Progress Towards a Gravimetric Geoid for New Zealand and a Single National Vertical Datum

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Abstract. New Zealand, unlike most countries, does not have a single national vertical datum. Instead, twelve separate and poorly linked primary levelling networks tied to twelve different tide gauges are used. The current vertical datums are based on the false assumption that mean sea level, measured at the twelve tide gauges, corresponds to the same equipotential surface. Due to the effects of long-period tides and sea surface topography, offsets of up to 0.5 metres between vertical datums are possible. In addition, no regional geoid model is available for New Zealand.

A consequence of the multiple vertical datums is that the gravity observations are downward continued to twelve different surfaces. This will cause the anomalies to be distorted, especially in the medium wavelengths, which will propagate into the geoid solution, meaning that the initial solution will not completely unify the datums. An iterative process to achieve a better unification is proposed. It is expected that this approach will be more successful than using a geoid model based on distorted gravity anomalies. The results of a regional gravimetric geoid computation using land based gravity observations, satellite altimetry and a global geopotential model are also presented.

Keywords. Gravimetric geoid, vertical datum unification, New Zealand

1 Introduction

New Zealand does not currently have a single national vertical datum. Instead, twelve separate datums are used (Figure 1). Each datum is based on mean sea level (MSL) observed at a different tide gauge, often over a very short time period (less than two weeks in some cases). These tide gauges form the origin of each levelling network. Despite some early evidence to the contrary, the datums were assumed to be stable and thus capable of being connected by precise levelling, so that they could eventually form a national vertical datum (e.g., Hannah 2001). Due to computational limitations at the time of calculation, each levelling network was adjusted independently. Even if a national adjustment was performed, New Zealand consists of two primary islands (Figure 1); so at least two vertical datums would result.

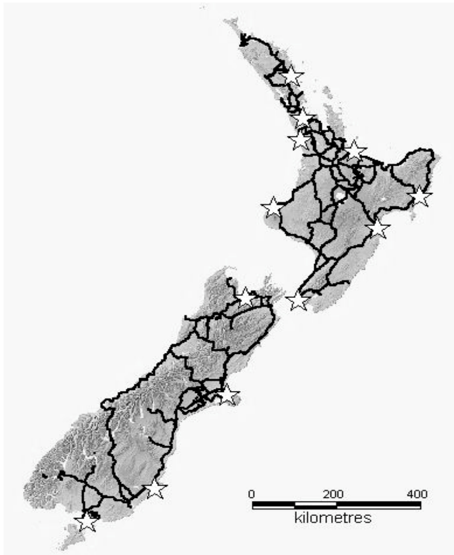

Figure 1. Tide gauges and levelling networks used to provide vertical geodetic control in New Zealand

The New Zealand precise levelling observations (Figure 1) had been reduced to give 'normal orthometric' heights at each benchmark (e.g., Heiskanen and Moritz 1967) because gravity data were not collected while the levelling observations were made. While levelling is a precise means of transferring heights between points, the very slow speed at which it is done, i.e. a period of 40 years in New Zealand, means the network will most certainly be subject to uplift or subsidence due to a variety of processes, mostly tectonic. New Zealand is situated at the active boundary of the Australian and Pacific plates. This uplift/subsidence can be as much as 8.5

m over ~40 years in localised areas, and ~10 mm/yr over larger areas (Walcott 1984).

The effect of sea surface topography (SST) means that MSL measured at tide gauges departs from a single equipotential surface, which will create offsets between adjacent or overlapping vertical datums been based on such measurements (cf. Hipkin 2000). Due to the elongated shape of New Zealand, the broad-scale variation in SST is in the order of ~10 cm around the South Island and ~15-20 cm around the North Island. This, combined with steric sea level rise and long-period tides, has meant that MSL at the datum origins does not lie on a single equipotential surface, and offsets of ~0.5 m have been observed. Hannah (2001) points out that the ability to form a New Zealand vertical datum based solely on regional levelling networks referenced to MSL at a number of tide gauges is becoming more remote with time.

