A Comparison of Gridding Techniques for Terrestrial Gravity Observations in New Zealand

M.J. Amos[1,2] W.E. Featherstone[2]

1. Land Information New Zealand, Private Box 5501, Wellington, New Zealand; Fax: +64-4-460-0112; E-mail: mamos@linz.govt.nz
2. Western Australian Centre for Geodesy, Curtin University of Technology, GPO Box U1987, Perth, WA 6845, Australia; Fax: +61-8-9266-2703; E-mail: W.Featherstone@curtin.edu.au

Abstract. The horizontal positions of the terrestrial gravity observations in New Zealand (NZ) do not have a uniform spatial distribution. As such, they do not provide a representative sample for the computation of mean gravity anomalies, particularly in mountainous areas where the majority of data are along valley floors. A common technique to account for this irregular sampling is to grid the point Bouguer anomalies, and then to use a high-resolution digital elevation model to reconstruct the free-air anomalies onto this finer grid. Thus, a regular grid of free-air anomalies that are more representative of topography is obtained. There has been some contention as to whether refined or simple Bouguer anomalies should be used in this gridding phase. For instance, in Australia, simple Bouguer anomalies are appropriate, whereas in Canada, refined Bouguer anomalies are essential. Since the topography in NZ is rougher than in Australia, this raises the question of whether terrain corrections should be applied to the gravity anomaly before or after the gridding process occurs. This paper presents a comparison of how these different gridding scenarios have performed over NZ. From comparisons with GPS-levelling data, it is concluded that the refined Bouguer anomaly is the most appropriate for gravity gridding for future gravimetric geoid computations in the region.

Keywords. Geoid/quasigeoid, terrain corrections, Bouguer gravity anomalies, gravity gridding and interpolation, digital elevation model

1 Introduction

In order to compute a new gravimetric geoid model of New Zealand (NZ) to replace/unify the 12 existing local vertical datums (e.g., Amos and Featherstone 2003), gravimetric terrain corrections (TCs) are required. As well as playing an essential role in the solution of the geodetic boundary-value problem by Stokes's method, TCs are also useful for smoothing (i.e., removing the high frequencies from) the gravity field prior to interpolation. Reducing the high frequencies as much as possible makes the gridding process less prone to aliasing.

We acknowledge that the role of TCs in geoid determination remains somewhat controversial in the geodetic literature (e.g., Wang and Rapp 1990; Martinec et al. 1993, 1996; Jekeli and Serpas 2003). However, we choose not to enter this debate here, nor shall we consider the appropriateness of the planar versus the spherical Bouguer model (cf. Vaníček et al. submitted). Instead, we only aim to investigate the role of Moritz's (1968) TC in conjunction with the planar Bouguer gravity anomaly defined at the Earth's surface.

With regard to gravity gridding, there has been some [minor] contention as to whether refined (i.e., with TCs) or simple (i.e., without TCs) planar Bouguer anomalies should be used in this process. For instance, simple planar Bouguer anomalies are sufficient in Australia (Goos et al. 2003), whereas refined Bouguer anomalies are essential in Canada (Janak and Vaníček, submitted). Given that the topography in NZ is considerably rougher than in Australia, though possibly less so than the Canadian Rocky Mountains, this raises the question of whether TCs should be applied before or after gridding the gravity anomalies in NZ. At the conceptual level, however, we expect that TCs are required in NZ, but we aim to verify this by experimentation.

Another important, though often overlooked, consideration is the inclusion of height data during the gridding process (cf. Tscherning and Forsberg 1992; Reilly 1972). This is needed because, without downward continuation, the Bouguer gravity anomalies fundamentally refer to the topography (e.g., Vaníček et al. submitted; Reilly 1972). This probably important aspect will be neglected in this study, but remains for future investigation.

This paper briefly describes the computation of NZ TCs from a 1.8" (~56m) digital elevation model

(DEM) using the planar 2D-FFT implementation of Moritz's (1968) algorithm (Schwarz et al. 1990; cf. Kirby and Featherstone 1999). These TCs are compared with the Hammer-type TCs at the gravity observation stations, computed by Reilly (1970). We then replicate the experiments of Goos et al. (2003) in the NZ context, and compare the resulting geoid models with local GPS-levelling data.

