrections to 20-ft tape

The 20-ft tape was graduated throughout in tenths and hundredths of a foot, so that the slope length was obtained directly as the mean of the tape readings. This tape was standardised on the flat, so that a correction for sag was necessary, while the correction for inclination of scales was not required.

A combined sag and deformation correction is given by

$$- \left( 1 + \frac{2t}{aE} \right) \frac{w^2 l^3 \cos^2 \theta}{24 t^2}$$

where  $\theta$  is the angle of inclination and the other symbols have the same significance and (except for  $l$ ) the same numerical values as before. Angles of inclination were not measured, but, since  $\cos^2 \theta = (l^2 - h^2)/l^2$ , the formula reduces to

$$- 0.000\ 000\ 007\ 078\ l(l^2 - h^2).$$

The coefficient of linear thermal expansion and the standard length were given in the certificate of standardisation. Since the bay measured with this tape could be any length up to 20 ft, the corrections could not conveniently be tabulated, but were computed as required.

### Correction for variation of gravity

The N.P.L. standardisation was carried out at Teddington, England, where  $g$ , the acceleration due to gravity, is 981.19 cm/sec/sec. A change in the value of  $g$  will cause a change in the elastic elongation of the tape, and the correction to a 100-ft length, as evaluated at the N.P.L., is

$$+ 0.000\ 041\ \Delta g,$$

where  $\Delta g = g_{\text{base}} - g_{\text{NPL}}$ . This correction was applied to the total length of the base as

$$+ 0.000\ 000\ 41\ \Delta g \times \text{length of base.}$$

The value of  $g$  at each base was computed from the value of  $g$  at the Christchurch Magnetic Observatory (980.534 cm/sec/sec) by the 1941 formula of Jeffreys,

$$g = g_C \frac{1 + 0.005\ 2895 \sin^2 \phi - 0.000\ 0059 \sin^2 2\phi}{1 + 0.005\ 2895 \sin^2 \phi_C - 0.000\ 0059 \sin^2 2\phi_C}$$

where the subscript C refers to Christchurch. This reduces to

$$g = 978.117(1 + 0.005\,2895 \sin^2\phi - 0.000\,0059 \sin^2 2\phi).$$

The value thus obtained was then corrected for height above sea level, the correction being

$$- 1.75 \, gH/R_A.$$

#### Correction for difference of tension

With the Macca base measuring equipment, the laboratory standardisation should be carried out using the actual straining weights, cords, and swivel-hooks that are used in the field. However, the tension equipment was not sent to the laboratory with the tapes, and the standardisation was carried out under a tension of 20 lb at each end of the tape. The correction to a 100-ft length for departure from the tension of standardisation was evaluated by the N.P.L. as

$$+ 0.0027 \, \Delta t,$$

where  $\Delta t = t_{\text{base}} - t_{\text{NPL}}$ , in lb.

The masses of the weights, cords, and swivel-hooks were determined by the Weights and Measures Branch of the Department of Labour, Wellington, both before and after the field work. From these determinations the tensions applied in field measurement were estimated, making allowance for some attachments that were broken in the field, so that the adopted tension varied slightly from base to base, ranging from 20.1623 lb to 20.1377 lb. The correction to the length of the base was

$$+ 0.000\,027 \, \Delta t \times \text{length of base}.$$

#### Correction for height above mean sea level

The correction for reduction to mean sea level is given by

$$- \frac{H}{R_A + H} \times \text{length of base}$$

The observed differences of height between adjacent measuring heads were used to obtain the heights of all the measuring heads, the mean of which gave the mean height of a section, and from the heights of all the sections the mean height  $H$  of the whole base was derived. For the purpose of computing these mean heights, the height of a short bay or a short section was weighted in proportion to its length. The radius of curvature in the azimuth ( $A$ ) of the base was computed from

$$R_A = \frac{\rho v}{\rho \sin^2 A + v \cos^2 A}$$

where  $\rho$  and  $v$  are the radii of curvature of the international spheroid, in meridian and perpendicular respectively, at the mid-latitude of the base.

#### Correction for N.P.L. re-standardisation

A further correction was applied after the tapes had been returned to the N.P.L. for restandardisation. The lengths of the field standard tapes at the times of the field comparisons were found by interpolation between the two laboratory standardisations on the assumption that the lengths had varied uniformly with time. If  $S$  is the later mean length of the standard tapes, and  $S_0$  the mean length used in the field comparisons, then the correction for re-standardisation is

$$+ (S - S_0) \times \text{no. of 100-ft bays in the base}.$$

### Adopted limits of error and probable error

The mean of the two tape measurements of a section on the forward measurement was taken as the first measurement, and the mean of the two tape measurements on the return measurement was taken as the second measurement. The limit allowed for the difference between the first and second measurements was arbitrarily adopted as 1 in 250 000, or 0.013 200 ft in a section of 3300 ft. The mean of the first and second measurements was then taken as the length of the section.

The probable error of each section was computed as 0.6745 times one-half the difference between the first and second measurements. The probable error of the whole base was taken as the square root of the sum of the squares of the probable errors of the individual sections.\*

### Revision of North Island bases

The lengths of the Matamata and Waitemata bases as derived from the remeasurement with the Macca equipment in 1947 were accepted as the correct lengths, and these lengths were compared with those of the original measurements with Langmuir's equipment in 1911. The corrections, in links, necessary to obtain the 1947 values from the 1911 values were found to be:

Matamata base,    - 0.0000 1216 2 × length of base,  
Waitemata base,   - 0.0000 0733 4 × length of base.

The mean of these two values,

- 0.0000 0974 8 × length of base,

was adopted as the correction to be applied to the other North Island bases. The maximum correction, at the Kaingaroa base, was 0.8 ft (0.24m).

Except in the case of the Eltham base, the heights used in the original computations were all somewhat too great (55 ft in the case of the Kaingaroa base). The reductions to sea level were therefore recomputed.

### Final results

The final lengths of all the bases were as follows. Values in metres have been derived using the Sears ratio (see next page), not the SI ratio adopted on metrication in 1973.

	Links	Metres
Waitemata base	41 790.479 ± 0.0079	8 406.8925
Matamata base	54 799.085 ± 0.0081	11 023.8032
Kaingaroa base	91 198.176 ± 0.0147	18 346.1229
Eltham base	79 604.347 ± 0.0128	16 013.8195
Wairarapa base	64 776.063 ± 0.0219	13 030.8485
Culverden base	55 003.495 ± 0.0067	11 064.9239
Waitaki base	59 997.557 ± 0.0065	12 069.5676
Riversdale base	56 541.119 ± 0.0066	11 374.2441

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\*"Probable error" had long been used in statements of precision in surveying. Since that time it has been abandoned in favour of "standard error" – which in practice amounts to replacing the factor 0.6745 by 1.



## VII. Astronomical and Geodetic Computation

### Figure of the earth

The shape of the surface of mean sea level, assuming this to be extended over the land areas, is not known precisely but it is known to be irregular. It is called the geoid. As it is impossible to derive formulae for computation upon an unknown surface, it is necessary in practice to select some defined surface as the figure of the Earth, and the most satisfactory choice is an oblate spheroid, the surface generated by the revolution of an ellipse about its minor axis. Theoretical studies show that the Earth, if it were of homogeneous material, would assume such a figure, and it is found to be a close approximation to the true figure.

The decision to adopt the international (Hayford) spheroid as the figure of the Earth for New Zealand surveys was made by H.E. Walshe, Surveyor-General, about 1935. New Zealand surveys being hitherto regarded as plane, the choice of spheroid was unrestricted, and he was influenced in this decision by the recommendation of the International Association of Geodesy that all countries computing new triangulation or recomputing old should adopt the Hayford spheroid. The choice has since been found to be very suitable.

All measurements of length in surveys and on plans affecting land titles in New Zealand were required to be in terms of a chain of 100 links, having a length of 66 imperial feet. The conversion from metric to British units was made by the Sears (1927) metre-foot relation, leading to the following elements of the international spheroid:

major semiaxis:	$a$	= 31 706 827.42 links
minor semiaxis:	$b$	= 31 600 070.43 links
compression:	$(a - b)/a = f = 1/297$	= 0.003367
eccentricity:	$(a^2 - b^2)/a^2 = e^2 = 593/88209$	= 0.00672 26700 22333 ...
2nd eccentricity:	$(a^2 - b^2)/b^2 = e'^2 = 593/87616$	= 0.00676 81701 97224 ...

### Geodetic tables

When the computation of the triangulation was begun, no tables for machine computation on the international spheroid were available, and tables covering the New Zealand range of latitude were therefore computed by the Computing Branch of Head Office and were later published by the Government Printer under the title, *Geodetic and Transverse Mercator Projection Tables: for the computation of geodetic and transverse Mercator projection positions between latitudes 34° and 48° (international spheroid)*. Values are given for every minute of latitude, with interlinear first differences where necessary.

If  $\rho$  and  $v$  are the radii of curvature of the spheroid, in the direction of the meridian and of the decuman respectively, and  $\phi$  is the geodetic latitude, the factors required in the computation of geodetic positions are:

$$K = \frac{1}{2} \text{ arc } 1'' \cdot \frac{v}{\rho} \sin \phi \cos \phi,$$

$$M = \frac{1}{\rho \text{ arc } 1''}$$

$$N = \frac{1}{v \text{ arc } 1''}$$

$$O = \frac{1}{2\rho v \text{ arc } 1''}$$

$$Q = \frac{\tan}{2\rho v \text{ arc } 1''}$$

For convenient positioning of the decimal point in computation with a calculating machine, the tables give values of  $K \times 10^8$ ,  $M \times 10^5$ ,  $N \times 10^5$ ,  $O \times 10^{10}$ ,  $Q \times 10^{10}$ . Before a product with one of these functions is formed, a distance in links is therefore multiplied by  $10^{-5}$ , and a difference of longitude in seconds by  $10^{-4}$ .

#### Trigonometric and other tables

Trigonometric functions to eight figures, during most of the preliminary computation, were taken from Gifford's *Natural Sines* and *Natural Tangents*, until the American reprint of Peters' *Eight-place Tables of Trigonometric Functions for Every Sexagesimal Second of the Quadrant* became available. Peters' tables were used for all the final computation. In astronomical work, use was made of the Nautical Almanac Office's *Seven-figure Trigonometrical Tables for Every Second of Time*.

Corrections for astronomical refraction were taken from the tables to  $0''.01$  for every minute of zenith distance computed under the direction of W.T. Neill and published in *Records of the Survey of New Zealand*, 3:113-131. These give mean refraction at 30 in. pressure at  $50^\circ$  F, with provision for correcting them for changes in air density due to changes in temperature and pressure. In practice it was found simpler to take the correcting factors for temperature and pressure from Tables 295 and 296 in Hayford's *Geodetic Astronomy* (or other books which reprint the same tables).

#### Geodetic position computation

Preliminary values of geodetic positions — latitudes and longitudes of stations and azimuths of rays — had been computed at intervals up to 1947, mainly for use in control of topographical mapping. When the final values of the lengths of the base-lines became available, a "first adjustment" of the observations was carried out, giving adjusted spherical angles and spheroidal lengths throughout the net. Starting from some station to which geodetic values had been assigned, new values of latitude, longitude, and azimuth were then computed. Laplace equations were then formed and a "second adjustment" (more correctly a modification of the first adjustment) was carried out. Final values of latitude, longitude, and azimuth were then computed for all stations throughout the network.

The method of geodetic position computation, both in the preliminary adjustment and in the final work, was that due to Colonel A.R. Clarke, which gave adequate accuracy over the lengths of the lines observed. The notation and conventions adopted were:

- $\phi$  south latitude, reckoned positive,
- $\lambda$  east longitude, reckoned positive,
- $A$  azimuth, reckoned eastward from north,
- $s$  spheroidal distance.

The subscripts A and B denote the quantities at the standpoint and at the forepoint respectively. Latitude and longitude were expressed to  $0''.0001$ , azimuth to  $0''.001$ .

#### Direct computation by the Clarke formulae

If  $(\phi_A, \lambda_A)$  are the known latitude and longitude of the point A, and  $A_A$  and  $s$  are the azimuth and distance to the point B, the latitude and longitude  $(\phi_B, \lambda_B)$  and the reverse azimuth  $A_B$  are found as follows.

$$\text{Let} \quad x' = s \cos A_A, \quad y' = s \sin A_A;$$

then the midlatitude of the geodesic AB is given by

$$\phi_A - x' \cdot \frac{1}{2}M,$$

where  $\frac{1}{2}M$  is taken for the latitude region of the geodesic, the precise value not being critical. The geodetic factors  $M$  and  $O$  are then interpolated for the midlatitude.

