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Summary

Most mapping in New Zealand is based upon the NZGD49 datum. However positioning using GPS is based upon WGS84, a global datum. The NZGD49 and WGS84 datums differ due to the different ellipsoids (shape and position) used to model the earth, observational methods and the effects of earth deformation.

These differences give a combined apparent shift of approximately 180 metres between WGS84 and NZGD49 coordinates, ie. if WGS84 coordinates are plotted on a map based upon NZGD49 the position would have an apparent northing error of approximately 180 metres. The WGS84 coordinate can be converted to a NZGD49 coordinate by several different methods. The accuracy achievable depends on the method selected.

An accuracy of approximately 15 metres can be achieved by applying the nationwide mid-point value of the range of differences in the latitude and longitude. That is adjust the WGS84 latitude 6.1 seconds southwards and the longitude 0.5 seconds westwards.

A better agreement of approximately 4 metres can be achieved using a 7 parameter Molodenskii-Badekas transformation. Applying this transformation involves converting the WGS84 latitude and longitude to Cartesian XYZ coordinates relative to the centre of the ellipsoid, then applying a rotation, translation (shift), and scale change, and finally converting the resulting coordinates back to latitudes and longitudes on the NZGD49 ellipsoid. The recommended parameters for this transformation are based upon calculations by Mackie in 1982 using doppler observations at 18 control stations. Mackie's calculations determined the relationship between WGS72 and NZGD49. The parameters to transform from WGS84 to NZGD49 are derived by applying the published relationship between WGS72 and WGS84 to Mackie's results.

Recommended 7 Molodenskii-Badekas parameters for the transformation from WGS84 to NZGD49

Ellipsoid parameters

Ellipsoid	Semi major axis (metres)	1/flattening
WGS84	6378137	298.257223563
NZGD49	6378388	297

Transformation parameters:

Translation (metres)	$T_x = -59.47$	$T_y = 5.04$	$T_z = -187.44$
Rotation (seconds of arc)	$R_x = 0.47$	$R_y = -0.10$	$R_z = 1.024$
Scale change (part per million)	$\Delta s = 4.5993$		

These parameters should only be used to obtain NZGD49 (horizontal) coordinates and not heights.
The following test point can be used to confirm that the parameters have been applied correctly:

Datum	Latitude	Longitude
WGS84	41 00 00.000000 S	173 00 00.000000 E
NZGD49	41 00 06.203677 S	172 59 59.485406 E

Note: converted coordinate accuracy is approximately ±4 metres:

Recent GPS measurements of first order NZGD49 stations have enabled new Molodenskii-Badekas transformation parameters to be calculated. These new parameters result in accuracy of approximately 3.5 metres which is considered an insignificant improvement over the Mackie based parameters (approx. 4 m) which have been informally promoted by Land Information New Zealand (and its predecessor, the Department of Survey and Land Information) since 1990. It is recommended that the Mackie based parameters be officially adopted by the department to avoid the confusion which a different set of parameters could introduce.

For applications requiring a higher degree of accuracy, eg. surveying, it is generally necessary to reference observations to local NZGD49 control stations rather than transform WGS84 coordinates to NZGD49, as no simple national transformation can achieve an accuracy significantly better than 4 metres. Transforming GPS derived heights is a separate problem which is beyond the scope of this report.

1 Introduction

This report compares currently available transformation parameters for converting between NZGD49 (New Zealand Geodetic Datum 1949) and WGS84 (World Geodetic System 1984) coordinates. The need for transformation parameters between WGS84 and NZGD49 coordinates comes from the increasing use of the Navstar Global Positioning System (GPS) for measuring the spatial position of physical features. The reference system used by GPS is WGS84 which is a three dimensional geocentric system, unlike NZGD49 which is only a two dimensional (ie. horizontal) non-geocentric system. This report will therefore only compare the accuracy of the transformation parameters on the horizontal coordinates and not heights .

2 Definition of New Zealand Geodetic Datum 1949 (NZGD49)

NZGD49 is a horizontal datum based on geodetic observations that were primarily observed between 1923 and 1949. A detailed account of the establishment of NZGD49 is given in Lee (1978). The first order triangulation covered most of the North Island but only the eastern side of the South Island and comprised approximately 290 stations, with a braced quadrilateral joining the networks across Cook Strait. The scale of the network was controlled by five North Island and three South Island baselines.

With the aid of electro-mechanical calculating machines the adjustment of the geodetic observations was undertaken in 8 figures using the method of condition equations. The reference ellipsoid is the International (Hayford) ellipsoid which was positioned so as to approximate the geoid across New Zealand. Twelve Laplace stations were used to orientate the ellipsoid so the Z axis approximated the geocentric natural systems Z axis (eg the Earth rotation axis).