2 NZ Data and Computations

2.1 Gravity Data

The NZ gravity data used are (see Amos and Featherstone (2003) for a fuller description): 40,445 land and sea-bottom gravity observations; 2,401,932 crossover-adjusted ship-track gravity observations; and a 2' by 2' grid of marine gravity anomalies derived from satellite altimetry. The crossover adjustment of the ship-track gravity, selection of the altimeter-derived gravity anomalies, and the operational merging of these datasets is described in Amos et al. (this issue).

2.2 The DEM

The 1.8''-resolution DEM was supplied by *GeographX* (www.geographx.co.nz), which has been derived from Land Information New Zealand's (LINZ) source data. The estimated precision of this DEM is ±22m horizontally and ±10m vertically. The heights refer to local mean sea level and are mean values of the topographic height in each cell. The statistics of this DEM are given in Table 1, and the highest point in NZ (Mt Cook) is 3,754 m.

Note, however, that NZ uses 12 separate and unconnected vertical datums (e.g., Amos and Featherstone 2003). Therefore, the heights in this NZ DEM do not refer to the same vertical reference surface. The effect of this on the geoid will be studied in the future. Meanwhile, it will be assumed here that the DEM refers to a homogeneous vertical datum.

2.3 Moritzian TCs by 2D-FFT

A NZ-wide 1.8''-resolution grid of TCs (Fig. 1; Table 1) was computed from the abovementioned DEM using the planar 2D-FFT implementation of Moritz's (1968) algorithm (Schwarz et al. 1990; Fig. 2). The software and methods used are identical to those adopted by Kirby and Featherstone (1999). Since the TCs refer to the mean cell elevations in the DEM, there is a component missing due to the difference between the elevation of the gravity observation and the mean elevation of the corresponding DEM cell. There is also a component missing due to the near-gravimeter terrain effects out to one-half of the resolution of the DEM (cf. Leaman 1998). The Moritzian TCs were computed over a spherical cap radius of 50 km about each computation point (i.e., each DEM cell element), beyond which they are negligible (cf. Kirby and Featherstone 1999). This has resulted in a band-limited Moritz TC signal (i.e., ~28 m to 50 km).

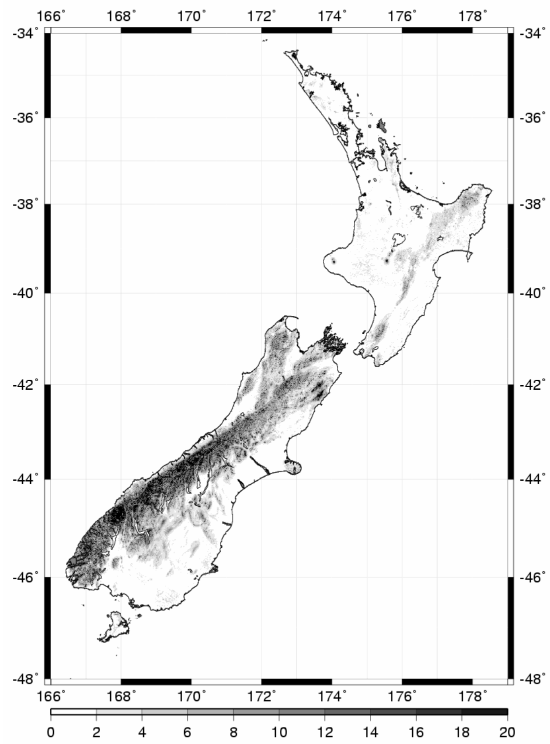

Fig 1. A generalised image of the 1.8''-resolution Moritz terrain corrections over New Zealand (mgal)

2.4 Hammer TCs

The NZ Institute of Geological and Nuclear Sciences (GNS) provided TCs with its land gravity observations (Reilly 1972). These were computed in three components (*ibid.*, Woodward 1982; Fig. 2):

- Inner- and outer-zone TCs were computed using Hammer (1939) charts and topographical maps available in the 1970s. These TCs are residual to a spherical Bouguer cap of thickness equal to the gravity observation elevation H and a radius of 21.994 km (Hammer zone M). The [variable] division between the inner and outer zones, given in Woodward and Ferry (1973), was based on the roughness of the residual topography.