Then

$$\begin{aligned}\epsilon &= x'y'O, \\ x &= s \cos(A_A - \frac{2}{3}\epsilon), \\ y &= s \sin(A_A - \frac{1}{3}\epsilon), \\ \phi'_B &= \phi_A - xM.\end{aligned}$$

The geodetic factor  $Q$  is interpolated for  $\phi'_B$  which is a first approximation to  $\phi_B$ . Then

$$\eta = y^2 Q, \quad \phi_B = \phi'_B - \eta.$$

The geodetic factor  $N$  is interpolated for  $\phi_B$ . Then

$$\begin{aligned}\Delta\lambda &= \frac{yN}{\cos(\phi_B + \frac{1}{3}\eta)} \\ \lambda_B &= \lambda_A + \Delta\lambda, \\ -\Delta A &= \Delta\lambda \sin(\phi_B + \frac{2}{3}\eta) + \epsilon, \\ A_B &= A_A \pm 180^\circ + \Delta A.\end{aligned}$$

The values of  $\phi_B$  and  $\lambda_B$  computed through different routes were required to agree within 0".0002, while the values of  $A_B$  for the two routes were required to agree with the contained spherical angle within 0".002. In the majority of cases, the values agreed exactly or differed by only one unit in the end-figure; in the latter case, the value derived from the shorter route was generally adopted. In the few cases where a disagreement of two units occurred, the mean value was adopted.

In all but a few cases, the azimuth of the geodesic was used in this computation. In the very large triangles of the Cook Strait quadrilateral and in the northern part of the South Island, it was necessary to correct the azimuth of the geodesic by the amount

$$\frac{e'^2 \cos^2 \phi \cdot s^2 \sin A \cos A}{6 \sqrt{2} \text{arc } 1''}$$

to give the azimuth of the normal plane section, before the required agreement in computation could be achieved.\*

Inverse  
computation  
by the  
Clarke formulae

For the inverse problem of finding the two azimuths and the distance between two points whose latitudes and longitudes are known, the geodetic factors  $K$  and  $N$  are interpolated for  $\phi_B$ . Then

$$\begin{aligned}\Delta\lambda &= \lambda_B - \lambda_A, \\ \eta &= \Delta\lambda^2 K, \\ \phi'_B &= \phi_B + \eta, \\ \text{midlatitude} &= \frac{1}{2}(\phi_A + \phi'_B).\end{aligned}$$

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\*Many different formulae have been proposed for point-to-point computation on the spheroid, and it is perhaps not sufficiently appreciated that they do not all use the same curve for the line joining two points. The Clarke formulae treat this curve as being the plane section containing the normal at the standpoint and passing through the forepoint, so that a different curve is used when standpoint and forepoint are interchanged. For most rays, both curves so nearly coincide with the geodesic that the difference can be ignored, but in long rays the difference becomes appreciable. Since the use of electronic calculators became general, the Department has used an extended form of formulae, originally due to Legendre, which use the azimuths and length of the geodesic.

The geodetic factors  $M$  and  $N$  are interpolated for the midlatitude. Then

$$x = \frac{\phi_A - \phi'_B}{M}$$

$$y = \frac{\Delta\lambda \cos(\phi_B + \frac{1}{2}\eta)}{N}$$

$$\epsilon = xyO$$

Corrections are next applied to  $x$  and  $y$ , as follows:

$$x' = x - \frac{2}{3} \epsilon y \text{ arc } 1'' = x - [323.2091 \epsilon y],$$

$$y' = y + \frac{1}{3} \epsilon x \text{ arc } 1'' = y + [161.6046 \epsilon x],$$

where the bracketed quantities are corrections to the eighth decimal place in  $x$  and  $y$ . The azimuth  $A_A$  is then found from

$$\tan A_A = \frac{y'}{x'}, \text{ or } \cot A_A = \frac{x'}{y'},$$

the usual practice being to divide the smaller value by the larger so as to avoid the region of the table of tangents and cotangents where differences are unwieldy. The reverse azimuth is found as before from

$$-\Delta A = \Delta\lambda \sin(\phi_B + \frac{1}{2}\eta) + \epsilon,$$

$$A_B = A_A \pm 180^\circ + \Delta A,$$

and the distance from

$$s = \sqrt{(x')^2 + (y')^2}.$$

## Star ephemerides

At the beginning of the astronomical observations, the apparent places of stars were taken from the *Nautical Almanac*, the ephemerides at that time being based on Eichelberger's catalogue. Occasionally stars were observed which did not appear in the *Nautical Almanac*; in such cases the apparent places were sought in the *American Ephemeris* which included a number of additional stars, or were computed from the mean places in Eichelberger's catalogue by means of independent day numbers (for which a computation form was printed) or of Besselian day numbers. When stars were observed which could not be identified in Eichelberger's catalogue – which happened more often in the case of time observations which were not planned in advance – use was made of other catalogues such as those of Boss and the Cape.

When the annual *Apparent Places of Fundamental Stars* appeared in 1941, a sufficient number of stars for all purposes could be obtained from it, and reference to other sources was rarely necessary. The ephemerides in these volumes were based upon the catalogue whose German title is usually abbreviated to FK3. The use of different catalogues at different times and at different stations has meant that there are systematic differences in the resulting astronomical positions of stations, but as these differences are in most cases less than the expected errors of observation, no allowance has been made for them in computation.

## Astronomical computation

For the three standard methods of astronomical observation – time by zenith distances of stars near the prime vertical, latitude by circummeridian zenith distances, and azimuth by hour angles of close circumpolar stars – computation forms were printed which provided for all the necessary entries in a compact space and, as far as possible, contained also the formulae to be used. Computation was done by calculating machine, and the formulae for the solution of spherical triangles were derived from the fundamental formulae and did not follow the logarithmic formulae which appear in most textbooks.

All astronomical computations were done independently by two computers, the results being later compared and any differences investigated. The results were accepted only when complete agreement had been reached.

In astronomical computation, the following symbols and conventions were used:

- $\phi$  latitude of station, negative south,
- $\lambda$  longitude of station, positive east,
- $a$  right ascension of star,
- $\delta$  declination of star, positive north, negative south,
- $t$  hour angle of star, positive west, negative east, or positive west from 0<sup>h</sup> to 24<sup>h</sup>,
- $z$  zenith distance of star,
- $A$  azimuth of star, reckoned eastward from north,
- $T$  chronometer time

Most computations required initial approximations to the latitude and longitude of the station, and these were obtained from the provisional solution of the triangulation net, or occasionally from field computation of the observations.

#### Computation of time and chronometer error

The printed form provided for three stars, with four observations on each star. If  $z$  is the zenith distance of a star corrected for refraction, the hour angle was computed from

$$\cos t = \frac{\cos z - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

Usually two observations were made to each star, sometimes three or more. In such cases, the values of  $\cos z$ , and the differences between consecutive values, were entered on the form. The values of  $\sin \phi \sin \delta$  and  $\cos \phi \cos \delta$  were also recorded. The simple procedure for machine computation was then:

- (1) Evaluate the reciprocal of  $\cos \phi \cos \delta$  and set it on the keyboard;
- (2) Multiply this reciprocal negatively by  $\sin \phi \sin \delta$ ;
- (3) Clear the multiplier register only, and multiply by the first value of  $\cos z$ , thus getting the first value of  $\cos t$ ;
- (4) Without clearing any registers, alter the multiplier to the second value of  $\cos z$ , thus getting the second value of  $\cos t$ . Similarly for the third and later observations.

When only one observation of each star is made, a better formula is

$$\cos t = \frac{\cos z \sec \delta - \tan \delta \sin \phi}{\cos \phi}$$

which can be evaluated without a transfer of figures from product register to keyboard and without the writing of intermediate results.

The local sidereal time was then given by

$$\text{L.S.T.} = a + t,$$

and the chronometer error by  $T - \text{L.S.T.}$

The mean result for the two or more observations on one star was computed, and the mean result derived from a star in the east and a star in the west at about the same time was then taken as the chronometer error at the mean of the times.

The chronometer was generally set to keep approximate local sidereal time. If it was set to keep approximate mean solar time, no allowance was made for this; the difference between sidereal and solar time was treated merely as part of the error and rate of the chronometer.

At least two determinations of the chronometer error were required each night, at or near the beginning and at or near the end of the night's observations. In most cases, intermediate determinations were also available, as time observations were made during any convenient break in other work. If observation had been abandoned before a second time determination had been made on any night, the determinations on the preceding and following nights were used to determine the rate of the chronometer. The results were, of course, required to be consistent, and time observations were very rarely rejected.

When a calculating machine is used, it is just as easy to interpolate the time as it is to interpolate the chronometer error, and although the other computation forms provided spaces for entering the chronometer error at stated times, it was generally found convenient to interpolate the L.S.T. at a given chronometer time, or the chronometer time at a given L.S.T. as the case might be, without directly interpolating the chronometer error. Such computation as was needed for this was recorded on the back of the computation form.

**Latitude by  
circummeridian  
zenith  
distances**

The computation form provided for the reduction of all the observations on one star, together with a summary of the results obtained from all the star pairs, this last being entered on the first sheet when the sheets were bound together.

The approximate latitude being known, the approximate zenith distance was computed from

$$z = \phi - \delta, \text{ or } z = \phi + (180^\circ + \delta),$$

according as the transit was observed above or below the pole. This gives a positive zenith distance for a star to the south of the zenith and a negative zenith distance for a star to the north. However, the signs of both the zenith distance and the latitude were usually ignored in computation, only the numerical values being important. The factors,

$$A = \cos \phi \cos \delta \operatorname{cosec} z, \quad B = A^2 \cot z,$$

were next computed. The chronometer time of transit was obtained by adding the chronometer error to the right ascension of the star.

The form provided columns for the chronometer time of each observation, the difference between this time and the chronometer time of transit, the factors  $m$  and  $n$  corresponding to this interval of time, the correction to the zenith distance, the observed circummeridian zenith distance, the corrected meridian zenith distance, and the residual. The values of  $m$ , and where necessary  $n$ , were taken from Chauvenet's *Spherical Astronomy*, vol. II, or from other books which have reprinted the same tables.

The meridian zenith distance for each observation was computed as  $z - Am + Bn$  for a star at upper transit, or  $z + Am + Bn$  for a star at subpolar transit. The second correction term was so rarely needed that it was possible to express the first term in the equivalent form,

$$(\pi/5760) A \Delta t^2, \text{ or } 545''.4154 A (\Delta t \times 10^{-3})^2,$$

where  $\Delta t$  is in seconds of time. The constant 545''.41544 could then be set on the keyboard and multiplied by the squares of the successive values of  $\Delta t$ , these being read from tables such as Barlow's.

If face had been changed during the observations, a correction for the index error of the vertical circle was applied to the meridian zenith distance. This correction was obtained from the meridian zenith distances themselves, and was one-half of the difference between the mean of the face left and the mean of the face right observations; where an equal number of face left and face right observations had been taken, this became  $(\Sigma L - \Sigma R)/n$ . The correction was applied positively to the results on one face, negatively to the results on the other, so as to make the results roughly equal.

The mean meridian zenith distance was next computed. The observed and the corrected zenith distances were expressed to 0''.1, but the mean meridian zenith distance was

carried to 0".01. The residuals were then entered. It was found convenient to fix an arbitrary rejection limit at a residual of 4"; where a residual greater than this occurred, that observation was rejected and a new mean was taken. A correction for refraction was applied to the mean meridian zenith distance.

The latitude was then obtained from

$$\begin{aligned}\phi &= z + \delta && \text{for upper transit,} \\ \text{or } \phi &= z - (180^\circ + \delta) && \text{for subpolar transit.}\end{aligned}$$

These equations are correct with the conventions mentioned above, but in fact the signs were generally ignored, as the required result was known with sufficient accuracy to make the method of computation obvious.

The probable error of the latitude derived from observations on one star was taken as

$$\pm 0.6745 \sqrt{\frac{\sum v^2}{n(n-1)}}$$

where  $v$  denotes a residual in meridian zenith distance and  $n$  is the number of observations. The result from a star in the north was paired with that from a star in the south at about the same zenith distance, and the mean of the two results was taken. The probable error of the mean was taken as one-half of the square root of the sum of the squares of the probable errors of the separate results. The latitudes derived from the various pairs of stars were then weighted inversely as the squares of their probable errors, and a weighted mean was computed as the final astronomic latitude of the station. The probable error of the weighted mean was taken as

$$\pm 0.6745 \sqrt{\frac{\sum wv^2}{(n-1)\sum w}}$$

where  $w$  denotes a weight. Tables of  $0.6745/\sqrt{n(n-1)}$  and  $0.6745/\sqrt{(n-1)}$  were computed for values of  $n$  up to 100 to assist in this computation.