The adjusted first order station coordinates were assigned no formal errors so must be considered to be error free. These first order stations have subsequently been held fixed when adjusting lower order networks.

3 Definition of World Geodetic System 1984 (WGS84)

The United States Defense Mapping Agency (DMA) has been involved in the development of World Geodetic Systems since 1960. The World Geodetic System that was established in 1984 (WGS84) used Doppler data from the US Navy Navigation Satellite System (NNSS). WGS84 was primarily developed to support the US DMA's mapping, charting and geodetic products.

The WGS84 reference frame constitutes a mean or standard earth rotating at a constant rate around a mean pole of rotation fixed in time. Its origin is at the earth's centre of mass, and the axes are coincident with the Conventional Terrestrial System as defined by Bureau International de l'Heure (BIH) for the epoch of 1984.0. The fundamental parameters and reference ellipsoid of WGS84 are the same as the International Union of Geodesy and Geophysics (IUGG) sanctioned Geodetic Reference System 1980 (GRS80), as described by Moritz (1980a), except for one parameter, $\bar{C}_2$. WGS84 defines the normalised second degree zonal harmonic coefficient of gravitational potential constant, $\bar{C}_2$ (= $-J_2 / \sqrt{5}$), instead of the dynamical form factor, J_2, of GRS80. The indirect use of $\sqrt{5}$ thus introduced a truncated difference (after the eighth digit) in the flattening, *f*, for the WGS84 ellipsoid from the *f* of GRS80 ellipsoid (Kumar, 1993). DMA (1991) gives the full list of adopted constants for WGS84, while further information on GRS80 is given by Moritz (1980a).

The WGS84 reference frame, now over a decade old, was designed only to have an accuracy of 1-2 metres (1 sigma), which is more than adequate even for large scale mapping (DMA 1991). However recent geodetic requirements of the Department of Defense (DoD) has required accuracy at the decimetre level. By comparing the four defining parameters (a, GM, J_2 and ω) of WGS84 with the more recently adopted scientific community values of the International Earth Rotation Service (IERS) (McCarthy, 1992), it was established that the GM value was the only parameter warranting revision.

The main reason for updating the original WGS84 GM value was to reduce the 1.3 metre radial error (bias) in all DoD GPS orbit fits. The GM value of the IERS standards was adopted as the new WGS84 GM (Malys and Slater, 1994).

At the time of redefining the GM parameter for WGS84, the coordinates of the five Air Force and five DMA GPS monitor stations were updated. This was achieved by simultaneously processing GPS data from the Air Force, DMA and selected International GPS Service for Geodynamics (IGS) sites during the 1992 global IGS campaign. The adjustment of these Air Force and DMA sites was performed while constraining a selection of the IGS sites to their ITRF91 (International Terrestrial Reference Frame 1992) values. This resulted in a new realisation of WGS84 through the adoption of new coordinates for the 10 DoD GPS tracking stations. The new realisation of WGS84 is reported by Swift (1994) to be coincident with ITRF91 at the order of 10 cm. This refined WGS84 reference frame, along with the improved GM value, have been given the designation WGS84 (G730), and was placed into DMA's orbit processing from the first day of GPS week 730 which corresponds to 2 January 1994 (Malys and Slater, 1994).

4 Data used to test transformation parameters

The Land Information New Zealand Datum 2000 project has remeasured all the NZGD49 first order stations, as well as many lower order NZGD49 stations, using GPS. This has allowed Land Information New Zealand to compare WGS84 coordinates against NZGD49 coordinates at 261 NZGD49 first order stations and 117 NZGD49 second order stations. These control points form the data set used to calculate and test transformations between WGS84 and NZGD49.

The Datum 2000 GPS observations span a period from 1994 to 1996. They have been reduced as baselines using Trimble GPSurvey software.

The reference frame for the GPS data was introduced into the network using three fixed zero order 2000 stations, Whangaparoa [1334], Heaphy House [WELL] and the University of Otago Surveying School [OUSD]. Their coordinates were calculated in an adjustment of the Australian regional network in terms of ITRF92 at the epoch of 1994.0 using the GRS80 ellipsoid (Morgan *et al.* 1996). This adjustment included the 1993 observations of 26 New Zealand stations. The ITRF92 reference frame differs from WGS84 by less than 1 metre in New Zealand, which is better than the absolute accuracy of the WGS84 realisation (see Table B-1 for size of transformation parameters between WGS84 and ITRF92).