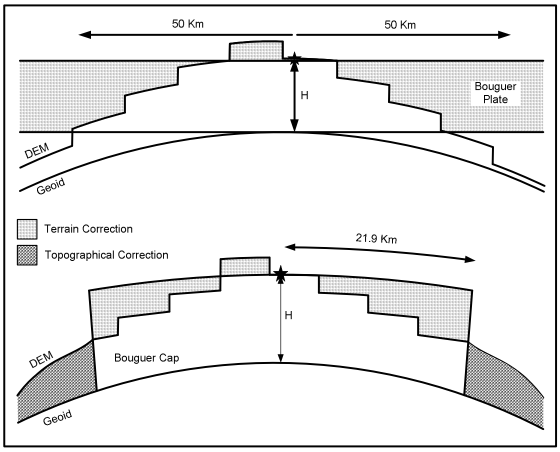

Fig 2. Schematics of the models used to compute Moritz TCs (top) and GNS TCs and topographical corrections (bottom).

The inner zone started at Hammer zone B (i.e., 2 m from the observation). The sum of the inner and outer zone TCs gives a band-limited TC signal (i.e., 2 m to 21.994 km).

- A so-called topographical correction (Reilly 1972) was computed from the mean heights in an [old] 300m-resolution DEM using a spherical Earth model out to 166.7 km (Hayford zone O). This topographical correction removes the total gravitational effect of the topography (not the topography residual to the Bouguer plate/shell) between radii of 21.994 km and 166.7 km. This topographical correction is not considered in our gridding of the Bouguer gravity anomalies.

3 Analysis of the TCs

3.1 GNS and Moritzian TCs

Both TCs are based on the same equation (Kirby and Featherstone 1999) with the Moritz TC version having several approximations applied to make it more suitable to implement with a Fourier transform. Each TC also refers to distinctly different conceptual models of the topography (Fig. 2). The Moritz TCs assume a planar Earth, while the GNS TCs use a spherical approximation. In addition they refer to different parts of the TC spectrum (i.e., ~28 m to 50 km for Moritz versus 2 m to 21.994 km for GNS). Conceptually, the GNS TCs are probably superior to the Moritzian TCs because they consider a spherical Earth model (cf. Vaníček et al. submitted) and provide more of the near-gravimeter effects. However, this must be balanced against the additional need to perform downward continuation prior to geoid computation (cf. Jekeli and Serpas 2003), whereas the downward continuation is implicit to the Mortiz TCs (under some assumptions; e.g., Martinec et al. 1993, 1996). Nevertheless, it remains instructive to compare the different TCs (Table 1; Fig. 3).

	Max	*Min*	*Mean*	*STD*
56m DEM (all NZ)	3737	0	229	390
Moritz TC (all NZ)	174.98	0.00	1.84	3.95
Moritz TC (interpolated to gravity obs)	36.20	0.00	1.72	2.89
GNS TC (at gravity obs)	49.12	0.00	2.05	3.66
Diff. GNS-Moritz (at gravity obs)	27.95	-10.71	0.33	1.35

Table 1. Statistics of the DEM (m) and TCs (mgal) over NZ [40,445 gravity observations]

