NEW ZEALAND MAP GRID

Department of Lands and Survey
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The following document is a scanned image of the Department of Land and Survey Technical Circular 1973/32 that describes the technical details of the New Zealand Map Grid.

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1. New projection for topographic maps. The present series of topographic maps with a yard grid will eventually be replaced by a new series with a metre grid. The North Island and the South Island will be mapped on one projection with one grid, to be known as the New Zealand Map Grid (NZMG).

The projection adopted was derived from mathematical analysis by Mr W. I. Reilly, Geophysics Division, Department of Scientific and Industrial Research, to give a small range of scale variation over the land area of New Zealand. This has been achieved at the expense of abandoning the orderly arrangement of scale curves. Such a projection has no recognized name, and it is simply called the New Zealand Map Grid projection.

2. Nature of the projection. The projection is conformal, but is otherwise unlike any other projection used for detailed mapping. The pattern of scale curves is shown in Fig. 1. The range of scale enlargement is from +0·023 % to -0·022 %, considerably less than that of any other projection previously used for New Zealand. Fig. 1 shows also the curves of constant convergence.

3. New Zealand Map Grid coordinates. With the projection of Fig. 1 as a basis, a system of coordinates in metres has been established, with the following characteristics:

(1) The true origin of the projection is placed at latitude 41° south, this being the whole degree nearest to the middle latitude of the country.

(2) The true origin is placed at longitude 173° east, this being the whole degree nearest to the meridian to which equal perpendiculars can be drawn from the easternmost and westernmost points of the country.

(3) The meridian of longitude 173° east, which is not represented by a straight line, is oriented so that its tangent at the origin is the north-south axis of coordinates.

(4) The true origin is assigned arbitrary coordinates sufficiently large to render all coordinates positive, or east and north of a so-called "false origin".

(5) In a metre coordinate system sufficiently extensive to cover the whole country, the northing must at some stage reach seven integral figures. In the scheme now adopted, the coordinates have seven integral figures in all cases.

(6) The easting is always less than 5 000 000 metres, the northing always greater then 5 000 000 metres, so that no confusion between easting and northing can arise whichever one is stated first.

(7) The coordinates assigned to the true origin are also sufficiently large for the grid to be extended a considerable distance out to sea without departing from the characteristics stated in (5) and (6).

(8) Thus, the true origin is placed at latitude 41° south, longitude 173° east, and the coordinates of this point are 2 510 000 metres east, 6 023 150 metres north.

The land area of New Zealand is thus fitted into a rectangle 1 000 000 metres wide by 1 500 000 metres from north to south. Coordinate values have been allotted to the nominated origin so that the easting ranges from 2 000 000 m to 3 000 000 m (East Island is just east of the 3 000 000 m line), and the northing ranges from 5 300 000 m to 6 800 000 m, with the 6 000 000 m line at about the middle of the country. The seemingly strange values assigned to the origin have been chosen also to fit into a scheme of map sheets at a scale of 1:50 000 with boundaries at 10 000 m values.

The New Zealand Map Grid scheme of coordinates is shown in Fig. 2.

4. Computation of isometric latitude. If $\Delta\phi$ is the difference of geodetic latitude in seconds from latitude 41°, positive northward, then the corresponding difference of isometric latitude $\Delta\psi$ is given by a Maclaurin series with coefficients (for the International Spheroid) as follows.

$$\begin{aligned}
\Delta\psi = {} & 0{\cdot}63991\ 75073\ (\Delta\phi \times 10^{-5}) \\
& - 0{\cdot}13587\ 97613\ (\Delta\phi \times 10^{-5})^2 \\
& + 0{\cdot}06329\ 4409\ (\Delta\phi \times 10^{-5})^3 \\
& - 0{\cdot}02526\ 853\ (\Delta\phi \times 10^{-5})^4 \\
& + 0{\cdot}01178\ 79\ (\Delta\phi \times 10^{-5})^5 \\
& - 0{\cdot}00551\ 61\ (\Delta\phi \times 10^{-5})^6 \\
& + 0{\cdot}00269\ 06\ (\Delta\phi \times 10^{-5})^7 \\
& - 0{\cdot}00133\ 3\ (\Delta\phi \times 10^{-5})^8 \\
& + 0{\cdot}00067\ (\Delta\phi \times 10^{-5})^9 \\
& - 0{\cdot}00034\ (\Delta\phi \times 10^{-5})^{10}
\end{aligned}$$