**Latitude by  
ex-meridian  
zenith distances  
of a close  
circumpolar  
star**

A printed form was not available for this computation, as the method was not often used, and a blank sheet of paper was ruled into columns as required. From the chronometer times of the observations the corresponding local sidereal times were obtained, and the hour angles  $t$  were then derived from the differences between the right ascension and the local sidereal times. As the sign of  $t$  did not affect the computation, the values less than  $12^h$  were taken, without regard to sign. The corresponding zenith distances, corrected for refraction, were converted into altitudes  $h$ . From the declination, the polar distance  $p$ , expressed in seconds of arc, was derived, and the constants  $\frac{1}{2}\text{arc } 1''.p^2$  and  $\frac{1}{3}\text{arc } 1''.p^3$  were computed. For each observation, the latitude was then given by

$$\phi = h - p \cos t + \frac{1}{2} \text{arc } 1'' \cdot p^2 \sin^2 t \tan h + \frac{1}{3} \text{arc } 1'' \cdot p^3 \cos t \sin^2 t.$$

Columns were provided for recording  $\sin t$ ,  $\cos t$ , and  $\tan h$  for each observation, and for entering the three correction terms in the above formula. The latitudes so derived were corrected for index error of the vertical circle as explained above, and the mean latitude, the residuals, and the probable error were computed as before.

**Latitude by  
meridian  
zenith  
distances**

This method was used at only two stations. The only correction to be applied to the observed zenith distances was that for refraction, after which the latitude was at once derived by the formulae given above under circummeridian zenith distances. The stars were paired in approximately equal zenith distances to the north and to the south of the zenith, and the mean latitude from each pair was free from index error as the same face was used for all observations. In some cases, two stars had been observed at about the same zenith distance on the same side of the zenith; in such cases, the mean of the two results was taken before it was combined with the corresponding

observation on the other side of the zenith. The mean of the latitudes given by the various pairs, the residuals, and the probable error were computed as before.

**Azimuth by  
hour angles of  
close  
circumpolar stars**

Twelve sets of observations were made in this case, and a printed form provided for the computation of six sets. As the right ascension and declination of the star were not necessarily constant throughout the period of observation, they were interpolated for each set. The local sidereal time of each observation was derived from the chronometer time and the chronometer error. The hour angle was then given by

$$t = \text{L.S.T.} - \alpha.$$

Hour angle in this case was reckoned westward and could have any value up to  $24^{\text{h}}$ .

To avoid the troublesome interpolation of  $\tan \delta$  where  $\delta$  was near  $90^\circ$ , the fundamental formula of the spherical triangle was put in the form,

$$\tan A = - \frac{\cot \delta \sin t}{\cos \phi - \sin \phi \cot \delta \cos t}$$

$A$  being the azimuth of the star. Successive values of  $\cot \delta \cos t$  were recorded, and the differences between them, from which successive values of the denominator  $\cos \phi - \sin \phi \cot \delta \cos t$  were obtained. The azimuth  $A$  was always near  $180^\circ$ , greater if  $\tan A$  were positive, less if  $\tan A$  were negative.

The recorded chronometer time was the mean of the times of transit across the vertical thread in several positions of the telescope, usually four, the observations being made in quick succession. A diaphragm with several threads was not used. The difference  $\Delta t$  between the mean and each separate time was computed, and the corresponding value of  $m$  was found from tables for correcting circummeridian observations or was computed from

$$m = 0.00054 \ 54154 \Delta t^2,$$

where  $\Delta t$  is in seconds of time. A correction to  $A$  for the curvature of the star's path was then computed from

$$\text{mean } m \times (-\tan A).$$

The readings of the striding level were recorded for each set, and the level correction,

$$(E - W) \tan h \times (\frac{1}{2} \text{ value of one division of level}),$$

was applied to the observed horizontal circle reading of the star. The altitude  $h$  (or the zenith distance  $z$ ), to the nearest minute, was recorded at the time of observation, or could easily be computed. The angle measured clockwise from the star to the referring mark was then found, and this, added to the azimuth of the star, gave the azimuth of the referring mark.

The mean of the twelve sets, with the residuals and the probable error, were then found. Where a set had been repeated, a mean was taken as the value of that set unless there was reason for rejecting either the original observation or the repeat. The final azimuth was the mean of not more than twelve sets. The correction for diurnal aberration,

$$0''.3196 \cos A \cos \phi \sec h,$$

was applied to the final mean azimuth of the referring mark. For a circumpolar star,  $\phi \simeq h$  and  $\cos A \simeq -1$ , so that this correction was in fact always equal to  $-0''.32$ .



### Longitude by meridian transits of stars

The computation in this case was done in the field. Two cyclostyled typewritten forms were used, the first providing for the computation of the longitude by each of the six stars in one half-set. The chronometer time of transit was corrected for chronometer error, obtained by interpolation between the radio time signals received, for diurnal aberration by

$$0^{\text{s}}.021 \cos \phi \sec \delta,$$

which is negative for stars at upper transit and positive at lower transit, and for level error by  $Bb$ , where

$$B = \cos z \sec \delta,$$

and  $b$ , the inclination of the horizontal axis in seconds of time, was derived from the striding level readings and the value of one division of the level scale,  $0''.92$  or  $0^{\text{s}}.0613$ . For each set of level readings with level both direct and reversed, the difference of level readings,  $\Sigma W - \Sigma E$ , was found. The mean difference for observations with the telescope north, and the mean with telescope south, were found, and then the mean of these two means. The last mean, multiplied by one-quarter of the value of a level division,  $0^{\text{s}}.0153$ , was the level factor which was assumed constant for the half-set. The star coefficient  $B$  is negative for stars at lower transit but is otherwise positive. The application of these corrections to the recorded chronometer time gave the G.M.T. of transit.

The G.S.T. of transit was then computed from

$$\text{G.S.T.} = \text{G.S.T. of } 0^{\text{h}} \text{ G.M.T.} + \text{sidereal equivalent of G.M.T.,}$$

and the longitude  $\lambda'$  from

$$\lambda' = \alpha - \text{G.S.T.}$$

The second computation form summarised the results from two half-sets, and provided for the computation of corrections for collimation and for azimuth,  $Cc$  and  $Aa$ , where

$$C = \sec \delta, \quad A = \sin z \sec \delta,$$

and  $c$  and  $a$  were found from the observations themselves,  $c$  being the error of collimation and  $a$  the error in azimuth, both in seconds of time.  $A$  is negative for stars between the zenith and the pole but otherwise positive; at upper transit,  $C$  is positive for "clamp west", negative for "clamp east". For each star, an observation equation,

$$\Delta\lambda + Cc + Aa + (\lambda - \lambda') = 0,$$

was written,  $\lambda$  being the mean longitude (rounded off to the nearest second in the field computation) as given by the twelve stars, and  $\Delta\lambda$  being a correction to this mean longitude. The twelve equations were then combined in groups to give a number of equations equal to the number of unknowns. Thus, in each half-set the equations for the three most northerly stars were added algebraically, and also those for the three most southerly stars. Each set therefore gave four equations from which the four unknowns,  $\Delta\lambda$ ,  $c$ ,  $a_E$ ,  $a_W$ , were found. The final longitude was then  $\lambda + \Delta\lambda$ , and the residuals for each star were found by substituting these values in the observation equations.

### Accuracy of astronomical and geodetic positions

Astronomical latitudes, longitudes, and azimuths were computed to  $0''.01$ . The probable errors were as follows:

	Average	Maximum	No. of stns
	"	"	
Latitude	$\pm 0.28$	$\pm 0.69$	65
Longitude	$\pm 0.20$	$\pm 0.30$	11
Azimuth	$\pm 0.31$	$\pm 0.48$	39

No account was taken of the variation of the pole, the error from this cause being less than the average probable error of the observations.

Geodetic latitudes and longitudes were computed to  $0''.0001$  and geodetic azimuths to  $0''.001$ . Some criticism has been made of these degrees of accuracy, on the grounds that they are well beyond the limits of observation. Such criticism misses the point — that these are not observed positions, but are computed positions on an idealised spheroid; in other words, they are not “real” positions but are hypothetical positions which give internal consistency over the network. As such, they may justifiably be expressed to any degree of accuracy attainable by computation, and the limits adopted were found to be convenient in practice.

## VIII. Adjustment of the Observations.

### Prologue

Computation of the adjustment of a triangulation network today is done by electronic computer, and generally by the method of variation of coordinates. When the adjustment of the first-order net was done, however, electronic computers were not available. The computation was done with the aid of electromechanical calculating machines and by the then usual method of condition equations. Textbooks were of little use – such as were available were written for users of logarithms – and techniques were developed by the staff of the Computing Branch.

### Purpose of adjustment

The directions in a triangulation figure are subject to certain conditions – for example, the three angles of a triangle should total to  $180^\circ +$  the spherical excess of the triangle. In practice, the observed values never satisfy all of the necessary conditions, and it is customary to adjust them so that they do. The application of adjustment “corrections” is sometimes regarded as the removal of errors of observation, but this is a misunderstanding of the situation. A different selection of observations would lead to different adjusted values, so that the “true” or errorless values are not attainable. The purpose of adjustment is to produce internally *consistent values*. If the observations are carefully made, such values should be close to the “true” ones, but it is not claimed that they are identical.

### Adjustment figures

For the adjustment of the observations, each of the eight base nets was regarded as a separate figure and adjusted independently. The remainder of the net was divided into four figures, two in each island. The earliest part of the North Island net, extending from the Kaingaroa base to East Cape, had been observed to a slightly lower degree of accuracy than the later part, and it was considered that it should not influence the main adjustment. The net was therefore divided along the lines between the successive stations, Putauaki, Hikurangi, Taupiri, Pohokura, and Mohaka. The main network, to the west and south of the dividing line, was then adjusted as a unit; the smaller network, to the north and east, was then adjusted to fit the main network along the dividing line. The South Island network was already divided into two parts by the Culverden base net; the part to the north, including the connection to the North Island net along the line from Kapiti No 2 to Papatahi, was adjusted as a unit, the part to the south of the Culverden base net was adjusted as another unit. In both islands, some subsidiary networks were omitted from the main adjustment and were later adjusted to fit the main net.

### Schedules of directions and of triangles

The direction method of adjustment was used throughout. For each part of the net to be adjusted as a unit, the observed directions were numbered consecutively, proceeding in clockwise sequence at each station. Two computation books were used, one for the North Island and one for the South Island, and diagrams of the net, showing the station names and the direction numbers, were drawn in these. A schedule of the observed directions was entered following the diagram, this schedule providing for the observed value of the direction, the correction to be derived from the figure adjustment, and the seconds of the corrected direction.

A schedule of triangles was then prepared, the angles of each triangle being arranged in the numerical order of their direction numbers and the triangles being also arranged in the numerical order of their first angles. All the triangles of the figure were listed. In preparing such a list of triangles, a preliminary step was to consider each direction

in turn, and to list, on a piece of scrap paper, all the angles of triangles in which that direction was the first or negative one. The first angle on this list, and the other two angles of the same triangle, were then entered in the schedule of triangles and the angles were crossed off the list. The first angle of those remaining, and the other two angles of the same triangle, were then entered in the schedule of triangles and the angles were crossed off the list; and so on until all the triangles had been listed. This procedure ensured that triangles were neither repeated nor omitted, and that they were arranged in a systematic order for ease of reference.

The schedule of triangles allowed for further information to be entered later, columns being provided for: the two direction numbers of the angle, the observed angle, the correction to the angle derived from the figure adjustment, the corrected spherical angle, one-third of the spherical excess, the Legendre or "plane" angle, the sine of the Legendre angle, and the length of the side opposite the angle.

#### Spherical excess

The first-order triangulation was adjusted on the spheroid, the spherical excess of each triangle being computed from the two-sides-and-included-angle formula,

$$\epsilon = 0.ab \sin C,$$

where  $a$  and  $b$  are the side lengths derived from a preliminary approximate solution,  $C$  is the observed angle to the nearest second, and  $0$  is the geodetic factor taken from *Geodetic and Transverse Mercator Projection Tables* for the mean latitude of the triangle. It was found convenient to have the values of  $0$  shown by a set of lines on a diagram of the net, so that the requisite value for any triangle could be readily found without a direct evaluation of the latitude.

The spherical excesses were verified by the balancing condition that in two sets of triangles covering the same area the sums of the spherical excesses must be identical. The error in any spherical excess, as revealed by this test, was never more than  $0''.001$ , and was removed by altering any spherical excess not exactly divisible by 3, where such occurred, or by a quite arbitrary adjustment.