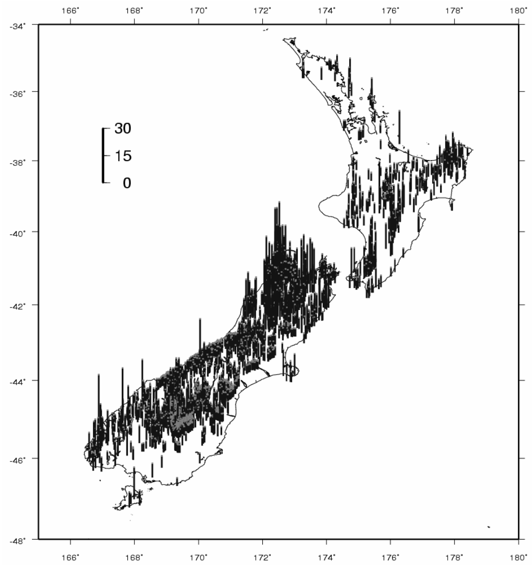

Fig 3. Magnitude of the differences greater than 2.5 mgal between Moritzian and GNS terrain corrections (mgal)

Comparing Figs. 3 and 1, the differences between TCs appear to be highly correlated with the areas of NZ where the topography is particularly rugged and steep. This correlation can be attributed to one or all of incorrect horizontal positioning of the gravity observations (thus affecting the station height), errors in the DEM, errors in the GNS TCs, or errors in the Moritzian TCs, e.g., due to the numerical instability in areas of steep slopes (see Section 3.2). The horizontal positioning errors of the gravity observations are unknown, but Woodward (pers. comm. 2003) points that the large differences in the South Island (Fig. 3) are probably due to positioning errors because the heights of the gravity observations do not agree with topographic maps.

The absolute values of the differences between the GNS and Moritz TCs were also plotted versus decreasing magnitude, which resulted in a remarkable asymptotic property (Fig. 4). At present, we are unable to give a plausible explanation for this, but it may be a by product of the different mathematical models, or the instabilities in Moritz's TC, or both. Moreover, we are unaware of this strange property being reported elsewhere.

3.2 Instabilities in the Moritzian TCs

It is well documented that numerical instabilities occur in Moritz's (1968) algorithm for steep (>45°) topographical gradients and densely sampled DEMs (e.g., Martinec et al. 1996; Tsoulis 2001; Kirby and Featherstone 2001). Given the rugged topography in NZ (Table 1), it is important to ascertain whether the steep slopes (up to 86°; Table 2) do cause spurious TC values. Since the GNS TCs refer to a different mathematical model (Fig. 2), they cannot be used to unequivocally detect spurious Moritzian TCs, but they can give some indication.

A complementary approach is to compute the topographical gradients (Table 2) and use these in conjunction with the computed Moritzian TCs and the differences with the GNS TCs to get some indication as to whether spurious values exist. However, this analysis did not show that the outlying values were a function of increasing topographic gradient. As such, further work is needed in this regard.

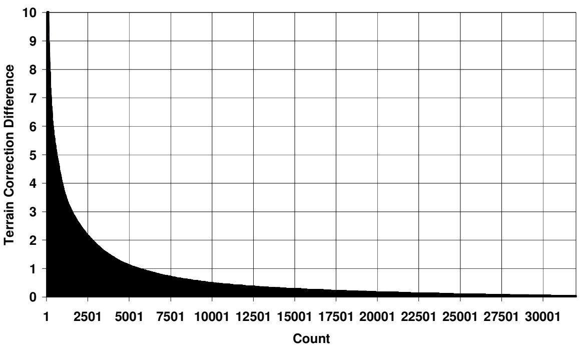

Fig 4. Descending magnitude of differences between Moritzian and GNS TCs over NZ (mgal)

DEM gradient	*Max*	*Min*	*Mean*	*STD*
East-west	86°	0°	4.8°	9.1°
North-south	85°	0°	4.4°	8.5°

Table 2. Statistics of the 56m DEM gradients (degrees)

4 Quasi-Geoid Computations

We replicated the experiments of Goos et al. (2003) to determine the role of TCs in gridding the NZ gravity data and thence the geoid. Goos et al. (2003) investigated the effect of including DEM information to reduce the aliasing of gravity observations caused by irregular or biased sampling (i.e. only making observations on hill tops or in valleys). It specifically compared the performance of gridding refined and simple Bouguer anomalies in Australia. However, they omitted that the refined or simple Bouguer anomalies are defined at the Earth's surface (cf. Vaníček et al. submitted). Accordingly, the interpolation should strictly take into account the three-dimensional distribution of the anomalies (Tscherning and Forsberg 1992; Reilly 1969). As stated, this point will not be addressed here, but will be the subject of future investigations.

The gravity gridding process used by Goos et al. (2003) is schematically shown in Fig. 5. To explain, the simple Bouguer (SB) technique involves the following stages:

(1) Compute SB anomalies at the observation points from the observed gravity data;
(2) Interpolate the point SB anomalies onto a regular 2' grid using the GMT tensioned spline algorithm (Smith and Wessel 1990);
(3) Reconstruct free-air anomalies (FAA) from the SB grid using the 1.8" DEM by applying a 'reverse' Bouguer plate correction (Featherstone and Kirby 2000);
(4) Convert the 1.8" FAAs to Faye gravity anomalies by applying the 1.8" Moritz TCs;
(5) Interpolate the 1.8" Faye anomaly grid to a 2' grid (by tensioned spline).

The refined Bouguer (RB) technique is largely similar to the SB procedure, as follows:

(1) Compute RB anomalies at the observation points using the observed gravity data and (a) the GNS Hammer TCs (HRB) and (b) the 1.8" Moritz TCs (MRB) that were bi-cubically interpolated to the observation points;
(2) Interpolate the point HRB and MRB anomalies onto regular 2' grids (as above);
(3) Reconstruct Faye anomalies from the 1.8" RB grids (as above)

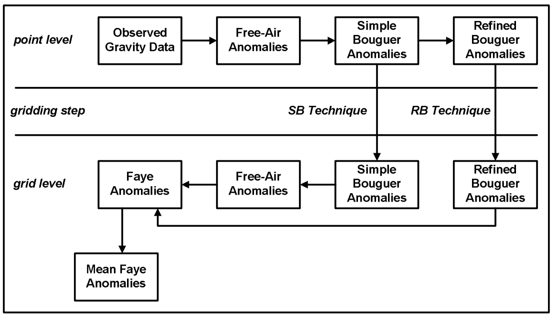

Fig 5. Flowchart of the techniques tested to compute grids of mean Faye anomalies (from Goos et al. 2003)

(4) Interpolate the Faye anomaly grids to 2' grids (by tensioned spline)

The terrestrial FAA grids were then augmented with a 2' by 2' grid of KMS02 (Andersen et al. 2004) satellite altimetry and crossover adjusted ship-track anomalies (Amos et al. this issue) in the marine areas around NZ.

The GGM01S (Tapley et al 2004) and EGM96 (Lemoine et al. 1998) global geopotential models were combined (up to degree and order 90, and 91 to 360, respectively) and used to remove the low-frequency gravity anomalies from the above augmented anomaly grids. The residual anomaly grids were then subjected to a 1D-FFT gravity-to-geoid transformation with the unmodified spherical Stokes kernel (1° integration cap radius) to evaluate the residual co-geoid. This was then restored with the GGM01S/EGM96 quasigeoid, and the primary indirect effect (computed from the 1.8" DEM and averaged onto a 2' grid) applied to give three quasigeoid solutions. These are:

- MSB: quasigeoid model from gridded SB anomalies with Moritz TCs applied after the gridding (cf. Kirby and Featherstone 2001);
- HRB: quasigeoid model from gridded RB anomalies with GNS's Hammer TCs applied before the gridding;
- MRB: quasigeoid model from gridded RB anomalies with Moritz's TCs applied before the gridding (Fig. 6).

The quasigeoids were then compared with a nation-wide set of 1371 GPS-levelling points (Table 3). It is clear from Table 3 that the RB quasigeoids give a better fit to the GPS-levelling points than the MSB quasigeoid. The MRB solution is slightly better than the HRB solution, but the issues surrounding the different TCs in Section 3 (particularly the Moritzian TCs) make it difficult to make a conclusive decision in this regard. As mentioned earlier, the levelled heights are based on 12 different vertical datums, which will bias the differences computed. When the comparisons in Table 3 are repeated on a datum-by-datum basis, the above findings are confirmed (albeit with lower standard deviations). Therefore, it can be concluded that refined Bouguer anomalies should be used for gravity gridding prior to future geoid computations in NZ.