Since GPS is nominally referenced to a geocentric ellipsoid, heights that are obtained from it are not related to potential surfaces of the Earth's gravity field. In most cases, users need heights that are referenced to the gravity field, primarily to determine fluid flows and to be consistent with the existing geodetic infrastructure. To convert GPS-derived ellipsoidal heights to normal/orthometric heights, it is necessary to use a quasi/geoid model.

Global geoid models that are available to degree 360 (e.g., EGM96; Lemoine et al. 1998) have an equivalent spatial resolution of ~55 km. This is insufficient for localised GPS surveys, as it will not provide much of the high-frequency variations in the geoid (i.e., the omission error). Unlike many other countries, such as Australia (Featherstone et al. 2001) and the United States (Smith and Roman 2001), New Zealand does not currently have a high-resolution regional geoid model. Gilliland (1990) produced a gravimetric co-geoid model for New Zealand on a 0.25° grid by combining gravity data and the OSU81 (Rapp 1981) global model to degree 180. Unfortunately, this geoid model is no longer available. Therefore, there is a pressing need to compute a higher resolution regional gravimetric geoid model for New Zealand.

Mackie (1982) estimated 'geoid' heights at 18 stations distributed across New Zealand by subtracting spirit-levelled heights from Doppler-derived WGS-72 ellipsoidal heights. Gilliland (1990) compared his co-geoid model with Mackie's results, giving rms differences of ~1.3 m. This was a reasonable result given the accuracy of the Doppler heights, but GPS techniques should yield ellipsoidal heights that are at least an order of magnitude better.

An evaluation of geoid user groups in New Zealand by Pearse (2001) determined that an initial gravimetric geoid model with an accuracy of ~10-15 cm would meet a majority of their requirements. Those users that would not be satisfied required a high accuracy of 5 mm; a level of accuracy that is unachievable with the data sets currently available in New Zealand. Instead, these users will still have to rely upon precise geodetic spirit levelling.

2 Vertical Datum Unification

An examination of international experience carried out by Hannah (2001), and built upon by Pearse (2001), shows that there is currently no clear international standard for vertical datum unification. However, one common feature among most of the countries studied was that all of their levelling networks have been adjusted together to yield a single vertical datum. This is not yet a desirable option in New Zealand because the levelling networks often have poor connections with one another, making a combined adjustment problematic. In addition, the vertical deformation in New Zealand over the 40-year period that the levelling has been conducted is poorly known in many areas.

The unification of vertical geodetic datums is a major focus of current geodetic research. Featherstone (2000) cites a number of studies that have investigated this issue. He lists a number of factors that render the unification of vertical datums problematic. These include the appropriate formulation and solution of the geodetic boundary-value problem, and the spatially varying accuracy of the geoid and SST models. Solutions to the geodetic boundary-value problems have been proposed, initially by Colombo (1980) and more recent developments provided in Rummel and Teunissen (1988). Rummel (2001) lists a number of problems that affect height datum unification after the launch of dedicated satellite gravity missions. All these issues will need to be examined for the 12 New Zealand vertical datums if they are to be properly unified.

At the present time, the most viable technique to approach vertical datum unification in New Zealand is through a regional gravimetric geoid model (cf. Kumar and Burke 1998). Simply by comparing the GPS-levelling heights on each vertical datum with the gravimetric geoid model will yield an offset that can be used to unify the datums (cf. Featherstone 2000). In this scenario, the gravimetric geoid model would form the 'official' national vertical datum with all 12 local vertical datums being referenced to it. This approach has the additional benefit of allowing GPS observations to be directly transformed into heights in relation to each local vertical datum. Also, users can continue to utilise the local datums for day-to-day operations, and the geoid model only when transformations need to be made.

3 Geoid Development Processes

The calculation of regional gravimetric geoid models is arguably becoming a relatively routine procedure, with readily available software that can generate regional solutions. The situation in New Zealand would allow for the generation of a geoid by this technique, however it would result in the geoid not being connected to any of the existing levelling networks. Such a solution would greatly diminish the benefits of using the geoid model, as offsets would still need to be calculated to relate the calculated orthometric heights to the different local vertical datums.