$$\begin{aligned}
\Delta\phi \times 10^{-5} = {} & 1{\cdot}56270\ 14243\Delta\psi \\
& + 0{\cdot}51854\ 06398\ \Delta\psi^2 \\
& - 0{\cdot}03333\ 098\ \Delta\psi^3 \\
& - 0{\cdot}10529\ 06\ \Delta\psi^4 \\
& - 0{\cdot}03685\ 94\ \Delta\psi^5 \\
& + 0{\cdot}00731\ 7\ \Delta\psi^6 \\
& + 0{\cdot}01220\ \Delta\psi^7 \\
& + 0{\cdot}00394\ \Delta\psi^8 \\
& - 0{\cdot}0013\ \Delta\psi^9
\end{aligned}$$

The coefficients for the inverse series are also shown above. These series give 10-figure accuracy for a range of ±7°, i.e. from 34° to 48° latitude.

5. Series defining projection. A conformal projection of the spheroid is given by a series in powers of the complex variable ζ, where

$$\zeta = \Delta\psi + i\Delta\lambda,$$

$\Delta\psi$ being the difference of isometric latitude obtained from the series in 4, and $\Delta\lambda$ being the difference of longitude in radians from 173°, positive eastward. That is, coordinates from the true origin are given by

$$z = a(B_1\zeta + B_2\zeta^2 + B_3\zeta^3 + \cdots).$$

For the New Zealand Map Grid, the series is terminated after the sixth term, and constants are added to the coordinates, which are then given by

$$N + iE = N_0 + iE_0 + z.$$

For the International Spheroid, the major semiaxis is given by

$$a = 6\ 378\ 388\ \mathrm{m},$$

and the coefficients of the series defining the New Zealand Map Grid are as follows, all except B_1 being complex numbers.