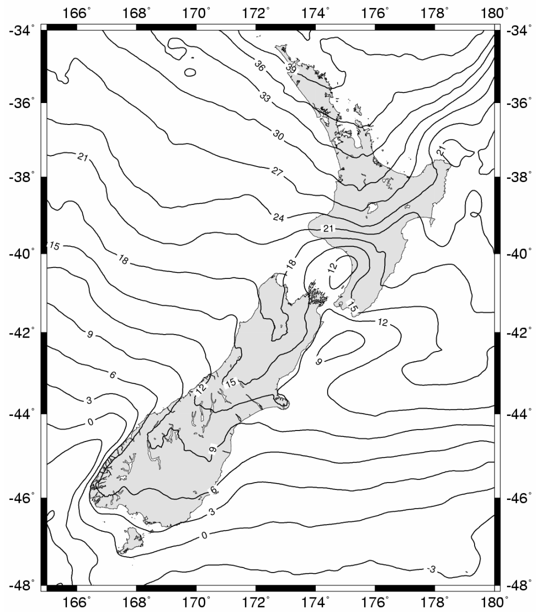

Fig 6. Refined Bouguer (Moritz TC) quasigeoid (metres)

	Max	*Min*	*Mean*	*STD*
GGM01S/EGM96	2.959	-1.414	-0.389	0.547
MSB quasigeoid	3.210	-0.666	0.038	0.461
HRB quasigeoid	1.870	-0.937	-0.329	0.260
MRB quasigeoid	1.938	-1.012	-0.343	0.227

Table 3. Comparisons of geoid solutions with 1371 GPS-levelling points (metres)

5 Conclusions and Future Work

This paper has presented a comparison of different gridding techniques for terrestrial gravity observations in NZ. One method is to grid SB anomalies, and the other to grid RB anomalies.

The Moritz and Hammer TC computation methods were also compared. Noting that they refer to conceptually different topographic models and different parts of the TC spectrum, the Hammer TCs are probably superior because they assume a spherical Earth and provide more near-gravimeter effects. The Moritz TCs benefit from the fact that (under some assumptions) downward continuation is implicit so does not need to be performed. It was speculated that, due to the steep topographic gradients (up to 86°) in NZ, the Moritz TCs may be unreliable in certain locations (particularly in the Southern Alps). This possible unreliability was also indicated by the differences between the two TCs. Future work will recompute TCs over NZ with prism integration (as opposed to the planar 2D-FFT).

Quasigeoid models were computed using 'reconstructed' MSB, HRB and MRB Faye gravity anomalies. A comparison with 1371 GPS-levelling points showed that, while the MSB quasigeoid was better than the GGM01S/EGM96 GGM, the RB models were better still; the difference between the MRB and HRB quasigeoids was less clear.

Overall, it is concluded that the refined Bouguer gravity anomaly should be used in future geoid computations, which concurs with our original expectation and the finding of Janak and Vaníček (submitted).

Acknowledgements: This research is funded by LINZ, a Curtin Univ Postgraduate Scholarship, and as part of ARC grant DP0211827. We are greatly indebted to the suppliers of data. We would like to thank Derek Woodward of GNS for providing information on the GNS TCs, as well as other discussions on the Bouguer gravity anomaly. Finally, we would like to thank the reviewers for their time taken to consider this manuscript.