#### Error of closure

For each triangle, the numerical values of the three observed angles were entered in the schedule of triangles, and the spherical excess was shown below the angles. The error of closure of the triangle,

$$A + B + C - \epsilon - 180^\circ,$$

where  $A$ ,  $B$ ,  $C$  denote the observed angles, was then entered below the spherical excess. The summation of the three angles was done by calculating machine, and the spherical excess was subtracted from the sum, so that the error of closure was obtained without recording the sum of the angles. The sum, as it appeared in the register of the machine, however, was inspected for gross errors in the degrees or minutes. The errors of closure were checked by a balancing condition exactly similar to the balancing of the spherical excesses.

#### Condition equations

Three kinds of condition equations were required to be satisfied in the first adjustment of a figure:

- (a) *angle equations*, ensuring that the sum of the three angles of every triangle should be equal to  $180^\circ + \epsilon$ ,
- (b) *side equations*, ensuring the correct relations between the angles used in computing sides, so that any side has the same length when computed through different routes,
- (c) *length equations*, when the figure contains more than one fixed length, ensuring the correct relations between the angles used in computing lengths, so that any

fixed length retains its fixed value when computed from another fixed length.

Angle and side equations controlled the shape of the figure, length equations controlled the scale. At a later stage of the work, Laplace azimuth equations were introduced to control the orientation.

### Angle equations

If the angles of a triangle, in terms of the direction numbers, are, say,  $-1+2$ ,  $-15+16$ ,  $-19+20$ , and  $v$  denotes the correction to a direction, then the angle equation is

$$-1-v_1+2+v_2-15-v_{15}+16+v_{16}-19-v_{19}+20+v_{20}=180^\circ+\epsilon,$$

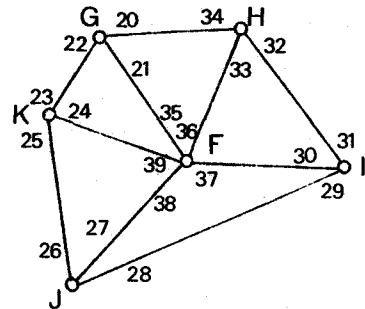
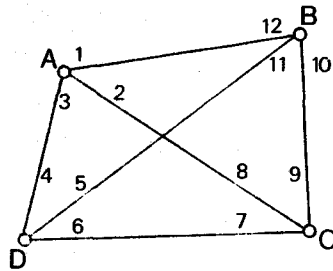
which can be rearranged as

$$-v_1+v_2-v_{15}+v_{16}-v_{19}+v_{20}+(-1+2-15+16-19+20-\epsilon-180^\circ)=0,$$

where the  $v$ 's are the unknowns to be found from the adjustment, and the bracketed term is the absolute term or the error of closure of the triangle.

### Side equations

The side equation can be found by equating two values of the one side computed from the same initial side through two different chains of triangles. In practice, side equations are formed for two main types of figure, a quadrilateral with both diagonals observed and a polygon with an interior station. In both cases, the side equation can be written mechanically without going through any formal derivation. One point is selected as the "pole" of the equation; in the case of the polygon it is the interior station, in the case of the quadrilateral it may be one of the vertices or the intersection of the diagonals. The lines radiating from the pole are written down in order, as the numerator of a fraction. The denominator is formed by writing the same lines, each moved one place to the left, with the first side transferred to the end. Each side is then replaced by the sine of the opposite angle, giving the side equation in terms of the angles.



Thus, for the quadrilateral ABCD, with the pole at A, the side equation is

$$\frac{AB \ AC \ AD}{AC \ AD \ AB} = 1,$$

or 
$$\frac{\sin(-8+9) \sin(-4+6) \sin(-11+12)}{\sin(-10+12) \sin(-7+8) \sin(-4+5)} = 1,$$

and the side equation for the polygon GHIJK is

$$\frac{FG \ FH \ FI \ FJ \ FK}{FH \ FI \ FJ \ FK \ FG} = 1,$$

or 
$$\frac{\sin(-33+34) \sin(-30+31) \sin(-27+28) \sin(-24+25) \sin(-21+22)}{\sin(-20+21) \sin(-32+33) \sin(-29+30) \sin(-26+27) \sin(-23+24)} = 1.$$

The step of writing down the sides may of course be omitted.

### Linear form of the side equation

The side equation as given above is the condition which must be satisfied by the adjusted angles. In terms of the observed angles and their corrections, it is of the form,

$$\frac{\sin(-4-\nu_4+5+\nu_5) \sin(-7-\nu_7+8+\nu_8) \dots}{\sin(-4-\nu_4+6+\nu_6) \sin(-8-\nu_8+9+\nu_9) \dots} = 1,$$

and to put this into a form suitable for solution the  $\nu$ 's, which are unknown, must be separated from the observed angles, which are known. The sine of an adjusted angle may be expanded by Taylor's theorem, as

$$\begin{aligned} \sin(-4+5-\nu_4+\nu_5) &= \sin(-4+5) + (-\nu_4+\nu_5) \cos(-4+5) - \dots \\ &= \sin(-4+5) [1 + (-\nu_4+\nu_5) \cot(-4+5) - \dots], \end{aligned}$$

where the squares and higher powers of the  $\nu$ 's have been discarded. Expanding all the sines in the side equation in this way, we have

$$\frac{\sin(-4+5) \sin(-7+8) \dots}{\sin(-4+6) \sin(-8+9) \dots} = \frac{[1 + (-\nu_4+\nu_5) \cot(-4+6)] [1 + (-\nu_8+\nu_9) \cot(-8+9)] \dots}{[1 + (-\nu_4+\nu_5) \cot(-4+5)] [1 + (-\nu_7+\nu_8) \cot(-7+8)] \dots}$$

Performing the multiplication and division on the right-hand side, and again discarding second-degree and higher products among the  $\nu$ 's, we have

$$\begin{aligned} \frac{\sin(-4+5) \sin(-7+8) \dots}{\sin(-4+6) \sin(-8+9) \dots} &= 1 + (-\nu_4+\nu_6) \cot(-4+6) - (-\nu_4+\nu_5) \cot(-4+5) \\ &\quad + (-\nu_8+\nu_9) \cot(-8+9) - (-\nu_7+\nu_8) \cot(-7+8) + \dots, \end{aligned}$$

where the  $\nu$ 's are in radians. Expressing the  $\nu$ 's in seconds of arc, we finally have

$$\begin{aligned} &(-\nu_4+\nu_5) \cot(-4+5) - (-\nu_4+\nu_6) \cot(-4+6) + (-\nu_7+\nu_8) \cot(-7+8) \\ &- (-\nu_8+\nu_9) \cot(-8+9) + \dots + \left[ \frac{\sin(-4+5) \sin(-7+8) \dots}{\sin(-4+6) \sin(-8+9) \dots} - 1 \right] \frac{1}{\text{arc } 1''} = 0. \end{aligned}$$

The absolute term may be readily evaluated by calculating machine in the form,

$$\frac{(L - R) \times 206265''}{R},$$

where  $L$  and  $R$  denote the products of the sines in the left-hand (numerator) and right-hand (denominator) columns of the lay-out respectively. By making slightly different approximations in this derivation, we can find another form of the absolute term in which the divisor is  $L$  instead of  $R$ . In the first-order triangulation, the values of  $L$  and  $R$  were always so nearly equal that change of the divisor did not change the numerical value of the absolute term. In less accurate work, however, there may be a slight difference and, in such cases, a better result is obtained by using the mean divisor  $\frac{1}{2}(L + R)$ .

### Lay-out of side equation

In the computation of the side equation, panels were provided for the angles occurring in the numerator, on the left-hand side, and those occurring in the denominator, on the right-hand side. The direction numbers were entered in the first column of each panel, but, as the coefficients of the corrections to angles occurring in the denominator are negative, the signs of the angles in the right-hand panel were reversed. The sine of each angle to eight decimals, and the cotangent to three decimals, were then entered in adjoining columns, care being taken to enter a negative sign with the cotangent of an angle greater than  $90^\circ$ .

Angle	sin	cot	Angle	sin	cot
-221+222	.4408 0658	2.036	+219-221	.8165 5771	0.707
-245+246	.9920 6846	0.127	+244-246	.6349 2370	- 1.217
-251+253	.9903 8601	-0.140	+251-252	.8353 7889	0.658

$$\begin{array}{r} .4331\ 0600\ 81 \\ \hline .4331\ 0372\ 47 \end{array}$$
 $+1''.087$ 

$$+0.707(219) -2.743(221) +2.036(222) -1.217(244) -0.127(245) +1.344(246) \\ +0.798(251) -0.658(252) -0.140(253) +1''.087 = 0.$$

The continued product of the sines in each column was then found. To minimise end-figure errors, these products are best carried to the full capacity of the keyboard and, especially in long equations where there are many factors in each product, the factors are best used in decreasing order of magnitude. The absolute term was then evaluated in the form given above.

Each direction symbol was then multiplied by the corresponding cotangent and like terms were collected, giving the final linear form of the equation as shown in the example, where a bracketed symbol denotes a correction to an observed direction.

### Selection of angle and side equations

If  $L$  denotes the number of lines and  $S$  the number of stations in a figure, then the number of angle equations and the number of side equations needed are given by the well-known formulae,

$$\text{number of angle equations} = L - S + 1,$$

$$\text{number of side equations} = L - 2S + 3.$$

These formulae apply, not only to the complete network, but to every part of it, and they were applied in a systematic build-up of the figure line by line. As each new triangle was completed, both formulae were used to determine the number of each kind of equation demanded at that stage, and an equation was written as soon as the formulae indicated that one was needed, such equation of necessity involving the line last added and any of the earlier lines. Lines were added as far as possible in the order in which triangles would be computed; after a new station was introduced, all lines joining that station to earlier stations were added before the next new station was introduced.

In many cases a choice of equation was possible. Textbook advice about the selection of side equations relates to equations in logarithmic form, but it is equally valid applied to the equations involving natural sines: that small angles should be used for preference but that the same small angles should not be used more than once. Side equations should in general be written for the poorly-shaped figures; if side agreement can be achieved in these, the better-shaped triangles will also be in agreement. For a quadrilateral, the side equation with the pole at a vertex involves fewer directions and fewer sines than the equation with the pole at the intersection of the diagonals, and so in general is to be preferred.

One-way lines, or lines observed in one direction only, do not normally occur in a primary control network, but in the first-order net there were some instances of such lines, the observation of the reverse direction having been omitted because of unfavourable weather or the difficulty of re-occupying a station. The occurrence of one-way lines in a net does not affect the number of side equations but it reduces the number of angle equations because any triangle with a one-way line as a side contains only two observed angles and therefore an angle equation cannot be written for it. In the derivation of the angle equations by the systematic build-up of the figure, the easiest way of dealing with this case was found to be, not to modify the formula, but simply to write "one-way line" against the equation number whenever the triangle for which an angle equation would otherwise be written had a one-way line as side.

**Length  
equations**

The length equation is similar in form to the side equation, but it involves only one chain of triangles linking two fixed sides, and a fixed length is included as one factor in the continued product of the sines on each side of the equation when the absolute term is formed. When spherical angles are involved in the length equation, two courses are possible: a correction may be applied to each fixed length to convert it from linear to spherical arc measure, or the spherical angles may be converted to Legendre angles by subtracting one-third of the appropriate spherical excess from each before the sines are entered. The second method was the one followed in the first-order triangulation.

A length equation is required for each independent pair of fixed lengths in the figure. It is not required in a base net or in any other figure where only one length is fixed. In the adjustment of the first-order triangulation, a length equation was required for the chain connecting two contiguous sides of a base net, and for the chain connecting any other pair of sides previously fixed. A length equation is not normally required where a fixed triangle occurs, since the sides in this case are not independently fixed, but in the adjustment of some of the subsidiary first-order figures, a length equation was used in substitution for a side equation in such cases.

**Number of  
equations**

The numbers of condition equations in the figure adjustments are set out below:

	Angle equations	Side equations	Length equations	Total	Laplace azimuth equations
<b>Base nets</b>					
Waitemata	13	8	...	21	
Matamata	12	7	...	19	
Kaingarua	10	5	...	15	
Eltham	12	5	...	17	
Wairarapa	10	5	...	15	
Culverden	13	8	...	21	
Waitaki	17	11	...	28	
Riversdale*	21	15	...	36	
<b>North Island</b>					
Main figure	141	48	14	203	5
East Cape figure	32	11	3	46	1
Subsidiary figures	54	30	2	86	
<b>South Island</b>					
Northern figure	55	27	1	83	2
Southern figure	108	52	7	167	3
Subsidiary figures	86	28	23	137	—
	584	260	50	894	11

For each figure, apart from the base nets, previously fixed directions were held fixed, so that it was stipulated that the corrections to be applied to them were zero. Such directions were therefore omitted from the matrix of coefficients of the condition equations.