The adjustment of the 1994 to 1996 GPS data did not take into account any earth deformation that occurred during the period of the surveys. A few observations were rejected as being inconsistent with the majority (in particular some observations demonstrate problems with the station heights).

5 Transformation parameters

5.1 DMA derived similarity transformation parameters

DMA (1987a) lists four different sets of similarity transformation parameters for converting from NZGD49 to WGS84, based on different combinations of parameters being solved for in the least squares solution. The values for the parameters (Table 1) were derived from 14 Doppler stations. No description of the location of these sites is contained in DMA (1987a or b). The only indication of the accuracy of the transformation parameters in Table 2 was given by DMA (1987b, p. 10-9) for the three parameter solution. The accuracies for the T_X, T_Y and T_Z parameters were ± 5, 3 and 5 m, respectively, though the confidence interval was not specified.

The similarity transformation parameters need to be applied to Cartesian coordinates. The local datum coordinates available to DMA consisted of orthometric heights rather than ellipsoidal heights. To convert orthometric height to ellipsoidal height the geoid height (N) needs to be known. DMA determined N values in terms of NZGD49 by assuming the local geoid height at each Doppler station was zero. Then using the Abridged Molodenskii Datum Transformation formula for ΔH (DMA, 1987a, Table 7.8), determined ΔH values (≡ ΔN). Then the ΔN values were subtracted from the WGS84 geoid height (degree and order 180) DMA (1987a).

Number of Parameters solved for	T_X m	T_Y m	T_Z m	ε_x ”	ε_y ”	ε_z ”	Δs ppm
3	-84	22	-209	-	-	-	-
7	-55	17	-184	0.773	-0.122	0.745	-5.9218 *

Table 1 : **DMA Similarity Transformation Parameters to convert from WGS84 to NZGD49 (DMA, 1987a, p. 7-47). *: Δ*s* probably printed with wrong sign (see Sections 5.3 and 5.5).**

5.2: DMA Multiple Regression Equations

The DMA (1987b, p. 20-20) MRE for the conversion of NZGD49 coordinates to WGS84 coordinates are in (1). The number of stations used to determine the MRE is unclear as tests for determining the accuracy of the equations used either 14 points (DMA, 1987a, p. 7-51) or 31 points (DMA, 1987b, p. 20-20).

$$\Delta\phi = 6.18012 + 0.18236U + 0.10785V - 0.15566UV + 1.36545U^3 - 0.44813UV^2 + 0.16518V^3 - 1.66408U^5 + 0.44854U^9$$

$$\Delta\lambda = 0.55131 + 0.18193U + 0.29501V - 0.36522UV - 0.12613U^3 - 1.13550U^2V^2 + 3.31705U^3V^3 + 3.40098U^9V^6 \quad (1)$$

where

parameter definitions are the same as (eqn A7)

$\Delta\phi$ and $\Delta\lambda$ units are arc seconds

$U = K(\phi + 41)$, with ϕ being the geodetic latitude in decimal degrees

$V = K(\lambda - 173)$, with λ being the geodetic longitude in decimal degrees

$K = 0.15707963$, the combined scale factor and degree-to-radian conversion

To convert from WGS84 to NZGD49 the offsets $\Delta\phi$ and $\Delta\lambda$ are calculated using WGS84 latitude and longitude in these formulae, and then subtracted from the latitude and longitude.

5.3: Mackie derived similarity parameters

Mackie (1982) determined transformation parameters between WGS72 and NZGD49 based on 18 Doppler stations that were well distributed across both the North and South Islands of New Zealand. Mackie computed a geoid referenced to NZGD49, by integrating the components of the deviation of the vertical (ie. from Astro-geodetic levelling). This allowed the orthometric heights to be converted to ellipsoidal heights and when combined with the NZGD49 horizontal coordinates, provided a local three dimensional set of coordinates that were compared with the Doppler coordinates to establish the transformation parameters.

Mackie solved for what he termed a “3-parameter solution to the transformation vector” and also a 7-parameter solution. The seven parameter solution for the transformation parameters was obtained by a least squares solution based on the Molodenskii-Badekas method (Appendix A.3). The parameters to transform from NZGD49 to WGS72 were stated by Mackie (1982, p. 22) as:

$$
\begin{array}{lllll}
T'_x & = & +83.217\text{ m} & \pm 0.27\text{ m} & \\
T'_y & = & -9.026\text{ m} & \pm 0.27\text{ m} & \\
T'_z & = & +202.024\text{ m} & \pm 0.27\text{ m} & \\
\Delta s & = & -4.8256 \times 10^{-6} & \pm 6.32 \times 10^{-7} & \\
\varepsilon_x & = & -2.2650 \times 10^{-6} & \pm 1.32 \times 10^{-6} & = -0.47'' \pm 0.27'' \\
\varepsilon_y & = & +4.6660 \times 10^{-7} & \pm 1.06 \times 10^{-6} & = +0.10'' \pm 0.22'' \\
\varepsilon_z & = & -2.2558 \times 10^{-6} & \pm 1.36 \times 10^{-6} & = -0.47'' \pm 0.28''
\end{array} \tag{2}
$$