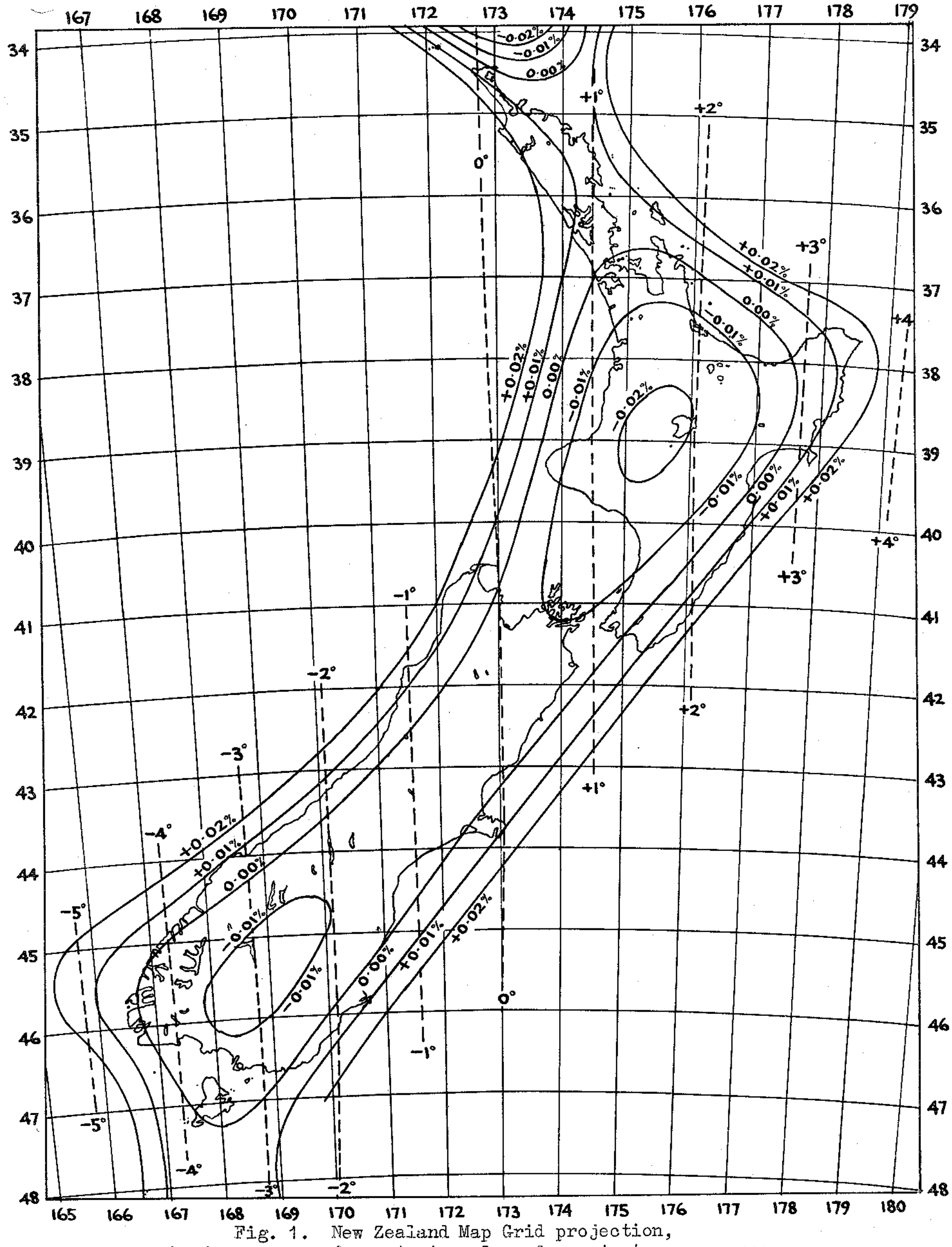

Fig. 1. New Zealand Map Grid projection, showing curves of constant scale and constant convergence.