\* The Riversdale base net contains eight stations and every station was observed from every other, so that it has the maximum possible number of angle and side equations.



## Solution of the equations

In the condition equations, the corrections to the observed directions are the unknowns and they are greater in number than the number of equations. To solve these by the method of least squares, correlate equations, equal in number to the number of unknowns, were formed from the condition equations. A set of normal equations was then formed from the correlate equations, and solution of these gave the values of the correlates. The corrections were then found by substitution in the correlate equations.

Solution of the normal equations was done by the Gauss-Doolittle method. At that time, the Cholesky method had been tested and had been adopted for small figures, but it was thought that for large systems, such as the 203 equations for the main North Island net, there would be less risk of accident with the Gauss-Doolittle method. Since that time, the Cholesky method has been adopted for all figures, regardless of size, its principal advantages being greater stability in the solution and economy of space, each equation requiring only one line instead of two lines as in the older method.

Finding sufficient space to write the assembly of equations was one of the main troubles encountered during the solution of the larger figures, although a system of "telescoping" the equations was used where one equation extended over several sheets of paper. Other troubles were met with that would not normally occur. For example, an error in the formation of a side equation was found during a check of the condition equations, but, owing to the disruption caused by moving to new quarters followed by a change of duties among the computers, it was not corrected at the time. Later, when the solution of the equations had been completed and the solution of the triangles was undertaken, agreement among the sides could not be achieved in one part of the figure and the error in the side equation was rediscovered. A large part of the solution had to be done again because of this error.

It was found that the order in which the equations are solved is of great importance in lightening the labour of solution. An unsuitable order may cause the matrix of coefficients to 'block up' "solid" at too early a stage, while a changed order may result in many zero coefficients and so tend to keep the solution "open". The order ultimately adopted involved beginning with an equation with relatively few terms. This was followed by as many equations as possible which involved the same unknowns without introducing too many new unknowns. The selection was aided by a diagram of the equations, with dots to represent the coefficients, which clearly showed the interlocking of the equations. As each equation was selected, it was ruled through on the diagram, and all the unknowns occurring in it were also marked in the other equations. In effect, this method of selection meant that all equations for a particular figure of the net were grouped together, each side equation being placed with the angle equations involving the same directions. Where long length equations occurred, solution was begun from a part of the net which did not involve these, and the long equations were often best inserted near the middle of the work instead of being left till the end.

The corrections derived from the solution were tested by substitution in the condition equations. The angle equations were required to be satisfied exactly, and an end-figure change of one unit was sometimes necessary to achieve this.

## Side computation

The lengths of the sides were computed by Legendre's theorem after one-third of the spherical excess of the triangle had been subtracted from each adjusted spherical angle. Where the spherical excess was not exactly divisible by 3 to three decimals, one angle received a share one unit greater or one unit less than the other two angles; the odd angle was selected arbitrarily. With one side of a triangle known, the other two sides were then given by

$$b = \frac{a}{\sin A} \sin B, \quad c = \frac{a}{\sin A} \sin C,$$

The quotient  $a/\sin A$  was evaluated first and set on the keyboard, being then multiplied in turn by  $\sin B$  and by  $\sin C$ . The work was recorded in the schedule of triangles prepared earlier.

As the work progressed, triangles would be met with in which two, or even three, sides were already known, but only one side, generally the greatest side, would be regarded as fixed, the other two being computed afresh. The agreement in two or more values of the same side computed from different triangles was the final test of the consistency of the adjustment, and the side lengths were systematically compared as the values become available. Generally, agreement within one or two units of the last decimal (0.001 link) was obtained, the value computed from the shorter route or from the better-shaped triangles being preferred, and other values amended where necessary to agree with it. All the triangles of the figure were used in side computation, except for some unusually "flat" triangles, more particularly in base nets, where a larger discrepancy than usual in the computed sides could be expected.

#### Laplace equations

At stations where latitude, longitude, and azimuth have been observed, the differences between the astronomic (observed) values and the geodetic (computed) values are a measure of the deflection of the vertical – the displacement of the astronomic zenith with respect to the geodetic zenith, or the departure of the geoid from the adopted spheroid. If the subscripts, A, G, are used to denote astronomic and geodetic values respectively, the deflection in the meridian is given by  $\phi_A - \phi_G$ , the deflection in the prime vertical by  $(\lambda_A - \lambda_G) \cos \phi$  or by  $(A_A - A_G) \cot \phi$ . It follows that

$$A_A - A_G = (\lambda_A - \lambda_G) \sin \phi,$$

a minus sign being prefixed to the right-hand side if south latitude is reckoned positive. This is the Laplace equation, which should be satisfied at every station where both azimuth and longitude have been observed – exactly, if the observations are free from error, or approximately, if it is conceded that the observed values may be subject to error.

The azimuth of a ray at any station can be computed from the azimuth of a ray at another station through the azimuths of the rays in a chain linking one station with the other, i.e. by adding the spherical angles in this chain (considered positive for a clockwise rotation) and adding also the convergences, the differences between the azimuths at the two ends of each ray. If there are two independently fixed azimuths in a net, there will therefore be another angle equation to be satisfied if these azimuths are to be held fixed. If the observed azimuths are not held fixed, the Laplace equation should still be satisfied approximately so as to curb any tendency for the net to swing away from its correct orientation. For the route from one Laplace station to another, a further angle equation is therefore necessary and may be called the Laplace azimuth equation.

From the difference in longitude,  $\lambda_A - \lambda_G$ , at any station, we can compute the difference in azimuth,  $A_A - A_G$ , and can therefore find what  $A_G$  should be to go with an observed  $A_A$  if the Laplace equation is to hold. The value of  $A_G$  computed through the chain from the  $A_G$  at another station will not agree with this, and the difference (chain minus Laplace) is the absolute term of the angle equation. The first adjustment of the net had already produced corrections to the directions involved in this equation, so that it was known by how much the angles were changed because of the geometrical condition equations and by how much they would be changed by the rigid Laplace condition. A "relaxed absolute term" was then calculated, so that the Laplace azimuth discrepancy would be divided between the triangulation and the astronomical observations, and the final adjusted angles therefore would not satisfy the Laplace equation exactly.

Eleven azimuth equations were thus formed for the chains connecting the twelve Laplace stations, distributed as shown in the last column of the table above. The adjustment had therefore to be recomputed with these equations added to the conditions. Much of the work of the first solution could be used again, the new work beginning with the addition of the new azimuth equations to the "forward solution" of the normal equations. The "back solution" had to be done afresh. The new solution gave the final adjustment of the directions, and the triangles were then re-solved.

# **Precision of the results**

The effect of the adjustment is shown in the following table, where the probable error of an observed direction was computed from the formula,

$$\pm 0.6745\sqrt{(\Sigma v^2/c)},$$

where  $c$  is the number of condition equations. In the case of the subsidiary figures, the values shown are totals or means for a number of figures adjusted separately.

	Total no of triangles	Average closure of triangles	Maximum closure of a triangle	Mean error of an angle	Prob. error of an obsvd. direction
<b>Base nets</b>		"	"	"	"
Waitemata	26	0.617	-1.405	0.340	$\pm 0.258$
Matamata	22	0.690	+1.662	0.402	$\pm 0.345$
Kaingaroa	16	0.997	+2.512	0.621	$\pm 0.506$
Eltham	18	0.802	+1.773	0.569	$\pm 0.443$
Wairarapa	16	1.139	-2.599	0.472	$\pm 0.369$
Culverden	26	1.211	-3.717	0.646	$\pm 0.465$
Waitaki	36	1.132	-2.684	0.540	$\pm 0.399$
Riversdale	56	0.744	-2.600	0.528	$\pm 0.356$
<b>North Island</b>					
Main figure	163	0.913	+3.888	0.562	$\pm 0.479$
East Cape figure	35	1.462	+5.489	0.864	$\pm 0.678$
Subsidiary figures	88	1.162	+4.660	0.627	$\pm 0.505$
<b>South Island</b>					
Northern figure	77	0.987	-3.143	0.704	$\pm 0.473$
Southern figure	154	0.973	-2.862	0.692	$\pm 0.511$
Subsidiary figures	137	1.210	+4.869	0.775	$\pm 0.671$

The values in the last two columns are those derived from the final adjustment after the introduction of Laplace azimuth equations.

The errors of closure of length between base nets, as computed from the unadjusted observations used in forming the length equations, are as follows.

	No. of triangles in chain	Absolute term of length equation	Length closure
		"	
Waitemata to Matamata	6	-0.766	1/270 000
Matamata to Kaingaroa	4	+7.175	1/29 000
Matamata to Eltham	9	+2.276	1/90 000
Eltham to Wairarapa	11	-8.540	1/24 000
North Island to Culverden	12	+2.531	1/82 000
Culverden to Waitaki	15	+4.396	1/47 000
Waitaki to Riversdale	9	-3.561	1/58 000

## **Geodetic Datum 1949**

The final solution of the triangles gave the final spherical angles and lengths of sides, and from these the final latitudes, longitudes, and azimuths could be computed, beginning at some initial station of defined position. The station chosen as initial was Papatahi, a centrally situated station of the main net and one of the corner stations of the subsidiary net containing Kelburn. The values of  $\phi_A - \phi_G$  at the 65 latitude stations and of  $A_A - A_G$  at the 39 azimuth stations from the first adjustment

were known, and the latitude and azimuth at Papatahi were chosen so as to make the means of these differences equal to zero. The longitude adopted was that derived from Kelburn. The rounded-off values are

South latitude	41° 19' 08".9000
East longitude	175° 02' 51".0000
Azimuth to Kapiti No 2	347° 55' 02".500

and these define the Geodetic Datum 1949, on which all subsequent computation of horizontal position has been based.

Geodetic positions of stations in the main net were computed by the Clarke formulae, beginning from Papatahi. The geodetic positions in some of the subsidiary figures were computed from their transverse Mercator co-ordinates.

#### Datum for heights

The heights of stations are measured from mean sea level as established by the tide gauge record at the principal ports. The datum itself has usually been derived from the records over a period of at least three years, often a random selection of years from among those available. Connections between the tide gauges and the triangulation network have been made by precise spirit levelling.

For the first computation of the heights of first-order stations, eight starting points were available, four in the North Island and four in the South Island, as follows:

Station	Connected to tide gauge at
Mt Eden	Auckland
3rd order station	New Plymouth
Bluff Hill No 2	Napier
Kelburn	Wellington
Botanical Hill	Nelson
2nd order station	Lyttelton
Flagstaff	Dunedin
The Bluff	Bluff

The height quoted is always the height of the station mark, usually the top of the iron tube.

#### Computation of difference

If a zenith distance  $z$  is observed at each end of a ray, the difference in height from station A to station B is given by

$$\Delta H = s \tan \frac{1}{2}(z_B - z_A) + \frac{1}{2}(i_A + g_A - i_B - g_B),$$

where  $s$  is the horizontal distance between the stations,  $i$  is the height of the instrument above the station mark at the occupied station, and  $g$  is the height of the observed point of the beacon above the station mark at the observed station. This formula assumes that the refraction is the same at each end of the ray and can therefore be eliminated by combining the two observations. For a one-way observation, the difference in height is given by

$$\Delta H = s \tan(90^\circ - z + K) + i_A - g_B,$$

$$\text{where } K = s(1 - 2k)/R \text{ arc } 1'',$$

$k$  being the coefficient of refraction and  $R$  the radius of the Earth. Assuming  $k = 0.07$ , and with a mean value of  $R$  for New Zealand, we have

$$K = 0''.00424 s, \text{ where } s \text{ is in feet,}$$

$K = 0''.00280 s$ , where  $s$  is in links.

These formulae are approximations, and, although they can be successfully applied to the short lines occurring in second-order and third-order triangulation, they are not always valid when applied to the longer lines in first-order nets. More elaborate formulae may be found in various textbooks, but trials of these did not indicate that they could significantly improve the closures in the condition equations in height nets. The difficulty lies, not in the lack of an accurate formula, but in the lack of a precise knowledge of the effect of refraction, which can vary considerably with the time of day and from place to place. In some instances, the assumption that the refraction was the same at each end of the ray was quite clearly wrong – notably in the Culverden base net where the refraction at the lower stations was obviously greater than at the higher stations. It was decided, however, to use the simpler formulae and to tolerate the large errors of closure until observations over shorter rays were available.

#### Condition equations in height nets

The adjustment of the height net was done in relatively small figures, convenience in computation being the principal consideration. Initially, the eight points connected by levelling to tide gauges were the only points of fixed height, but, when the figures containing these points had been adjusted, further points of fixed height were available for later figures.