References

Amos MJ, Featherstone WE (2003) Preparations for a new gravimetric geoid model of New Zealand, and some preliminary results, NZ Surveyor 293: 9-20

Amos MJ, Feathersone WE, Brett J (this issue) Crossover adjustment of New Zealand marine gravity data, and comparisons with satellite altimetry and global geopotential models, Proceedings GGSM 2004, Porto

Andersen OB, Knudsen P, Trimmer R (2004) Improving high-resolution altimetric gravity field mapping (KMS02). Proc IUGG General Assembly, Sapporo, Japan 2003, Springer, Berlin, in press

Featherstone WE, Kirby JF (2000) The reduction of aliasing in gravity anomalies and geoid heights using digital terrain data, Geophys J Int 141: 202-212

Goos JM, Featherstone WE, Kirby JF, Holmes SA (2003) Experiments with two different approaches to gridding terrestrial gravity anomalies and their effect on regional geoid computation, Surv Rev 37(288): 92-112

Hammer S (1939) Terrain corrections for gravimeter stations, Geophys 4: 184-194

Janak J, Vaníček P (submitted) Mean free-air gravity anomalies in the mountains, Stud Geophys e Geod

Jekeli C, Serpas JG (2003) Review and numerical assessment of the direct topographical reduction in geoid determination. J Geod 77: 226-239

Kirby JF, Featherstone WE (1999) Terrain correcting the Australian gravity database using the national digital elevation model and the fast Fourier transform, Aust J Earth Sci 46: 555–562

Kirby J, Featherstone WE (2001) Anomalously large gradients in the "GEODATA 9 Second" digital elevation model of Australia, and their effects on gravimetric terrain corrections, Cartography 30: 1-10

Leaman DE (1998) The gravity terrain correction - practical considerations. Explor Geophys 29: 476–471

Lemoine FG, Kenyon SC, Factor RG, Trimmer RG, Pavlis NK, Chinn DS, Cox CM, Klosko SM, Luthcke SB, Torrence MH, Wang YM, Williamson RG, Pavlis EC, Rapp RH, Olson TR (1998) The development of the joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) geopotential model EGM96, NASA Tech Rep NASA/TP-1998-206861, Goddard Space Flight Center, Greenbelt

Martinec Z, Matyska C, Grafarend EW, Vaníček P (1993) On Helmert's second condensation reduction, manuscr geod 18: 417-421

Martinec Z, Vaníček P, Mainville A, Véronneau M (1996) Evaluation of topographical effects in precise geoid computation from densely sampled heights, J Geod 70: 746-754

Moritz H (1968) On the use of the terrain correction in solving Molodensky's problem, Rep 108, Dept Geod Sci & Surv, Ohio State Univ, Columbus

Reilly WI (1969) Interpolation, smoothing and differentiation of gravity anomalies, N Z J Geol & Geophys 12: 609-627

Reilly WI (1970) Topographic-isostatic gravity corrections for New Zealand, Geophysical Memoir 7, Dept of Scientific & Industrial Research, Wellington

Reilly WI (1972) New Zealand gravity map series, NZ J Geol & Geophys 15: 3-15

Schwarz K-P, Sideris MG, Forsberg R (1990). The use of FFT techniques in physical geodesy, Geophys J Int 100: 485-514

Smith WHF, Wessel P (1990) Gridding with continuous curvature splines in tension, Geophysics 55: 293-305

Tapley BD, Bettadpur S, Watkins M, Reigber Ch (2004) The gravity recovery and climate experiment: mission overview and early results. Geophys Res Lett 31, L09607: doi10.1029/2004GL019920

Tscherning CC, Forsberg R (1992) Harmonic continuation and gridding effects on geoid height prediction, Bull Géod 66: 41-53

Tsoulis D (2001) Terrain correction computations for a densely sampled DTM in the Bavarian Alps, J Geod 75: 291-307

Vaníček P, Tenzer R, Sjöberg LE, Martinec Z, Featherstone WE (submitted) New views of the spherical Bouguer gravity anomaly, Geophys J Int

Wang YM, Rapp RH (1990) Terrain effects on geoid undulation computations, manuscr geod 15: 23-29

Woodward, DJ, Ferry LM (1973). Gravity terrain correction tables, Wellington, Department of Scientific and Industrial Research**:** 27

Woodward DJ (1982) Reduction of gravity observations on the VAX11-780 computer, Wellington, Department of Scientific and Industrial Research**:** 12