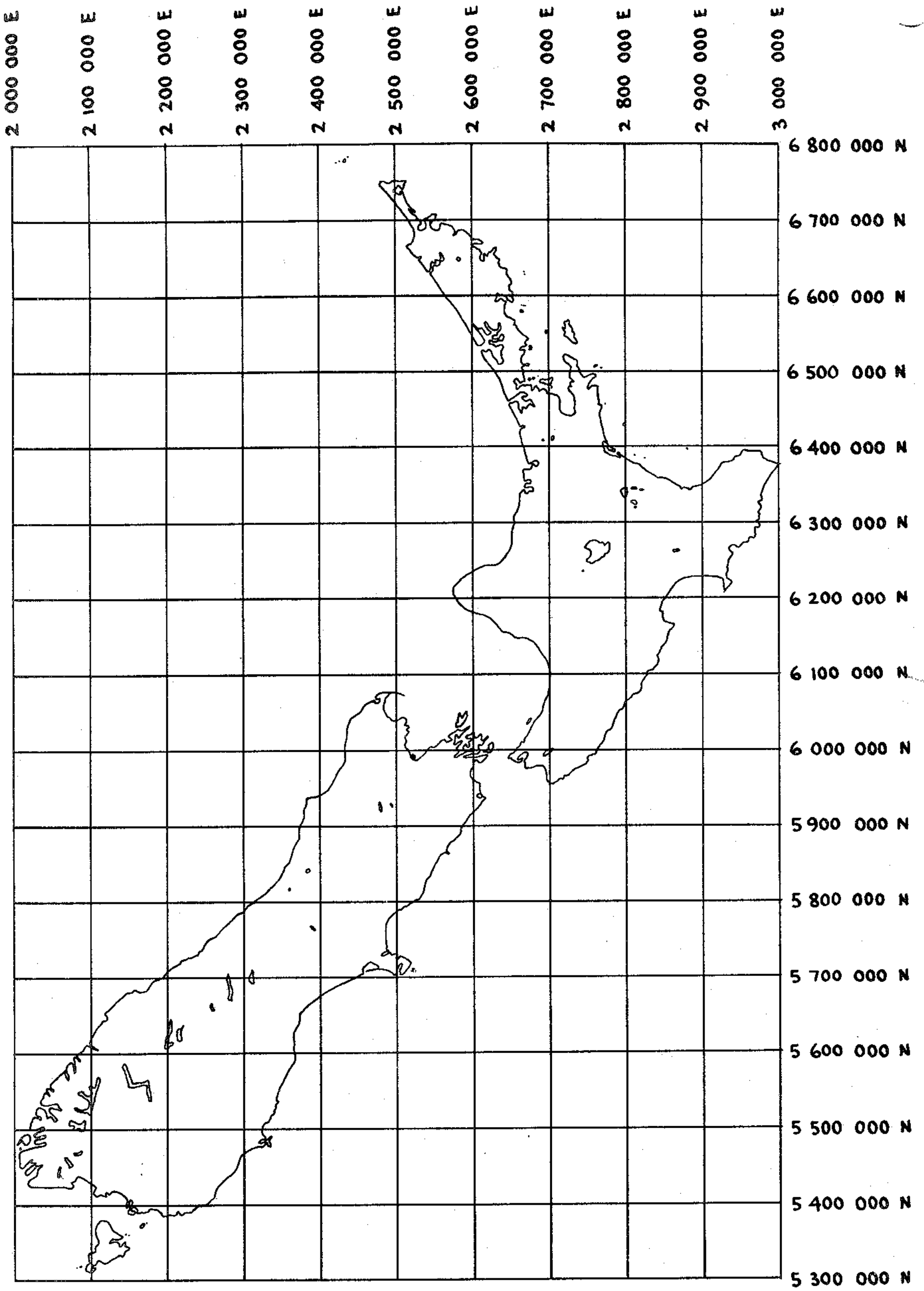

Fig. 2. The New Zealand Map Grid.

		i
$N_0 + iE_0 =$	6 023 150	+ 2 510 000
$B_1 =$	0·75578 53228	
$B_2 =$	+ 0·24920 4646	+ 0·00337 1507
$B_3 =$	− 0·00154 1739	+ 0·04105 8560
$B_4 =$	− 0·10162 907	+ 0·01727 609
$B_5 =$	− 0·26623 489	− 0·36249 218
$B_6 =$	− 0·68709 83	− 1·16519 67

6. Inverse computation. If

$$z = \frac{N - N_0 + i(E - E_0)}{a},$$

then

$$\zeta = b_1 z + b_2 z^2 + b_3 z^3 + \cdots.$$

Although the direct formula is a six-term polynomial, the inverse formula is an infinite series, of which the coefficients of the first six terms are

		i
$b_1 =$	1·32312 70439	
$b_2 =$	− 0·57724 5789	− 0·00780 9598
$b_3 =$	+ 0·50830 7513	− 0·11220 8952
$b_4 =$	− 0·15094 762	+ 0·18200 602
$b_5 =$	+ 1·01418 179	+ 1·64497 696
$b_6 =$	+ 1·96605 49	+ 2·51276 45

The value of ζ given by six terms of the inverse series is a first approximation from which a closer approximation can be obtained by

$$\zeta = \frac{z + B_2\zeta^2 + 2B_3\zeta^3 + 3B_4\zeta^4 + 4B_5\zeta^5 + 5B_6\zeta^6}{B_1 + 2B_2\zeta + 3B_3\zeta^2 + 4B_4\zeta^3 + 5B_5\zeta^4 + 6B_6\zeta^5}.$$

A second application of this formula gives sufficient accuracy at any point within the land area of New Zealand.

When the final value of ζ is obtained, the latitude can be derived from the inverse series given in 4 above.

7. Scale coefficient and convergence. By differentiation of the direct series, we have

$$\mathrm{d}z/\mathrm{d}\zeta = a(B_1 + 2B_2\zeta + 3B_3\zeta^2 + \cdots),$$

whence the scale coefficient is given by

$$m = \frac{a}{\nu\cos\phi}\,\frac{|\mathrm{d}z/\mathrm{d}\zeta|}{a},$$

where $|\mathrm{d}z/\mathrm{d}\zeta|$ denotes the square root of the sum of the squares of the numerical coefficients of the real and the imaginary parts of $\mathrm{d}z/\mathrm{d}\zeta$, i.e.

$$|dz/d\zeta| = \sqrt{(R^2 + I^2)}.$$

The convergence is given by

$$\tan\gamma = I/R.$$

The latitude factor of the scale can be computed from a Maclaurin series with the following coefficients:

$$\begin{aligned}\frac{a}{\nu\cos\phi} = \; & 1{\cdot}32309\ 46238 \\ & - 0{\cdot}86802\ 81742\ \Delta\psi \\ & + 0{\cdot}66299\ 99306\ \Delta\psi^2 \\ & - 0{\cdot}14371\ 346\ \Delta\psi^3 \\ & + 0{\cdot}05551\ 665\ \Delta\psi^4 \\ & - 0{\cdot}00729\ 966\ \Delta\psi^5 \\ & + 0{\cdot}00170\ 8\ \Delta\psi^6 \\ & - 0{\cdot}00021\ \Delta\psi^7\end{aligned}$$

There is no simple formula for computing scale or convergence from NZ Map Grid coordinates. It is necessary to compute the latitude and longitude first, and then compute scale and convergence from latitude and longitude.