The condition equations were all of the same type. The differences in height which form a closed circuit must total to zero, while the differences in height in a circuit which begins and ends at points of known height must total to the correct difference between the known heights. The number of independent condition equations is fixed by the formula,

$$\begin{aligned} &\text{number of height condition equations} \\ &= (\text{number of lines}) - (\text{number of points to be fixed}). \end{aligned}$$

The figure was built up line by line and this formula was applied as each circuit was completed, a condition equation being written whenever the formula indicated that one was needed. Where a choice of equation was possible, the shorter circuit was generally chosen.

As the change in vertical angle due to refraction is proportional to the length of the line, it follows that short lines are more reliable than long ones for the determination of heights. The correlate equations were therefore weighted in proportion to the lengths of the lines, before the normal equations were formed. The correlate equations associated with one-way lines were given twice the usual weight.

#### Revision of heights

Some of the large errors of closure in the condition equations indicated corrections of the order of 10 ft (3m) to the differences in height between first-order stations. The Canterbury Plains, in particular, provided large errors of closure more frequently than elsewhere. Heights of stations derived from first-order rays alone are therefore regarded as provisional, to be revised when second-order and third-order observations are made, such observations involving much shorter rays.

In the Rotorua-Taupo district, for example, extensive second-order and third-order networks were observed in the years following the completion of the first-order work. A line of precise levelling was also carried from the tide gauge on Moturiki Island inland to Taupo, connecting to three second-order stations on the way. This made it possible to recompute the heights of all the stations in this region, using the three second-order stations as origins and using the shorter rays of the second-order and third-order nets. The errors of closure rarely exceeded 1 ft. The corrections to the first-order rays, derived from the second-order and third-order nets, averaged 1.70 ft, with a maximum of 5.95 ft. The new heights were described as being on the Moturiki Datum 1953.

**In the Canterbury Plains, several first-order stations have been connected to a line of precise levels. As second-order and third-order nets have been observed, and as precise levelling routes have been extended, further stations have had heights fixed by levelling, and the computation of height nets has been revised.**

## IX. Coordinates

### Projection coordinates

The transverse Mercator projection – a conformal projection with a constant scale along the straight central meridian – is used for the systematic coordination of the stations upon a plane representation. Two systems are in use – National Grid coordinates and meridional circuit coordinates.

The National Grids comprise one projection in each island, the initial meridian being at longitude 175° 30' east in the North Island and 171° 30' east in the South Island. In the latter case the initial meridian is to the east of the true central meridian; this was chosen to keep the scale error small in the settled areas, the maximum scale falling in the uninhabited Fiordland region. The National Grids are used for topographical maps and in general for all maps on a national rather than a local scale; they are also used for the adjustment of second-order and third-order triangulation connected to the first-order net.

Coordinates for cadastral survey are based on one transverse Mercator projection for each meridional circuit, the old initial stations still serving as origins.

### Projection tables

The fundamental formulae of the transverse Mercator projection, giving the northing  $x$ , the easting  $y$ , the convergence  $\gamma$ , and the scale coefficient  $m$ , corresponding to a given latitude  $\phi$  and difference of longitude  $\Delta\lambda$  in seconds from the initial meridian, were put in the form,

$$x = (M_0 - M) + \Delta\lambda^2 G + \Delta\lambda^4 I,$$

$$y = \Delta\lambda F + \Delta\lambda^3 H + \Delta\lambda^5 J,$$

$$\gamma = \Delta\lambda \sin \phi + \Delta\lambda^3 C + \Delta\lambda^5 E,$$

$$m = 1 + \Delta\lambda^2 B + \Delta\lambda^4 D,$$

where  $M$  is a meridional distance measured from the equator,  $M_0$  being that of the origin, and the co-efficients,  $B, C, D, \dots J$ , are functions of the latitude  $\phi$  and of the dimensions of the spheroid. The formulae are adapted to southern hemisphere computation, with south latitude and east longitude considered positive, the coefficients then being:

$$B = \frac{1}{2} \text{arc}^2 1'' \cdot \cos^2 \phi (1 + e'^2 \cos^2 \phi)$$

$$C = \frac{1}{8} \text{arc}^2 1'' \cdot \sin \phi \cos^2 \phi (1 + 3e'^2 \cos^2 \phi)$$

$$D = \frac{1}{192} \text{arc}^4 1'' \cdot (11 + 20 \cos 2\phi + 9 \cos 4\phi)$$

$$E = \frac{1}{240} \text{arc}^4 1'' \cdot (2 \sin \phi + 5 \sin 3\phi + 3 \sin 5\phi)$$

$$F = \text{arc} 1'' \cdot \nu \cos \phi$$

$$G = -\frac{1}{2} \text{arc}^2 1'' \cdot \nu \sin \phi \cos \phi$$

$$H = \frac{1}{6} \text{arc}^3 1'' \cdot \nu \cos^3 \phi (1 - \tan^2 \phi + e'^2 \cos^2 \phi)$$

$$I = -\frac{1}{24} \text{arc}^4 1'' \cdot \nu \sin \phi \cos^3 \phi (5 - \tan^2 \phi + 9e'^2 \cos^2 \phi)$$

$$J = \frac{1}{120} \text{arc}^5 1'' \cdot \nu \cos \phi (2 \cos 2\phi + 3 \cos 4\phi)$$

For the inverse computation, or finding the latitude  $\phi$ , the longitude  $\lambda$ , the convergence  $\gamma$ , and the scale coefficient  $m$ , corresponding to given coordinates  $(x, y)$ , the latitude  $\phi_1$  of the foot of the ordinate on the initial meridian can be found at once since it is the latitude for which  $M_1 = M_0 - x$ . The formulae then are:

$$\phi = \phi_1 + y^2 \mathcal{C} + y^4 \mathcal{J}$$

$$\lambda = \lambda_0 + y \mathcal{F} + y^3 \mathcal{H} + y^5 \mathcal{J}$$

$$\gamma = y \mathcal{A} + y^3 \mathcal{C} + y^5 \mathcal{E}$$

$$m = 1 + y^2 \mathcal{B} + y^4 \mathcal{D}$$

where the coefficients are functions of  $\phi_1$ :

$$\mathcal{A} = \frac{\tan \phi}{\nu \text{ arc } 1''}$$

$$\mathcal{B} = \frac{1}{2 \rho \nu}$$

$$\mathcal{C} = -\frac{\tan \phi (1 + \tan^2 \phi - e'^2 \cos^2 \phi)}{3 \nu^3 \text{ arc } 1''}$$

$$\mathcal{D} = \frac{1}{24 \rho^2 \nu^2}$$

$$\mathcal{E} = \frac{\tan \phi (2 + 5 \tan^2 \phi + 3 \tan^4 \phi)}{15 \nu^5 \text{ arc } 1''}$$

$$\mathcal{F} = \frac{\sec \phi}{\nu \text{ arc } 1''}$$

$$\mathcal{G} = \frac{\tan \phi}{2 \rho \nu \text{ arc } 1''}$$

$$\mathcal{H} = -\frac{\sec \phi (1 + 2 \tan^2 \phi + e'^2 \cos^2 \phi)}{6 \nu^3 \text{ arc } 1''}$$

$$\mathcal{J} = \frac{\tan \phi (5 + 3 \tan^2 \phi)}{24 \rho \nu^3 \text{ arc } 1''}$$

$$\mathcal{J} = \frac{\sec \phi (5 + 28 \tan^2 \phi + 24 \tan^4 \phi)}{120 \nu^5 \text{ arc } 1''}$$

Tables of all of these coefficients, including the meridional distances, for every minute of latitude from  $34^\circ$  to  $48^\circ$  were computed by the Head Office Computing Branch and later published by the Government Printer as *Geodetic and Transverse Mercator Projection Tables*.

The tabulated coefficients are intended for forming products with the powers of a distance in links or of a difference of longitude in seconds and have been multiplied by some convenient power of 10 to bring the decimal point of both factors within range of the dials of a calculating machine. The distance or the difference of longitude must therefore be divided by the same power of 10. Specifically, distances in links must be multiplied by  $10^{-6}$  and differences of longitude in seconds must be multiplied by  $10^{-4}$ . For example, the  $x$ -coordinate of the transverse Mercator,

$$x = (M_0 - M) + \Delta \lambda^2 G + \Delta \lambda^4 I,$$

for computation becomes

$$x = (M_0 - M) + (\Delta \lambda^2 \times 10^{-8}) (G \times 10^8) + (\Delta \lambda^4 \times 10^{-16}) (I \times 10^{16}).$$

The values of  $G \times 10^8$  and  $I \times 10^{16}$  are the values actually tabulated under the symbols  $G$  and  $I$ .



The coordination of all the first-order stations was done with the aid of these tables.\*

### National Grid coordinates

The tables of transverse Mercator coefficients were originally intended for the computation of meridional circuit coordinates within  $3^\circ$  of longitude from the initial meridian. Terms which were not appreciable within this limit were discarded, so that sixth-order and higher-order coefficients were not computed and small spheroidal expressions in the fourth-order and fifth-order coefficients were also omitted. At the time, it was supposed that second-order and third-order triangulation would be adjusted and computed in terms of the meridional circuits. Later, it was decided that all triangulation should be adjusted in terms of the National Grids, the grid coordinates of control stations being computed using the tables of transverse Mercator coefficients already published. This has meant that the tables have been used with greater distances from the initial meridian than was originally intended, and the omission of terms not provided for in the tables has resulted in a slight, though unimportant, departure from strict conformality in the outer regions of the projection.

Coordinates computed with the tables are in links, but can be converted to any other units as required. Originally, it was intended that National Grid coordinates should be in metres, and some brief tables in these units were computed some time before 1938. At the request of the military authorities, National Grid coordinates since that time have been in yards. Provisional values in use before the establishment of the Geodetic Datum 1949 were called "Military Grid", but revised values in terms of the Geodetic Datum are called "National Grid".

In each island, a so-called "false origin" has been used so that National Grid coordinates are always positive. In the North Island, the true origin is at longitude  $175^\circ 30'$  and latitude  $39^\circ$ , and this position is 300 000 yards east and 400 000 yards north of the false origin. In the South Island, the true origin is at longitude  $171^\circ 30'$  and latitude  $44^\circ$ , and this position is 500 000 yards east and 500 000 yards north of the false origin. Following military custom, it is customary to state the easting first.

The National Grids are used for sheet boundaries and position references on topographical maps. A grid reference to the nearest hundred yards on any one topographical sheet is given as a six-figure number, derived by discarding the 100 000-yard figures and giving the next three figures of the easting and the northing. Thus a point whose full coordinates are 372 412.8 yards east, 468 035.5 yards north, is given as grid reference 724 680.

### Meridional circuit coordinates

For cadastral survey, the old meridional circuits are still used as a basis of coordinates, the old initial stations still serving as origins, although some minor modifications have been made to the circuit boundaries. The new coordinates are again on the transverse Mercator projection, and have been computed from the same tables, but values are in links and false origins have not been used.\*\* In New Zealand survey custom, the northing is stated first.

\*The coefficients, F, H, J, occurring in the expression for y, chanced to coincide with the initials of the Chief Computer, F.H. Jennings, at the time when the tables were compiled. Computation using these tables was therefore referred to as "the F.H.J. method" by the computing staff.

Since the introduction of electronic computers, computation has been done by means of series in powers of a complex variable involving constant coefficients, and tables are no longer needed.

\*\*When coordinates were metricated in 1973, every initial station was assigned the coordinates, 700 000 m north, 300 000 m east.

To simplify the procedure of computing meridional circuit coordinates from National Grid coordinates, double-entry tables are used, giving coefficients for a two-dimensional Bessel interpolation formula as advocated by Dr L.J. Comrie. Such tables are used in pairs, one table enabling meridional circuit coordinates to be computed from National Grid coordinates and a companion table enabling National Grid coordinates to be computed from meridional circuit coordinates. The meridional circuit coordinates obtained by the use of the first table are checked by using them as the arguments for entry into the second table, the second computation being required to reproduce the original national Grid values within two units of the second decimal.

#### Tangent-to-chord corrections to bearings

The spheroidal azimuths of selected rays at every station were derived from the computation of geodetic positions by the Clarke formulae, so that, with the corrected spheroidal angles from the figure adjustment, the azimuths of all rays were found. In practice, this was done by using the known azimuths to find a "station constant", the rotation necessary to convert the adjusted directions into azimuths. This constant was then added to each of the adjusted directions at that station.

The projection convergence at every station, reckoned positive eastward from projection north, was also computed, so that the projection bearing of every ray was found from

$$a = A + \gamma,$$

where  $A$  is the azimuth,  $\gamma$  the convergence, and  $a$  the projection bearing.