To achieve the unification of the multiple vertical datums in New Zealand, it is proposed to compute a regional gravimetric geoid that has each datum offset calculated subsequent to the determination. This technique will require an iterative approach that should eventually converge on the optimum offsets between the various datums. These offsets will able to be checked by the observation of GPS positions at the datum origins so that the ellipsoid/levelling datum difference can be determined.

Featherstone et al. (2001) discuss the procedures used to compute the Australian model. To summarise, AUSGeoid98 used a hybrid Fast-Fourier-Transform (FFT) and modified Stokes integral technique to evaluate regional geoid undulations. They found that a spherical cap was necessary for the Australian data, however in other parts of the world using the full rectangular grid in the calculations appears to give better results (cf. Forsberg and Featherstone 1998). The procedure should be applicable to the New Zealand situation, however it will be necessary to trial the use of a spherical cap (and modified kernel) to determine its effectiveness, among many other things.

The determination of terrain corrections and downward continuation is an important step in the geoid computation procedure that has received a reasonable amount of coverage in the literature recently. For instance, Tsoulis (2001) found that by varying the modelling technique of the FFT algorithm the convergence of solutions was regained, especially on slopes that have a gradient over 45°, and it approximates the correction computed by the classical, but computationally intensive, prism summation method. This is important to the New Zealand situation because in a number of places the topography exceeds a 45° angle and numerical convergence in these areas is essential.

When reducing gravity observations to compute the geoid, it is necessary to allow for the topographical mass density (e.g., Huang et al. 2001). This has traditionally been done with an approximation of the average value of 2.67 g/cm^3 due to problems with obtaining the actual density. Huang et al. (2001) show that the topographical density effect on the geoid ranges from –7.0 to 2.8 cm in the Canadian Rocky Mountains, and that the variation is significant enough to be accounted for when aiming to determine a 'one-centimetre' geoid. Based on the topography in New Zealand, it is expected that the effects will be comparable, although the geoid model will probably not be accurate to the centimetre level with the current New Zealand data sets.

It was also shown in Tziavos and Featherstone (2001) that the use of density data in the gravity reduction has a significant influence on the geoid model. They recommend that even the most simplistic density model should be used in all steps of the geoid computation. Like Western Australia, where the Tziavos-Featherstone study was performed, New Zealand has areas of rapid density change. Therefore, it is expected that the use of even simple models will be of benefit to the New Zealand geoid solution.

The enhancement of gravimetric geoids and unification of vertical datums are popular areas of research at the present time. The proposed novel approach of iteratively resolving the geoid to combine the multiple vertical datums is significant both scientifically and practically. The scientific merit of such a procedure is that it will prove a potentially useful technique for local vertical datum unification. The practical merit is that it will provide both a unified vertical datum and a regional gravimetric geoid model for the people of New Zealand. The iterative approach will be based on the theory of Rummel and Teunissen (1988), where the GPS-levelling to geoid offsets will be used to reduce the gravity data to the computed geoid as opposed to the different (offset) vertical datums.

4 Preliminary Co-geoid Computations

A preliminary gravimetric co-geoid model has been computed for the New Zealand region using the EGM96 global geopotential model (Lemoine et al. 1998), terrestrial gravity data, and satellite-altimeter-derived marine gravity anomalies (Hwang et al. 1998). This is nearly the same as the procedure as was used for the determination of AUSGeoid98 in Australia (Featherstone et al. 2001).

4.1 Regional Data Sets

The 2001 release of the Institute of Geological and Nuclear Sciences (GNS) national gravity database was used for the computation of the regional gravimetric co-geoid. This release consists of 40,000 terrestrial (Figure 2) and approximately 1.5 million marine gravity observations. The horizontal positions of the terrestrial gravity observations were transformed onto a geocentric datum using a seven-parameter model.

The satellite altimetry grid of Hwang et al. (1998) was used to provide gravity information in the marine areas surrounding New Zealand. A comparison of the GNS marine gravity data with these satellite altimeter data (cf. Featherstone 2002 this issue) revealed that numerous gross errors exist in the GNS marine gravity data as a result of no crossover analysis being performed. For the purposes of the initial co-geoid computation, these marine gravity data were excluded, and the satellite-altimeter-derived marine anomalies used instead. These discrepancies will be investigated, through a crossover analysis and adjustment, to enable inclusion in future models.