Note that the sign of Δs as given by Mackie in (2) has the opposite sign to the Δs given by DMA (Table 1). It would appear from a comparison of the DMA (Table 1), Mackie (2) and calculated (Table 4) 7 parameters that the DMA value for Δs has been printed with the incorrect sign.

The values that were used for the centroid ($\overline{\mathbf{X}}_A \ \overline{\mathbf{Y}}_A \ \overline{\mathbf{Z}}_A$) when deriving the parameters in (2) were not specifically stated. However if one assumes that the centroid was calculated using Mackie's equation 5 (1982, p. 22), which is equivalent to that given in (eqn A5), the coordinates of the centroid can be calculated from the Cartesian coordinates given in Mackie's table 1 (*ibid.*, p. 22) to be:

$$
\begin{array}{lll}
\overline{\mathbf{X}}_A & = & -4774224.01\text{ m} \\
\overline{\mathbf{Y}}_A & = & 545802.12\text{ m} \\
\overline{\mathbf{Z}}_A & = & -4159198.36\text{ m}
\end{array} \tag{3}
$$

To compare the parameters derived by Mackie with parameters contained in Section 5.5, the parameters need to be converted from the Molodenskii-Badekas method (Appendix A.3) to the Bursa-Wolf method (Appendix A.2). This can be achieved by combining equations (A1), (A4) and (A6) to give the Bursa-Wolf translation parameters:

$$
\begin{bmatrix} \mathbf{T_X} \\ \mathbf{T_Y} \\ \mathbf{T_Z} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{X}}_A \\ \overline{\mathbf{Y}}_A \\ \overline{\mathbf{Z}}_A \end{bmatrix} - s_F \begin{bmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{bmatrix} \begin{bmatrix} \overline{\mathbf{X}}_A \\ \overline{\mathbf{Y}}_A \\ \overline{\mathbf{Z}}_A \end{bmatrix} + \begin{bmatrix} \mathbf{T}'_x \\ \mathbf{T}'_y \\ \mathbf{T}'_z \end{bmatrix} \tag{4}
$$

Solving (4) using the values in (2) and (3) results in (the sense, from NZGD49 to WGS72):

$$
\begin{array}{lll}
T_X & = & +59.47\text{ m} \\
T_Y & = & -5.04\text{ m} \\
T_Z & = & +182.94\text{ m}
\end{array} \tag{5}
$$

The scale and rotation parameters remain unchanged (see Appendix A.3). Having now converted Mackie's Molodenskii-Badekas based parameters to Bursa-Wolf parameters it is now possible to convert these parameters so that they for transforming from WGS84 to NZGD49, rather than transforming from NZGD49 to WGS72. This is achieved, due to the small rotation angles, by adding algebraically the parameters for converting WGS84 to WGS72 (Table B-1), with the rotation and scale parameters from (2) and the translation parameters from (5) (Table 2).

From	To	T_X m	T_Y m	T_Z m	Δs $x10^{-6}$	ε_x "	ε_y "	ε_z "
WGS84	WGS72	0.0	0.0	-4.50	-0.2263	0.0	0.0	0.554
WGS72	NZGD49	-59.47	5.04	-182.94	4.8256	0.47	-0.10	0.47
WGS84	NZGD49	-59.47	5.04	-187.44	4.5993	0.47	-0.10	1.024

Table 2 : **Seven parameter similarity transformation parameters derived from Mackie (1982) for conversion from WGS84 to NZGD49.**

5.4 Calculated latitude and longitude offset

A very simple approach to converting WGS84 latitude and longitude to NZGD49 is to determine an offset to apply to the latitude and longitude. One method of calculating the offset is to find the value which minimizes the maximum remaining error after the offset is applied. This is equivalent to the mean of the minimum and maximum observed offsets.

The offset calculated from the test data is 6.1 seconds south and 0.5 seconds west (to be applied to WGS84 latitudes and longitudes).

5.5 Calculated similarity parameters

Similarity transformation parameters have been calculated using the control points in the test data set. These points only provide horizontal control, since NZGD49 is only a two dimensional coordinate system.

In order to adequately constrain the solution, Mackie's approach has been followed. This approach endeavours to obtain a geoid height of approximately zero at the locations of the baselines measured for the 1949 adjustment.

The EGM96 global geopotential model has been used to calculate the geoid heights relative to WGS84 at the baseline stations. This is the WGS84 ellipsoidal height at which the orthometric height should be zero (to within the accuracy of the geoid model - of the order of 1 metre). Because the geoid height relative to NZGD49 is constrained to be zero, it follows that the NZGD49 ellipsoidal height of these points is close to zero.

Two sets of parameters have been determined based upon these criteria - one model comprises only translations (3 parameters), and the other is a full 7 parameter model. The parameters are:

Number of Parameters solved for	T_X m	T_Y m	T_Z m	ε_x "	ε_y "	ε_z "	Δs ppm
3	-54.4	20.1	-183.1	-	-	-	-
7	-39.5	-60.6	-211.8	-1.075	-1.096	2.545	4.692

Table 3 : **Calculated similarity parameters to convert from WGS84 to NZGD49**

6 Results and discussion

In order to test the proposed transformations the WGS84 coordinates of the control stations have been converted to NZGD49 using each transformation. These and the NZGD49 coordinates for the points have been converted to the NZMG map projection. The residual horizontal error in the transformation is the distance between the coordinates derived from WGS84 and those from NZGD49.

The residuals are summarised in Table 4. Seven transformation models are ordered from the smallest to largest maximum residual horizontal error.

Model	Residual Horizontal error (metres)			
	Average	RMS	95 Percentile	Maximum
Calculated 7 parameter	1.3	1.5	2.3	3.4
Mackie 7 parameter	1.8	2.0	3.4	4.0
Calculated 3 parameter	2.4	2.5	3.9	4.2
DMA MRE formula	1.9	2.2	3.4	5.9
DMA 3 parameter	3.7	3.9	5.8	7.7
Latitude/longitude offset	8.5	9.4	14.5	14.8
No transformation	191.4	191.6	202.4	202.6

Table 4 : Comparison of the residual horizontal errors due to applying different transformation models.

The most appropriate method to use depends upon the accuracy required and the software available. There is little to choose between the first three methods in terms of accuracy - Mackie's parameters are recommended as they are already in use.

The MRE method in principle should be able to give a better result since it can take account of distortion. However we have not recalculated these parameters using the new control points as the transformation tends to be unstable away from the control points and is not widely supported in software. Indeed, the DMA (now NIMA) have not pursued this method since the mid 1980's (G.Stentz, pers comm).

The DMA 3 parameter transformation has little to recommend it compared to the alternative transformation parameters.

For many navigation applications applying the latitude and longitude offset will be more than adequate if the equipment and software do not support a 3 or 7 parameter similarity transformation. This accuracy is well within that of stand-alone (non differential) GPS navigation.

Figure 1 shows the residual errors at the control points after the Mackie 7 parameter transformation has been applied to the WGS84 coordinates. As there are significant local trends, more accurate transformations could be determined for these local areas but this is beyond the scope of this paper.

Figure 1 also implies that surveys referenced to a local base station with known NZGD49 coordinates can obtain a significantly better accuracy. For example it should be straightforward to position to within 0.3m (commonly required in utility mapping) provided suitable equipment and procedures are used. Note that this assumes that relative consistency of NZGD49 is preserved in the breakdown from second order stations into the local cadastre.

The irregular distribution of error across the country also implies that simple transformations cannot provide a much better overall accuracy since they cannot adequately model the distortion in NZGD49.

Figure 1: Shows the remaining difference between WGS84 and NZGD49 after the Mackie 7 parameter transformation has been applied to the WGS84 coordinates.

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Appendix A: Transformation methods

There are a number of ways of defining the relationship between one reference system and another. The choice of the most appropriate network transformation model is influenced by:

i) the extents of the area for which it is to be applied

ii) the presence of distortion in either of the reference systems

iii) the dimensions of the reference systems (two- or three-dimensional), and

iv) the accuracy requirements.

This section focuses on those methods which have been applied to the New Zealand situation of converting between NZGD49 coordinates and global reference systems. It is noted that other theoretical transformation methods are available but parameters have not been developed (or at least widely disseminated) for New Zealand.

One of the most commonly used transformation methods in surveying is the similarity transformation, which preserves the shape, so angles are not changed, but lengths of lines and the position of points may be changed. It assumes that there are no systematic distortions within either network.