The bearing  $a$  is the bearing of the tangent to the projected geodesic at a particular station. The bearing  $\beta$  of the chord joining the projected positions of two stations can be found from

$$\beta = a + (\beta - a),$$

where the small angle  $\beta - a$  is computed from the coordinates of the two stations. The corrections at each end of the ray were computed together by the formulae,

$$\begin{aligned}\beta_A - a_A &= \frac{(x_A - x_B)(2y_A + y_B)}{6 \rho \nu \text{ arc } 1''} \\ \beta_B - a_B &= \frac{-(x_A - x_B)(y_A + 2y_B)}{6 \rho \nu \text{ arc } 1''}\end{aligned}$$

The function  $1/(6 \rho \nu \text{ arc } 1'')$  varies so slowly with the latitude that all the values required for New Zealand are contained in a small critical table.

There is unfortunately a lack of agreement in the terminology of the quantities involved, and the term "projection bearing" has been applied sometimes to the tangent, sometimes to the chord. In New Zealand, the terms used are "projection (tangent) bearing"  $a$  and "grid (chord) bearing"  $\beta$ . The angle  $\beta - a$  is sometimes known elsewhere as "the  $t - T$  correction", following the German notation for bearings.

The tangent-to-chord correction, applied at each end of a ray, should give grid bearings differing by exactly  $180^\circ$ . The above formulae, however, are approximations and may not give precise results at great distances from the initial meridian, in which case additional terms of the series are required. In a number of trial cases, however, it was found that these additional terms gave almost the same result as applying equal adjustments at each end of the ray. The latter procedure was therefore adopted as standard, the  $\beta - a$  corrections being computed from the first term of the series as given above, and equal adjustments being applied at each end of the ray where necessary to give grid bearings differing by exactly  $180^\circ$ .

Grid distances were computed from the differences in coordinates of the two stations, i.e.,

$$S = \sqrt{\Delta x^2 + \Delta y^2}.$$

Grid coordinates, bearings, and distances have been the control for the adjustment of second-order and third-order triangulation, such adjustment being done on the National Grid instead of on the spheroid.\*

*\*Subsequent events, not foreseen at the time, have shown this to be an unfortunate choice. With the introduction of metrication, a new grid covering both islands in one projection – the New Zealand Map Grid – has been introduced, and the National Grids will ultimately be abandoned. It is probable that, for future control surveys, there will be a return to adjustment on the spheroid.*

## PART IV. TABULATION OF RESULTS

### X. Geodetic Latitudes and Longitudes, and Heights

Some of the heights listed herein are provisional and may be revised when further survey is done. The symbols,  $\phi$ ,  $A$ ,  $\lambda$ , denote stations where observations for latitude, azimuth, and/or longitude respectively have been used.

Station		South Latitude ° ' "	East Longitude ° ' "	Height m
Kerr Point	$\phi A$	34 24 20.2154	173 01 24.6150	234.2
Unuwahao		34 26 01.8358	172 53 17.2307	308.9
Tirikawa		34 26 38.6385	172 44 21.0973	285.5
Te Paki		34 28 06.2695	172 46 16.4463	310.5
Kohuronaki		34 29 24.7010	172 50 03.3086	291.7
Te Kao		34 39 37.3530	172 57 14.1111	105.3
Houhora No 2		34 49 20.4139	173 09 32.2575	235.9
Paraawanui		34 49 56.2685	173 26 06.8634	158.0
Kaiwhetu		34 57 26.5606	173 32 23.9860	177.3
Reference Point	$\phi A \lambda$	34 59 56.5736	173 28 55.3186	86.2
Akatere		35 00 25.2576	173 39 49.9867	376.7
Pararake		35 00 29.2055	173 24 32.3573	115.9
Ngatu		35 01 36.4502	173 12 10.7270	89.3
Whakarara		35 02 50.1684	173 52 53.7314	328.9
Oruru		35 03 18.0462	173 30 44.2689	262.8
Maungataniwha		35 10 08.9873	173 31 23.5568	567.2
Taumatamahoe		35 11 21.7256	173 15 51.5787	557.7
Cape Brett		35 11 27.5721	174 19 45.9969	361.7
Te Ahuahu	$\phi A$	35 20 29.7701	173 50 27.2485	373.2
Tauwhare		35 24 41.8039	173 20 29.1160	599.7
Huruiki No 2		35 26 32.0673	174 18 56.3691	459.3
Ngapukehaua		35 30 27.2581	173 32 58.3813	762.3
Motatau No 2		35 35 35.5067	174 02 25.2867	559.5
Pukearenga No 2		35 37 08.6250	174 30 51.0224	136.1
Parakiore No 2	$\phi A$	35 39 39.2709	174 16 53.1885	390.9
Tutamoe No 2		35 45 43.2046	173 48 07.6781	769.8
Manaia		35 49 12.5648	174 31 00.5709	419.6
Tangihua		35 51 30.9709	174 05 26.5007	627.4
Mokohinau	$\phi A$	35 54 28.6359	175 06 53.4718	111.9
Mahora	$\phi A$	35 57 50.1199	173 46 09.2457	128.8
Taranga		35 57 53.6188	174 43 04.8376	334.9
Maungaraho		36 01 23.3806	173 58 33.2358	221.3
Tataweka		36 04 22.0594	175 22 41.7582	526.4
Cattlemount		36 05 41.1847	174 27 29.3749	429.9
Hauturu		36 11 55.8223	175 04 42.9651	722.5
Whangaparapara	$\phi A$	36 14 09.0887	175 22 37.4141	309.5
Muarangi No 2		36 17 06.4275	174 03 59.2169	213.9

Station		South Latitude			East Longitude			Height m
		°	'	"	°	'	"	
Muarangi		36	17	20.8458	174	04	02.5919	220.0
Tamahunga	$\phi A$	36	18	02.1274	174	42	51.1315	436.6
Wharehine		36	19	04.8170	174	24	17.3287	136.6
Ruahine		36	20	05.5344	175	31	06.2584	402.1
Moirs Hill		36	28	18.1877	174	35	59.7275	357.5
Te Rokotai No 2	$\phi A$	36	29	20.9736	174	14	44.2898	168.5
Moechau		36	32	31.1431	175	24	07.0423	891.8
Whangaparaoa		36	36	07.2969	174	49	54.6976	107.4
Paeohaka		36	42	27.3558	174	28	30.3271	182.0
Pukeatua No 2		36	42	52.6133	174	40	18.7772	145.3
Stony Batter		36	46	05.1651	175	10	04.9238	220.4
Kumeu		36	47	15.1623	174	32	17.5764	61.6
Waitemata		36	49	02.4175	174	37	29.3674	78.6
Mt Eden	$\phi A \lambda$	36	52	47.5150	174	45	51.6217	196.0
Ruaotuwheua		36	55	33.7454	174	33	11.9108	439.6
Papakai	$\phi A$	36	56	00.9391	175	35	02.1880	760.4
Kohukohunui No 2		37	02	24.1303	175	13	09.9395	687.8
Awhitu		37	03	58.0466	174	37	10.8629	126.4
Opaheke		37	06	57.1932	175	00	36.8671	329.1
Pakirarahi		37	10	54.4763	175	41	36.9264	786.9
Rataroa		37	14	20.8319	175	17	59.5732	322.9
Pukeotahinga No 2		37	20	19.1054	174	51	31.7935	301.7
Te Aroha		37	32	08.4880	175	44	31.7060	952.7
Taupiri		37	36	16.9022	175	11	09.9225	287.5
Maunganui		37	37	55.5551	176	10	18.3984	232.0
Motiti		37	38	04.6465	176	25	24.9384	56.8
Pukeamaru		37	39	25.8039	178	14	32.3600	990.1
Maketu	$\phi$	37	45	40.4993	176	27	58.3101	66.6
Waharoa	$\phi A \lambda$	37	45	58.8548	175	44	46.5025	55.5
Otawa	$\phi$	37	48	08.6574	176	14	46.1075	564.6
Maungakawa		37	48	36.7100	175	36	38.5216	495.4
Te Pohue	$\phi A \lambda$	37	49	34.7166	178	24	24.7943	338.7
Rangipoua		37	49	35.0022	177	46	06.5229	1040.8
Te Weraiti		37	49	49.2096	175	54	20.3829	764.7
Te Pareoterawahirua	$\phi$	37	50	51.4807	176	33	21.2463	229.5
Langmuir		37	51	41.1575	175	46	56.6750	75.6
Karioi		37	51	59.1996	174	48	05.7522	756.0
Hikurangi		37	55	14.5153	178	03	37.8877	1751.8
Whakatane		37	57	23.7354	177	00	31.2263	163.1
Opotiki		37	59	48.4633	177	18	27.2980	49.4
Maungatautari	$\phi A$	38	01	14.5133	175	34	32.8579	796.6
Tawhiti		38	04	37.3467	178	20	53.9365	519.6
Te Pakora No 2		38	04	38.2843	177	30	31.1976	854.2
Putauaki		38	06	31.1429	176	44	22.8707	820.8
Arowhana		38	06	38.9604	177	51	45.6304	1439.4
Ngongotaha	$\phi$	38	07	12.5117	176	11	51.7824	756.9

Station		South Latitude	East Longitude	Height
		° ' "	° ' "	m
Maungawhiorangi	$\phi$	38 10 35.8930	177 14 35.9830	815.3
Ruawahia No 2	$\phi$	38 13 43.8321	176 30 23.4652	1110.6
Maungahaumi		38 18 04.8218	177 39 50.2635	1212.9
Maungaongaonga		38 19 43.9827	176 20 36.8328	825.4
Hikurangi		38 21 06.9117	176 51 25.0523	928.8
Arakihi South	$\phi$	38 22 25.7915	178 03 22.9148	637.7
Maungamangero		38 23 23.9214	174 47 10.4051	805.6
Titirangi		38 23 53.7980	178 19 28.2999	343.1
Ahiwhakamura	$\phi$	38 25 03.5668	176 34 41.3095	546.1
Pukeahua		38 26 03.3872	176 00 08.3392	715.3
Kaingaroa		38 32 49.4424	176 26 50.5286	610.0
Pureora	$\phi$ A	38 33 10.4545	175 37 42.3725	1165.4
Pukeakura	$\phi$	38 33 34.1045	178 06 07.3235	496.7
Okahuatui	$\phi$	38 34 36.9116	177 48 16.6955	302.4
Maungapohatu No 2		38 34 42.7767	177 08 18.1413	1366.3
Taupiri		38 36 35.8432	176 38 39.3668	712.7
Tauhara		38 41 45.0999	176 09 46.7345	1087.9
Whakapunake	$\phi$	38 48 31.5379	177 35 50.3670	961.8
Peneta	$\phi$	38 55 54.1123	174 54 19.7025	596.1
Pohokura		38 56 39.7183	176 42 47.5898	1382.6
Pouiatoa No 2		39 04 58.0278	174 31 26.7003	391.6
Tongariro No 2		39 07 51.2413	175 38 13.1887	1962.5
Mohaka		39 07 54.7754	177 08 51.9963	248.7
Huirangi	$\phi$ A $\lambda$	39 08 08.7299	174 13 40.8423	241.1
Te Kapu	$\phi$	39 08 35.9041	177 53 03.6208	366.4
Kaweka		39 17 04.4652	176 22 44.8225	1724.2
Rainy Point	$\phi$	39 17 13.6001	174 23 40.2693	325.8
Jackson's Lookout		39 18 46.6483	174 06 07.9695	1131.9
Matemateaonga	$\phi$ A	39 23 05.4030	174 40 48.3014	713.0
Eltham	$\phi$	39 25 33.6542	174 16 32.4605	262.4
Waipuna	$\phi$	39 27 42.1778	175 11 52.2690	748.8
Bluff Hill No 2	$\phi$ A $\lambda$	39 28 50.1551	176 55 02.7253	101.3
Makino		39 29 00.4068	174 25 23.4367	369.0
Hihitahi	$\phi$ A	39 32 36.6766	175 44 06.9930	1116.4
Karimoi		39 33 20.7054	174 11 39.6646	102.8
Bluff No 2	$\phi$	39 37 53.5516	174 31 12.9348	354.3
Salisbury		39 41 15.6177	176 28 27.1303	475.0
Rangitatau	$\phi$ A	39 43 22.7898	174 57 02.4376	558.5
Mangamahu		39 45 24.3643	175 28 51.8690	698.8
Kahuranaki		39 47 49.4256	176 52 17.6563	645.7
Mangaweka		39 49 10.3054	176 05 06.7182	1733.3
Wairakai		40 00 10.8920	176 29 48.5712	310.0
Omakere		40 05 40.3085	176 46 59.2546	500.7
Mt Sarah	$\phi$ A	40 06 34.9372	175 16 53.2508	116.4
Ahiweka		40 10 58.7841	176 20 20.5165	734.8
Ross		40 12 44.0912	175 52 50.5802	1053.6

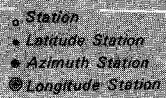
Station		South Latitude			East Longitude			Height m
		°	'	"	°	'	"	
Mt Stewart		40	14	31.0097	175	29	17.1586	131.4
Te Awaputahi	$\phi$	40	23	11.8319	176	27	10.5445	582.1
Oruarongo	$\phi$	40	29	40.8523	175	15	30.4822	45.8
Arawaru		40	30	42.7997	175	37	30.4717	767.5
Puketoi	$\phi$	40	34	54.5394	176	00	22.6422	763.9
Girdlestone		40	47	53.8283	175	26	35.3793	1547.2
Kapiti No 2	$\phi$	40	50	59.5096	174	54	54.8190	521.2
Attempt Hill	$\phi A$	40	51	20.7882	173	52	02.8675	728.7
Flagstaff No 3	$\phi$	40	59	40.8782	175	59	27.8848	483.5
Woodside		41	04	36.9025	175	23	23.8268	86.6
Rimutaka		41	07	14.7787	175	13	47.8849	725.5
Mt Campbell		41	07	32.3265	172	51	09.4576	1330.5
South Jury		41	07	47.2891	175	30	46.5828	276.5
Eringa	$\phi A$	41	10	54.7212	175	39	07.0772	574.7
Belmont		41	11	06.5281	174	52	25.6931	456.5
Bidwill		41	11	38.6267	175	23	55.2931	112.2
Kaukau	$\phi$	41	13	59.5311	174	46	40.0243	445.2
Botanical Hill	$\phi A \lambda$	41	16	28.3610	173	17	57.5405	147.3
Saddle Hill		41	16	38.1481	173	25	56.1369	1214.2
Terawhiti	$\phi$	41	16	41.8664	174	37	39.7692	457.6
Kelburn	$\phi A \lambda$	41	17	10.1916	174	46	05.0263	126.7
Mt Crawford		41	17	46.1884	174	49	41.8642	163.1
Papatahi	$\phi$	41	19	08.9000	175	02	51.0000	902.4
Jenkins Hill No 2		41	20	00.0791	173	16	50.6269	775.4
Dovedale	$\phi A$	41	20	05.2253	172	59	03.6435	368.5
Te Kopahou	$\phi$	41	20	34.1797	174	42	39.6840	483.7
Island Bay No 2		41	20	37.6190	174	46	43.1336	128.0
Mt Robertson		41	20	58.2830	174	01	19.1775	1036.0
Mt Gomorrah		41	22	41.8932	172	35	03.7985	1591.6
Te Maunga		41	24	02.7403	175	19	54.9878	1009.6
Mt Richmond		41	28	32.9762	173	23	44.7468	1760.1
Korere		41	31	37.3719	172	49	38.8940	664.3
Goulter Hill	$\phi A$	41	32	40.1520	173	48	07.4668	173.1
Mt Owen		41	33	13.4982	172	32	28.0877	1874.8
Dumgree		41	37	03.1804	173	59	50.8997	644.9
Red Hill		41	37	42.8313	173	03	27.6605	1790.4
Nestor		41	42	01.1836	172	45	43.2580	851.7
Haycock		41	42	06.5484	173	23	41.6678	1199.1
Mt Frederick		41	42	22.9319	171	50	47.4508	1105.0
Blairich		41	42	39.9315	173	47	48.3013	1503.7
Mt Newton		41	43	24.2743	172	16	46.9784	1359.1
Mt Murchison		41	43	50.9193	172	29	57.7094	1469.1
Tutaki No 2		41	49	21.2947	172	21	32.2188	953.1
Shingle Peak	$\phi A$	41	57	36.0586	173	20	45.0187	2088.6
Mt Gladstone		41	58	47.0510	173	36	05.2917	2371.2
Mt Travers No 2		42	00	54.8054	172	44	06.1434	2338.2

Station		South Latitude	East Longitude	Height
		° ' "	° ' "	m
Mt Victoria		42 01 52.3199	172 07 05.8949	1639.6
Mt Faraday		42 02 19.2153	171 34 30.8207	1484.8
Murphy		42 02 59.0840	173 13 06.2080	1827.3
St Bernard		42 08 11.1661	173 25 44.3513	2255.4
Faerie Queene No 2		42 15 56.8794	172 30 54.5313	2236.2
Dillon Cone		42 16 24.8470	173 13 26.0760	2173.7
Mt Fyffe		42 18 45.5027	173 36 42.1657	1602.0
Conical Hill		42 22 29.7684	171 59 38 1570	1601.6
Kaikoura Peninsula	$\phi A$	42 25 19.8665	173 41 48.2445	107.8
Mt Percival		42 28 30.8414	172 56 07.1411	1629.1
Mons Sex Millia		42 32 12.1458	172 26 58 9696	1834.8
Peaked Hill		42 33 25.5555	173 23 37.6221	806.5
Mt French No 3		42 40 47.2623	171 20 38.4998	1305.2
Isolated Hill		42 41 20.8197	173 00 36.4802	392.6
Charing Cross		42 44 45.5699	172 38 32.9077	999.5
Devil's Hill		42 47 15.9602	173 01 31.8274	869.4
Pahau	$\phi A \lambda$	42 47 27.5329	172 48 37.2935	189.4
Mt Koeti		42 47 29.6198	171 46 29.7156	1785.9
Balmoral		42 50 22.8753	172 41 32.4657	252.1
Mt Mason		42 53 45.2301	172 30 09.6349	852.8
Mt Alexander		42 54 31.5574	172 48 46.3602	748.1
Weka		42 59 20.8854	172 41 33.9720	427.1
Mt Cass		43 04 46.5101	172 49 26.4659	525.7
Okuku Hill		43 05 05.6389	172 19 12.7018	1143.2
Mt Grey		43 07 09.8060	172 32 53.6945	933.7
Leithfield		43 12 59.1107	172 44 59.6356	2.6
Oxford Hill		43 13 20.8526	172 05 40.8435	1339.6
Mairaki Downs No 2		43 16 16.1478	172 22 45.8295	282.7
Rangiora		43 17 56.2215	172 35 12.2582	71.6
Burnt Hill		43 22 52.4642	172 08 12.1663	368.1
High Peak		43 28 10.8247	171 44 29.1090	968.4
Harewood		43 28 18.6621	172 30 47.1350	43.6
Sandy Knolls		43 31 03.2478	172 20 05.6730	112.5
Survey Office		43 31 57.9122	172 38 20.7597	[34.8]
Mt Pleasant	$\phi A$	43 35 26.2953	172 43 37.8969	499.2
Burnham No 2		43 36 22.6914	172 18 23.7783	69.3
Mt Somers		43 37 16.0910	171 22 30.2597	1686.8
Highbank		43 37 34.0809	171 47 44.7970	263.0
Cass Peak		43 38 15.6732	172 37 26.2391	545.8
Lyndhurst		43 44 09.0343	171 40 11.9893	218.5
Gawler Downs	$\phi A$	43 44 55.3616	171 21 38.6945	639.7
Rakaia	$A$	43 47 10.8856	172 01 22.3972	103.1
Taumutu No 2		43 50 56.5261	172 21 42.9895	c.6
Little Mt Peel		43 52 54.8696	171 12 48.9508	1311.4
Ashburton	$\phi A \lambda$	43 53 57.6981	171 44 26.9825	105.0



Station		South Latitude	East Longitude	Height
		° ' "	° ' "	m
Hinds		44 00 34.2237	171 33 52.1717	83.2
Ealing		44 02 50.6647	171 24 50.5968	116.4
Mt Burgess		44 03 03.0390	170 37 59.7925	1430.5
Waitohi Hill		44 05 33.6442	171 06 10.7242	697.4
Chastleton		44 07 26.1946	170 55 13.2589	583.3
Coldstream	$\phi A$	44 07 41.3819	171 31 23.1028	35.6
Mt Mary		44 08 25.8262	170 16 39.0646	995.6
Winchester		44 12 24.2738	171 17 57.6557	33.3
Mt Dalgety		44 18 06.4055	170 36 10.9143	1751.9
Ohau Peak		44 19 09.2720	169 46 09.4520	1914.3
Mt Horrible	$\phi A$	44 24 07.9933	171 03 26.1030	388.2
Tuhawaiki		44 26 40.5034	171 15 29.3425	18.1
Dingle Peak		44 29 06.3842	169 22 43.1945	1832.7
Mt Studholme		44 38 35.1013	170 54 39.6182	1085.1
Mt St Bathans		44 44 07.9554	169 45 45.4575	2087.2
Tara		44 44 44.8901	170 41 41.8660	908.3
Kohurau		44 48 20.4026	170 19 31.5878	2007.8
Mt Harris		44 49 55.0629	170 57 52.0930	284.1
Mt Pisa No 2		44 52 25.3390	169 11 26.6443	1960.7
Awamoko		44 55 31.7864	170 52 15.1500	89.3
Waitaki	$\phi A \lambda$	44 56 42.4532	171 01 16.5139	48.5
Island Cliff	$\phi A$	44 58 11.4888	170 39 14.0014	385.3
Big Hill		44 59 02.4971	170 55 36.4359	317.5
Ben Lomond No 2		45 00 31.1918	168 36 55.8880	[1746.5]
Mt Nicholas		45 07 58.4493	168 23 55.1083	1458.4
North Rough Ridge		45 10 14.4705	169 49 31.5657	1058.8
Obi		45 10 58.7342	170 28 38.0382	1425.2
James Peak		45 14 53.7221	168 50 42.9306	2071.7
Symmetry Peaks		45 17 16.0078	168 34 00.3283	1904.5
Moeraki		45 22 32.2038	170 50 38.8789	154.4
Hyde Rock		45 23 21.3661	169 11 51.4273	1672.3
Summit Rock		45 25 37.6332	170 04 14.0313	1449.7
Puketapu No 3		45 29 32.1459	170 43 52.0064	343.1
Mt Cameron		45 30 18.9261	168 51 37.1581	1612.3
West Dome	$\phi$	45 35 07.7847	168.13 07.8545	1271.0
Whitecoomb		45 35 47.2459	169 04 58.9246	1453.3
Waikouaiti		45 36 06.3852	170 28 17.4794	660.0
Black Hill		45 39 27.0262	168 34 38.8846	1022.0
Lammermoor		45 43 19.0422	169 44 14.3649	1159.0
Silver Peak No 2		45 44 13.8275	170 26 47.1925	777.1
Etal	$\phi A$	45 47 59.4703	168 02 09.8950	931.7
Waiparu		45 48 06.7919	168 49 21.9724	451.0
Powder Hill		45 48 07.9197	170 23 46.1147	525.5
Mt Wendon		45 48 52.0256	168 59 52.3152	844.9
Flagstaff		45 50 03.3243	170 27 58.2880	667.5
West Base		45 50 19.9293	168 39 54.7080	150.9

Station		South Latitude			East Longitude			Height
		°	'	"	°	'	"	m
North Taieri		45	51	41.4481	170	16	57.3208	6.8
Maungatua		45	52	55.4010	170	06	43.9989	894.8
Tapanui No 2		45	53	22.1883	169	23	26.9708	1019.1
East Base		45	53	41.9677	168	47	15.6442	122.2
Saddle Hill		45	54	44.1757	170	21	19.6650	472.8
Jeffrey Hill		45	54	51.8604	170	20	46.2347	430.6
Sandstone	$\phi A \lambda$	45	55	09.0417	168	40	55.9988	181.0
Ben Callum No 2		45	56	56.2583	168	30	48.0121	646.4
Mt Stuart		46	03	27.8458	169	46	27.8039	432.5
Kelvin Peak		46	03	50.0769	168	46	26.2932	639.2
The Cone		46	04	11.5965	169	12	50.8757	313.2
Forest Hill		46	10	56.5492	168	25	40.5805	232.7
Mt Misery		46	10	59.9547	169	55	22.8521	332.1
Longwood		46	15	01.6002	167	50	21.9525	764.3
Kaihiku No 2		46	16	22.8937	169	27	01.3086	675.2
Mokoreta No 2		46	19	59.9248	169	07	52.9196	712.9
Glenomaru		46	20	55.3950	169	39	49.3241	362.6
The Bluff	$\phi$	46	36	59.5259	168	20	19.3355	265.1
Waikawa		46	39	04.6604	169	10	19.4246	163.4
Ruapuke		46	44	22.1429	168	31	56.8913	62.9
Mt Anglem		46	44	31.1790	167	54	59.5198	979.5
The Neck No 2		46	55	47.8864	168	10	39.6197	49.4
Pryse Peak		46	56	28.5912	168	00	20.1918	351.9











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