A digital elevation model (DEM) with a spatial resolution of 0.0005° (~55 m) is available for New Zealand. A generalised version is displayed as the backdrop in Figures 1, 2 and 4. It is derived from the Land Information New Zealand (LINZ) topographic database. It has an estimated accuracy of ±20 m horizontally and ±10 m vertically. However, this DEM was not used in these preliminary computations. Instead, previously computed gravimetric terrain corrections were taken from the GNS database. Therefore, there is probably an inconsistency between these and the topographical corrections (and downward continuation) routinely used in regional gravimetric geoid computations. As no indirect effects were computed, the result can be loosely termed a terrain-corrected free-air co-geoid.

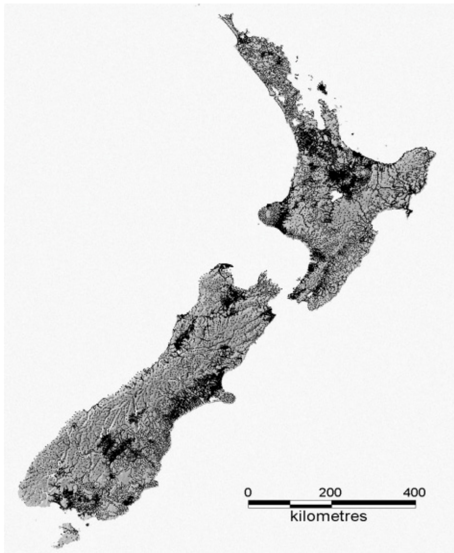

Figure 2. Terrestrial gravity coverage in New Zealand

4.2 Computation and Results

Terrain-corrected free-air gravity anomalies were calculated from the observed gravity data using a second-order free-air reduction and the supplied terrain corrections. The numerical solution of the modified Stokes integral used the 1D-FFT technique (Haagmans et al. 1993), thus requiring a regular grid of mean gravity anomalies. On land, the terrain-corrected free-air gravity anomalies were interpolated using tensioned splines (Smith and Wessel, 1990) to produce a 2-minute by 2-minute grid of mean anomalies. It is acknowledged that this interpolation is highly sensitive to aliasing; so future geoid models will use refined Bouguer gravity anomalies during the gridding stage.

Because of the discrepancies in the GNS marine gravity data (outlined above), the satellite altimetry grid was extracted for use in marine areas. This was placed around the land anomalies so that a continuous grid existed over the study area. However, no attempt was made to account for the discrepancies between the land and marine data due to the numerous problems with satellite altimeter data near the coasts (cf. Featherstone 2002 this issue). Future models will consider combination, probably via least-squares collocation (cf. Kirby and Forsberg, 1998) or filtering the satellite altimeter data.

Amos and Featherstone (2002 submitted) test a number of recent global geopotential models to determine the best fit to the New Zealand data. They found that EGM96 was very marginally better than the other models. The complete 360-degree expansion of EGM96 was therefore used to remove the low frequencies from the land and marine gravity anomalies. These residual gravity anomalies were then subjected to a 1-D-FFT transformation with a deterministically modified Stokes kernel to evaluate the residual geoid. This was then restored with the EGM96 geoid to determine the final co-geoid. As stated, no indirect effects, density models or downward continuation were included.

	Max	Min	Mean	STD
EGM96	3.712	-1.338	0.027	0.616
NZ co-geoid	4.201	-1.338	0.301	0.668

Table 1. GPS/levelling to co-geoid residuals (metres)

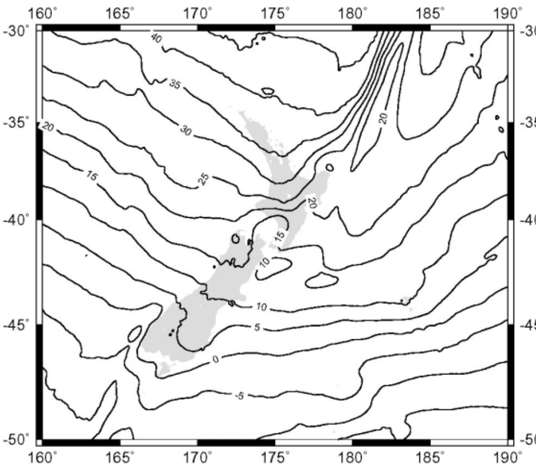

Figure 3. The 2002 New Zealand gravimetric co-geoid (contours in metres)

The resulting co-geoid (Figure 3) was compared with a set of 1017 points that have both GPS ellipsoidal and levelled heights (Figure 4). As mentioned earlier, the levelled heights are based on 12 different datums, which will bias the differences computed (Table 1). As such, even though the regional geoid model appears to be worse than EGM96, this could be an artefact of the different vertical datums. However, there are numerous deficiencies in this preliminary co-geoid model (outlined earlier), which are also plausible explanations for the results in Table 1.

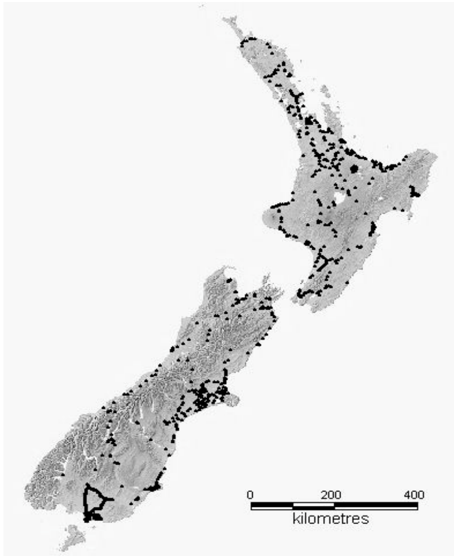

Figure 4. The 1017 GPS/levelling data over New Zealand

4.3 Preliminary vertical datum offsets

Datum	Pts	Max	Min	**Mean**	STD
One Tree	34	0.183	-0.027	**0.090**	0.062
Auckland	84	0.142	-0.383	**-0.133**	0.113
Moturiki	163	0.758	-0.480	**0.109**	0.257
Gisborne	57	0.661	0.041	**0.235**	0.172
Napier	26	0.595	-0.071	**0.075**	0.124
Taranaki	57	-0.080	-0.706	**-0.422**	0.180
Wellington	67	-0.266	-0.558	**-0.442**	0.077
Nelson	46	2.246	0.033	**0.684**	0.471
Lyttelton	164	4.201	-0.241	**0.848**	0.890
Dunedin	58	3.973	-0.307	**0.548**	1.222
Dud-Bluff	170	2.912	0.116	**0.786**	0.443
Bluff	91	0.693	-0.026	**0.175**	0.134

Table 2. Vertical datum offsets in bold (metres)

Next, the preliminary co-geoid model was compared with GPS-levelling heights on a datum-by-datum basis. As some of the levelled heights have not yet been unambiguously defined on each vertical datum (a work currently in progress), smaller subsets of points that were clearly in the regions of each vertical datum were used. Table 2 shows the results of the differences between the GPS-geoid and levelled heights. Though all the descriptive statistics are shown in Table 2, only the mean differences should be interpreted as the preliminary offsets (cf. Featherstone 2000). The values in Table 2 also agree reasonably well with observed height differences between adjacent vertical datums. However, the values presented in Table 2 are not always significant, with the standard deviations sometimes being greater than the computed differences.

5 Concluding Remarks and Future Work

The datum unification problem for New Zealand is to be resolved by an iterative approach that will unify the various vertical datums through a geoid model. The literature suggests that this technique will be valid in theory (e.g., Rummel and Teunissen 1988), however no practical implementation of this strategy has been attempted before (to the best of our knowledge). The preliminary gravimetric co-geoid that has been computed for New Zealand shows some vertical datum offsets, but the quality of the data and approximate techniques used to compute the co-geoid model render these as very preliminary results. Future computations will endeavour to account for these to produce both a better geoid and national vertical datum.

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