The general similarity transformation is given by:

$$\begin{bmatrix} \mathbf{X}_B \\ \mathbf{Y}_B \\ \mathbf{Z}_B \end{bmatrix} = s_{\mathrm{F}}\mathbf{R}\begin{bmatrix} \mathbf{X}_A \\ \mathbf{Y}_A \\ \mathbf{Z}_A \end{bmatrix} + \begin{bmatrix} \mathbf{T}_\mathbf{X} \\ \mathbf{T}_\mathbf{Y} \\ \mathbf{T}_\mathbf{Z} \end{bmatrix} \qquad \text{(A1)}$$

where

X_B, Y_B, Z_B	Cartesian coordinates in coordinate system B
X_A, Y_A, Z_A	Cartesian coordinates in coordinate system A
T_X, T_Y, T_Z	translations terms, are the coordinates of the origin of the XYZ_A coordinate system in the XYZ_B coordinate system, respectively
R	3 x 3 orthogonal rotation matrix (Section A.1)
s_F	scale factor = 1 + Δs, where Δs is the differential scale

There are seven parameters which are usually associated with a similarity transformation; three rotation angles, three translation components and one scale factor. If the rotations are small, as is expected when both coordinate systems refer to the same CTRS, then (A1) is approximately linear and the order of the rotations is unimportant.

The similarity transformation is popular due to:

i) the small number of parameters involved

ii) the simplicity of the model, which is more easily implemented into software, and

iii) the fact that it is adequate for relating two coordinate systems which are homogeneous (no local distortion in scale or orientation).

As a result similarity transformation parameters have been published to allow the conversion between coordinate systems used in New Zealand (Section 5). It is therefore necessary to outline some of the different models that are available for determining similarity transformations, especially those used for establishing New Zealand transformation parameters.

One disadvantage of the seven parameter similarity transformation method is that both networks are assumed to have only linear distortions (excluding shear components). Often older terrestrial networks do have non-linear distortions because of the adjustment and survey methodologies employed.

Multiple Regression Equations (MRE) are one method that attempts to account for non-linear distortions in either of the networks and is outlined in Section A.4.

A.1 Coordinate rotation methods

The Cardanian rotation matrix is the most commonly applied method of rotating a coordinate system. If the rotation angles are small, the order of applying the Cardanian angles to their respective axes does not influence the result. One of the six possible combinations is that contained in (A2).

$$\mathbf{R} = R_Z(\varepsilon_z) * R_Y(\varepsilon_y) * R_X(\varepsilon_x) \tag{A2}$$

where

R_X, R_Y, R_Z rotation matrices about the X, Y, Z axes respectively.

$\varepsilon_x\ \varepsilon_y\ \varepsilon_z$ rotation angles in radians about the X, Y, Z axes respectively. Positive rotations are clockwise rotations as viewed looking from the origin to the positive end of the axis in a right handed coordinate system.

and

$$R_Z(\varepsilon_z) = \begin{bmatrix} \cos\varepsilon_z & \sin\varepsilon_z & 0 \\ -\sin\varepsilon_z & \cos\varepsilon_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_Y(\varepsilon_y) = \begin{bmatrix} \cos\varepsilon_y & 0 & -\sin\varepsilon_y \\ 0 & 1 & 0 \\ \sin\varepsilon_y & 0 & \cos\varepsilon_y \end{bmatrix}$$

$$R_X(\varepsilon_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon_x & \sin\varepsilon_x \\ 0 & -\sin\varepsilon_x & \cos\varepsilon_x \end{bmatrix}$$

Therefore **R** can now be written as:

$$\mathbf{R} = \begin{bmatrix} \cos\varepsilon_z\cos\varepsilon_y & \cos\varepsilon_z\sin\varepsilon_y\sin\varepsilon_x + \sin\varepsilon_z\cos\varepsilon_x & \sin\varepsilon_z\sin\varepsilon_x - \cos\varepsilon_z\sin\varepsilon_y\cos\varepsilon_x \\ -\sin\varepsilon_z\cos\varepsilon_y & \cos\varepsilon_z\cos\varepsilon_x - \sin\varepsilon_z\sin\varepsilon_y\sin\varepsilon_x & \sin\varepsilon_z\sin\varepsilon_y\cos\varepsilon_x + \cos\varepsilon_z\sin\varepsilon_x \\ \sin\varepsilon_y & -\cos\varepsilon_y\sin\varepsilon_x & \cos\varepsilon_y\cos\varepsilon_x \end{bmatrix} \tag{A3}$$

For small rotation angles the rotation matrix (A3) can be approximated by

$$\mathbf{R} \cong \begin{bmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{bmatrix} \tag{A4}$$

A.2 Bursa-Wolf model

This model, presented by Bursa (1965) and Wolf (1963), solves for a seven parameter transformation - a scale factor, three rotation angles and three translation components. The Bursa-Wolf model is also known in Geodesy as the Seven Parameter Similarity model and takes the same form as the general similarity transformation of (A1).

One problem with the Bursa-Wolf model is that the adjusted parameters are highly correlated when the network of points used to determine the parameters covers only a small portion of the earth.

A.3 Molodenskii-Badekas model

The Molodenskii-Badekas model (Badekas, 1969) removes the high correlation between parameters by relating the parameters to the centroid of the network.

$$\begin{bmatrix} \mathbf{X}_B \\ \mathbf{Y}_B \\ \mathbf{Z}_B \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{X}}_A \\ \overline{\mathbf{Y}}_A \\ \overline{\mathbf{Z}}_A \end{bmatrix} + \begin{bmatrix} \mathbf{T}'_x \\ \mathbf{T}'_y \\ \mathbf{T}'_z \end{bmatrix} + s_{\mathrm{F}}\mathbf{R} \begin{bmatrix} \mathbf{X}_A - \overline{\mathbf{X}}_A \\ \mathbf{Y}_A - \overline{\mathbf{Y}}_A \\ \mathbf{Z}_A - \overline{\mathbf{Z}}_A \end{bmatrix} \quad \text{(A5)}$$

where

$\overline{\mathbf{X}}_A = \sum \mathbf{X}_{Ai} / n$ = centroid X coordinate for the points in coordinate system A

$\overline{\mathbf{Y}}_A = \sum \mathbf{Y}_{Ai} / n$ = centroid Y coordinate for the points in coordinate system A

$\overline{\mathbf{Z}}_A = \sum \mathbf{Z}_{Ai} / n$ = centroid Z coordinate for the points in coordinate system A

T'_x T'_y T'_z Molodenskii-Badekas translations terms

remaining terms are as defined for the Bursa-Wolf model (A1)

The adjusted coordinates, baseline lengths, scale factor, rotation angles, their Variance Covariance (VCV) matrices and the *a posteriori* variance factor computed by this model are the same as those from the corresponding Bursa-Wolf solution. However, the translations are different and their precisions are generally an order of magnitude smaller (Harvey, 1986). The difference between the translation terms of the Bursa-Wolf and Molodenskii-Badekas models is due to the different scaling and rotating of the centroid of the network. This can be seen clearly by expanding (A5) to give (A6), where $\mathbf{P_c}$ is a constant term for all points and obviously affects the translation terms.

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \mathbf{P_c} + \begin{bmatrix} T'_x \\ T'_y \\ T'_z \end{bmatrix} + s_{\mathrm{F}}\mathbf{R} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \quad \text{where } \mathbf{P_c} = \begin{bmatrix} \overline{X}_A \\ \overline{Y}_A \\ \overline{Z}_A \end{bmatrix} - s_{\mathrm{F}}\mathbf{R} \begin{bmatrix} \overline{X}_A \\ \overline{Y}_A \\ \overline{Z}_A \end{bmatrix} \quad \text{(A6)}$$

When transformation parameters from the Molodenskii-Badekas model are to be applied to transform coordinates of points, it is essential to know what values were used for the centroid ($\overline{\mathbf{X}}_A$ $\overline{\mathbf{Y}}_A$ $\overline{\mathbf{Z}}_A$) when deriving the parameters. However, in the past they have not always been published with the transformation parameters (eg. Mackie, 1982).

It should be noted that when working with a global network of points the Molodenskii-Badekas model has centroid coordinates that equal the centre of the ellipsoid (ie. $\overline{\mathbf{X}}_A = \overline{\mathbf{Y}}_A = \overline{\mathbf{Z}}_A = 0$) and therefore reduces to the Bursa-Wolf model.

A.4 Multiple Regression Equations (MRE) Method

Multiple Regression Equations method has the advantage over both the Bursa-Wolf and Molodenskii-Badekas models of being able to account for non-linear distortion in either of the networks. The main disadvantage of the MRE method is that outside the area of the control points used to determine the MRE the results can be extremely unreliable. Therefore the control points need to extend to the boundaries of the datum for which the transformation is to apply.

There are various forms that an MRE can take, but only the form used by DMA(1987a) will be presented, as this form has been used to determine transformation parameters between WGS84 and NZGD49 (Section 5.2). For each coordinate component a difference between datum values ($\Delta\phi$, $\Delta\lambda$,

Δh) is determined by an MRE, and this is then applied to the known datum coordinate component to obtain the unknown datum coordinate using:

$$\phi_B = \phi_A + \Delta\phi$$

$$\lambda_B = \lambda_A + \Delta\lambda$$

$$h_B = h_A + \Delta h$$

and

$(\phi\lambda h)_A$ known curvilinear coordinates of a station in terms of datum A

$(\phi\lambda h)_B$ unknown datum B curvilinear coordinates of the same station

The general form of the difference between the two datum, using an MRE, for the latitude component is (DMA, 1987a, eqn. 7-14):

$$\Delta\phi = A_0 + A_1U + A_2V + A_3U^2 + A_4UV + A_5V^2 + ... + A_{54}V^9 + A_{55}U^9V + A_{56}U^8V^2 + ... + A_{64}U^9V^2 + A_{65}U^8V^3 + ... + A_{72}U^9V^3 + A_{73}U^8V^4 + ... + A_{99}U^9V^9 \quad \text{(A7)}$$

where

$A_0, A_1, ..., A_{99}$	= 100 possible coefficients determined in a stepwise multiple regression procedure with U and V each limited to single digit exponents
$U = K (\phi - \phi_m)$	= normalised geodetic latitude of the computation point
$V = K (\lambda - \lambda_m)$	= normalised geodetic longitude of the computation point
K	= scale factor and degree-to-radian conversion
ϕ, λ	= local geodetic latitude and longitude, respectively, of the computation point (in degrees)
ϕ_m, λ_m	= mid-latitude and mid-longitude values, respectively, of the local geodetic datum area (in degrees).

Similar equations are obtained for $\Delta\lambda$ and Δh by replacing $\Delta\phi$ in the left hand side of (A7) by $\Delta\lambda$ and Δh, respectively.

The coefficients for the MRE were computed by DMA (1987a, p. 7-18) using the following approach. Prior to beginning the development process, individual $\Delta\phi$, $\Delta\lambda$, Δh coordinate differences are formed for each station within the datum area that has coordinates in terms of both datum. The multiple regression procedure of Appelbaum (1982) is then initiated to develop separate equations to fit the $\Delta\phi$, $\Delta\lambda$ and Δh coordinate differences. The first step of the procedure produces a constant and a variable. The variable will either be a function of ϕ or λ, or both. The procedure then sequentially adds one variable at a time to the equation. After a variable is added, all variables previously incorporated into the equation are tested and, if one is no longer statistically significant, it is removed. This stepwise addition or removal of variables ensures that only significant variables are retained in the final equation. In keeping with (A7), each variable consists of products of powers of normalised geodetic latitude (U), or normalised geodetic longitude (V), or both (ie. U^3V^4 is a single variable). The stepwise regression procedure continues until the precision desired for the equation is obtained.

The DMA derived MRE for transforming NZGD49 coordinates to WGS84 coordinates are contained in Section 5.2 for which the desired precision was that the rms difference be approximately less than 1.5 m (DMA, 1987a, p. 7-19).

Appendix B: Transformation parameters between ITRS and WGS realisations

The transformation parameters used in this report for converting between different realisations of ITRS and WGS are summarised in Table B-1. The application of these parameters was performed using the seven-parameter similarity transformation (Bursa-Wolf) formula given in (A1) using the simplified rotation matrix of (A4).

From	To	Reference	T_X m	T_Y m	T_Z m	Δs $x10^{-8}$	ε_x mas	ε_y mas	ε_z mas
WGS84	WGS72	preprint of DMA 1987a	0.000	0.000	-4.500	-22.63	0.0	0.0	554.0
WGS84	WGS72	DMA 1987a	0.000	0.000	-4.500	-21.9	0.0	0.0	554.0
WGS84	WGS84 (G730)	Swift 1994	-0.040	-0.010	-0.280	-21.8	4.2	-4.0	-15.6
WGS84 (G730)	ITRF92	Swift 1994	-0.070	-0.130	0.150	-0.03	-6.0	2.0	-2.4
ITRF91	ITRF92	Boucher *et al.* 1993	-0.011	-0.014	0.006	-0.14	0.0	0.0	0.0

Table B - 1 : Transformation parameters between ITRS and WGS realisations.

It is worth noting that the value given by DMA (1987a) for Δs, to convert WGS84 to WGS72 coordinates, is -21.9 x 10^{-8}. However, the preprint of DMA (1987a and b) stated that Δs = -22.63 x 10^{-8}, which is the value quoted in Australia (eg. Higgins, 1987 and Steed, 1990) and used within DOSLI to modify Mackie's parameters (Section 5.3). This difference of approximately 0.04 m at the Earth's surface can be considered insignificant when compared to the accuracy obtainable by the Doppler method used to establish WGS72 and WGS84.