8. Conversion of National Grid coordinates to N.Z. Map Grid coordinates. By using the series defining the transverse Mercator projection, incorporating the difference of origin between the National Grid and the N.Z. Map Grid, and substituting the resulting series in the polynomial defining the N.Z. Map Grid, a series can be derived whereby National Grid coordinates may be converted to N.Z. Map Grid coordinates.

Thus, if N_1, E_1 are the National Grid coordinates in yards, then for the North Island,

$$z_1 = \frac{(N_1 - 400\,000) + i(E_1 - 300\,000)}{697\,550{\cdot}2032},$$

and for the South Island,

$$z_1 = \frac{(N_1 - 500\,000) + i(E_1 - 500\,000)}{697\,550{\cdot}2032}.$$

The divisor is one-tenth of the major semiaxis in yards.

If coordinates in links from the true origin are used, then for both islands,

$$z_1 = \frac{x_1 + iy_1}{3\,170\,682{\cdot}742}.$$

N.Z. Map Grid coordinates of points on the North Island National Grid are then given by

$$\begin{array}{rlrl} & & i & \\ N + iE = & 6\,242\,099{\cdot}8520 & +\ 2\,726\,597{\cdot}6378 & \\ & +\ (+637\,445{\cdot}6192 & +18\,033{\cdot}6645) & z_1 \\ & +\ (-31{\cdot}2986 & -20{\cdot}6887) & z_1^2 \\ & +\ (+338{\cdot}5897 & +237{\cdot}3562) & z_1^3 \end{array}$$

Continued on next page

$$
\begin{array}{rr}
+\ (+26\cdot0948 & -315\cdot9783)\ z_1^4 \\
+\ (-35\cdot0361 & -141\cdot7149)\ z_1^5 \\
+\ (-12\cdot8414 & -2\cdot3307)\ z_1^6 \\
+\ (+3\cdot5982 & +2\cdot2930)\ z_1^7 \\
+\ (-0\cdot7649 & -0\cdot7083)\ z_1^8 \\
+\ (+0\cdot1281 & +0\cdot1356)\ z_1^9 \\
+\ (-0\cdot0199 & -0\cdot0225)\ z_1^{10} \\
+\ (+0\cdot0029 & +0\cdot0034)\ z_1^{11} \\
+\ (-0\cdot0004 & -0\cdot0005)\ z_1^{12},
\end{array}
$$

and N.Z. Map Grid coordinates of points on the South Island National Grid are given by

$$
\begin{array}{rrr}
 & & i \\
N+iE = & 5\,688\,827\cdot3670 & +\ 2\,389\,710\cdot0400 \\
 & +\ (+637\,699\cdot3640 & -11\,434\cdot9669)\ z \\
 & +\ (-93\cdot2734 & -148\cdot7144)\ z_1^2 \\
 & +\ (+511\cdot8874 & +546\cdot7385)\ z_1^3 \\
 & +\ (-51\cdot1870 & +77\cdot0733)\ z_1^4 \\
 & +\ (-39\cdot4372 & +60\cdot2889)\ z_1^5 \\
 & +\ (-20\cdot9858 & -68\cdot4688)\ z_1^6 \\
 & +\ (+6\cdot9582 & +18\cdot5013)\ z_1^7 \\
 & +\ (-1\cdot6305 & -3\cdot9043)\ z_1^8 \\
 & +\ (+0\cdot3063 & +0\cdot7042)\ z_1^9 \\
 & +\ (-0\cdot0530 & -0\cdot1164)\ z_1^{10} \\
 & +\ (+0\cdot0085 & +0\cdot0181)\ z_1^{11} \\
 & +\ (-0\cdot0013 & -0\cdot0027)\ z_1^{12}.
\end{array}
$$

The coefficients above will give an accuracy of 0·001 m in the transformed coordinates. As the maximum value of $|z|$ is nearly 1, it is an easy matter to decide which coefficients can be omitted when a lower degree of accuracy is required.

9. Double-entry tables for conversion of coordinates from other systems to the N.Z. Map Grid will be provided in due course, as the computations described above are laborious without an electronic computer. Scale and convergence may also be computed by means of double-entry tables.

Corrections to observed directions and to measured lengths, required in the adjustment of control surveys, can be computed from complicated formulae or derived with the help of graphs. It is unlikely, however, that the projection will be used for the adjustment of control surveys, as it will be easier to work directly on the spheroid.

I.F. Stirling
Surveyor-General

per: