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Preparations for a new gravimetric geoid model of New Zealand, and some preliminary results

ABSTRACT

The rationale is given for a new gravimetric model of the New Zealand geoid to support a new vertical geodetic datum based on entirely different principles to the conventional use of tide gauges and geodetic levelling. The geoid model is currently being computed by Land Information New Zealand (LINZ) in close collaboration with the Western Australian Centre for Geodesy. The data to be used in this new geoid model comprise >40,000 land and >1.3M ship-track gravity data points, a 56 m-resolution digital elevation model (DEM), a 2' × 2' grid of marine gravity anomalies derived from multi-mission satellite altimetry, and a hybrid global geopotential model derived from EGM96 and the new EIGEN-2 model. A preliminary geoid model has been computed from these data using spectral techniques with modified kernels, and comparisons with existing GPS-levelling data on the 13 different vertical datums used in New Zealand indicate an overall precision of ~ 35 cm, which can be improved with the more sophisticated data pre-processing currently underway. This preliminary geoid model has been used to estimate preliminary offsets among the 13 different vertical datums used throughout New Zealand. Importantly, the standard deviations are less than the computed offsets, which indicates that statistically significant offsets can be computed with the proposed approaches.

1. INTRODUCTION AND BACKGROUND

1.1 A Geoid for New Zealand

New Zealand does not currently have a national geoid. A discussion of the various options available is provided in, for example Pearse (2001) and Blick et al. (2001). There are two logical options for geoid determination, the GPS-levelling or gravimetric approaches. Given the poor spatial coverage of precise levelling and the fact that interpolation would be necessary in mountainous areas (where the geoid is more variable) the GPS-levelling geoid is not a preferred option. The gravimetric option utilises the better distribution of terrestrial gravity observations and a global geopotential model. This is the preferred method of geoid computation in New Zealand.

1.2 The Gravimetric Geoid

The geoid is essentially the equipotential surface of the Earth's gravity field that corresponds most closely with mean sea level (MSL) in the open oceans ignoring the effects of quasi-stationary sea surface topography. The primary practical application of the geoid height (N) in land surveying is to transform GPS-derived ellipsoidal heights (h) to orthometric heights

(H) using the simple *algebraic* relation $H = h - N$. This use of a geoid model in conjunction with GPS provides a very attractive alternative to geodetic spirit levelling, especially over long distances and throughout the steep terrain encountered in many parts of New Zealand.

A geoid model for this vertical coordinate transformation can be computed using the gravimetric method, provided that spatially dense and accurate gravity and terrain data are available. A modern gravimetric geoid uses a combination of three primary input data sources:

- a global geopotential model, which provides most of the long and intermediate wavelength (>100 km) geoid undulations;
- terrestrial gravity data (from land- and ship-based observations, or derived from satellite radar altimetry in open marine areas) in and surrounding the area of interest, which supply most of the intermediate wavelengths (>10-20 km), and;
- a high-resolution digital elevation model (DEM), which supplies most of the short wavelengths (>50 m), and is also required to satisfy theoretical demands of geoid computation from the geodetic boundary-value problem.

The pre-processing of these data is critically important, because if errors remain in any of these input data, they will directly propagate into the regional geoid model.

Unlike many other countries, New Zealand does not currently have a regional geoid model to support geodetic operations, including the transformation of GPS-derived heights (e.g., Reilly, 1990). Gilliland (1990) computed the first New Zealand gravimetric co-geoid, but this model is no longer available for use. Nevertheless, advances in theory and data availability would now render this model obsolete. Therefore, there is a need to compute a new geoid model for New Zealand. Amos and Featherstone (2003a) computed a very preliminary co-geoid model, but it has since been found that some incorrectly mapped and thus pre-processed gravity data had been used. In addition, the above two co-geoid models omit the primary indirect effect term, which may be greater than ~0.5 m in magnitude at the summit of Aoraki/Mount Cook, the highest mountain in New Zealand (~3754 metres above local MSL).

1.3 The Vertical Geodetic Datum

Probably the largest challenge to high-precision gravimetric geoid computation in New Zealand is its use of 13 separate vertical geodetic datums based on local MSL. Each levelling network (cf. Gilliland, 1987) is based on MSL observed at a different tide gauge, often over a very short time period (less than two weeks in some cases!). As well as not averaging out long-period tidal effects, these tide gauges are subject to sea surface topography, which is notoriously difficult to quantify and model in the coastal zone (e.g., Hipkin, 2000) or in harbours and estuaries where most of the tide gauges are located. Other oceanographic phenomena, such as storm surges or the outflow of fresh water, also act to bias the tide-gauge-measured MSL from the classical geoid. Accordingly, the 13 vertical datums in New Zealand are not unified and may be offset from one another by more than 0.23 metres (Pearse, 1998).

These different vertical datums also introduce two primary problems to practical geoid determination. First, the regional gravity and terrain data used to compute the geoid model refer to different reference surfaces. This causes long- and medium-wavelength errors in the computed gravity anomalies (cf. Heck, 1990), which then propagate into the gravimetric geoid model. Secondly, a single geoid model will not be suited for the direct transformation of GPS heights to these local vertical datums. Therefore, LINZ has proposed a new strategy for the New Zealand vertical datum. It will be based on a combination of ellipsoidal heights (in the three-dimensional NZGD2000) and a precise regional geoid model. This geoid model will then allow the existing vertical datums to be unified (cf. Kumar and Burke, 1998), but first it is necessary to contend with the above-mentioned practical and theoretical difficulties.

2. DATA USED FOR REGIONAL GEOID COMPUTATION

This section will describe the peculiarities of the New Zealand data and what pre-processing has been done, or is proposed, before computing the regional geoid model.

2.1 Global Geopotential Models

A global geopotential model (GGM) comprises a set of spherical harmonic coefficients that describe the long-wavelength characteristics of the Earth's gravity field. These are computed from the analysis of artificial Earth-satellite orbits (satellite-only GGMs), and higher resolution combined GGMs also include terrestrial gravity, terrain and satellite altimetry data. Notable examples of combined GGMs are OSU91A (Rapp *et al.*, 1991) and EGM96 (Lemoine et al., 1998), which are often used as the default geoid model in commercial GPS data processing and network adjustment packages. The differences between the OSU91A and EGM96 geoid models can be up to 2.5 metres over New Zealand. This means that some significant errors can be introduced if these models are mixed during the reduction of GPS and other survey data.

The geoid height is computed from a GGM using

$$N_M = \frac{GM}{r\gamma}\sum_{n=2}^{M}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\delta\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta) \tag{1}$$

and the gravity anomaly is

$$\Delta g_M = \frac{GM}{r^2}\sum_{n=2}^{M}\left(\frac{a}{r}\right)^n (n-1)\sum_{m=0}^{n}\left(\delta\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta) \tag{2}$$

where GM is the product of the Newtonian gravitational constant and mass of the Earth (assumed equal to that of the geocentric reference ellipsoid); γ is normal gravity on the surface of the reference ellipsoid; (r, θ, λ) are the geocentric spherical polar coordinates of the computation point; a is the semi-major axis length of the geocentric reference ellipsoid, $\overline{P}_{nm}(\cos\theta)$ are the fully normalised associated Legendre functions for degree n and order m; $\delta\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the fully normalised spherical harmonic coefficients of the GGM, reduced for the even zonal harmonics of the geocentric reference ellipsoid; and, M is the maximum degree of spherical harmonic expansion.

Amos and Featherstone (2003b, in press) evaluate the fit of recent GGMs to the New Zealand gravity field, which includes free-air gravity anomalies on land, GPS-levelling data, and vertical deflections. If the gravity field implied by a GGM is a close fit to these local gravity field parameters, it is then reasonable to expect that it is suitable as the basis for a regional gravimetric geoid model. Table 1 summarises their results for the EIGEN-2 satellite-only GGM (Reigber et al., 2002 submitted), OSU91A, EGM96, and a hybrid of EIGEN-2 and EGM96 where degrees 2-32 (inclusive) of EIGEN-2 are used to replace the corresponding low degrees of EGM96. EIGEN-2 is unique in that it uses data derived purely from the CHAMP dedicated satellite gravimetry mission, whose mission parameters and concepts are described in, for example, Rummel *et al.* (2002). On the other hand, OSU91A and EGM96 use pre-CHAMP ground-based satellite tracking data, terrestrial gravity observations and satellite altimetry.

model	*degree*	*max*	*min*	*mean*	*std*
raw data	N/A	195.785	-163.178	15.921	43.210
EIGEN-2	120	193.288	-182.501	-1.305	41.695
OSU91A	360	193.834	-175.097	-1.509	40.417
EGM96	360	192.494	-176.004	-1.770	40.438
EIGEN/EGM9632/360		192.369	-176.047	-1.847	40.457

Table 1a. Fit of GGMs to 40737 land gravity observations over New Zealand (mGal)

model	*degree*	*max*	*min*	*mean*	*std*
raw data	N/A	3.583	39.410	16.235	10.817
EIGEN-2	120	3.359	-7.595	0.647	2.259
OSU91A	360	4.211	-0.933	0.213	0.733
EGM96	360	3.712	-1.338	0.027	0.616
EIGEN/EGM9632/360		3.496	-1.376	-0.039	0.606

Table 1b. Fit of GGMs to 1055 GPS-levelling observations over New Zealand (metres)

From Table 1, it is difficult to unequivocally ascertain the best degree-360 GGM simply from the statistical fit to the local gravity field data, principally due to the error budget of the latter. A crude upper estimate of the error of the GPS-levelling is ~10 cm, the terrestrial free-air gravity anomaly is ~1-10 mGal (cf. Reilly, 1972), and the vertical deflections is ~2". Therefore, the high-degree GGMs are statistically insignificantly different from one another when using the New Zealand 'control' data. More importantly, long- and medium-wavelength errors in these terrestrial data may obscure the selection of the best GGM. Therefore, other considerations must be used in parallel. The argument in favour of the hybrid EIGEN/EGM model is that EIGEN-2 uses high-quality dedicated satellite gravity data, whereas EGM96 uses probably the best coverage of terrestrial gravity data. Therefore, the hybrid GGM probably represents the best-available long-wavelength (EIGEN-2) and medium-wavelength (EGM96) GGM data.

2.2 *Terrestrial Gravity Data*

2.2.1 Land Gravity Data

The Institute for Geological and Nuclear Sciences Ltd (GNS) supplied the land gravity data to Land Information New Zealand (LINZ) in 2001. The primary

		East-west vertical deflections				*North-south vertical deflections*			
model	*degree*	*max*	*min*	*mean*	*std*	*max*	*min*	*mean*	*std*
raw data	N/A	15.759	-27.797	-2.714	9.183	15.844	-16.798	0.154	8.289
EIGEN-2	120	15.858	-20.595	2.037	8.260	15.960	-14.337	-0.180	7.753
OSU91A	360	15.476	-5.932	2.541	4.996	10.570	-13.628	0.069	5.637
EGM96	360	13.671	-7.380	2.605	4.688	9.852	-12.596	-0.012	5.258
EIGEN/EGM9632/360		13.577	-7.500	2.559	4.681	9.840	-12.648	-0.040	5.254

Table 1c. Fit of GGMs to 33 Helmert vertical deflections over New Zealand (arc-sec)

database comprises 40,737 gravity "mapping" observations (Reilly, 1972; Woodward, 2001 pers. comm.) and auxiliary information (e.g., positions and computed gravity anomalies). The approximate average spatial density of these data is one observation per 7.5 km-squared, but this is higher in areas of scientific or commercial interest and lower in areas where it is impractical or difficult to collect ground gravity data (notably in Fiordland).

Since these 40,737 land gravity observations were originally reduced mainly for geophysical mapping purposes, gravity anomalies have been recomputed according to the more stringent geodetic requirements (e.g., Featherstone and Dentith, 1997). The horizontal positions of the gravity observations were supplied in terms of the New Zealand Yard Grid (i.e., on the New Zealand Geodetic Datum 1949). Therefore, they were transformed to geodetic latitude and longitude on the New Zealand Geodetic Datum 2000 (NZGD2000). As well as to correctly map these data on the new datum, geocentric geodetic coordinates are required to correctly compute normal gravity for the gravity anomaly (e.g., Featherstone and Dentith, 1997). Normal gravity was evaluated on the surface of the GRS80 ellipsoid at the geocentric latitude of the gravity observation using Somigliana's closed formula (Moritz, 1980).

The gravity values, and hence pre-computed gravity anomalies, in the GNS land gravity database are referred to the Potsdam (New Zealand) gravity datum. It has been known for a long time that the Potsdam datum contains an error (e.g., Torge, 1989). Therefore, a constant value of 15.27 mGal (Woodward, 2001 pers. comm.) was subtracted from all gravity values in the GNS database to convert them from Potsdam (New Zealand) to the International Gravity Standardisation Network 1971 (IGSN71) global gravity datum (Morelli *et al.*, 1971).

Free-air gravity anomalies were computed from the IGSN71-corrected gravity observations by subtracting the value of normal gravity at the geocentric observation latitude, then adding the second-order free-air correction and an atmospheric correction for the observation elevation (above local MSL). A second-order free-air correction is a more accurate representation of the vertical gradient of gravity for an ellipsoidal Earth because it takes into account the variation of normal gravity with latitude as well as higher order terms in height. The atmospheric correction accounts for the mass-inconsistency between the GRS80 normal ellipsoid and gravity observed on the Earth's surface, as well as the gravitational attraction of the atmosphere above the gravity observation point. The atmospheric correction is also needed to make the terrestrial gravity anomalies consistent with those derived from the GGM.

The relevant formulas for the above computations are given in Featherstone (1995) and Featherstone *et al.* (1997) so will not be duplicated here. The difference between the linear and second-order free-air gravity anomalies reaches 1.149 mGal at the summit of Aoraki/Mount Cook (H = ~3754 m above local MSL, NZGD2000 latitude = 43° 35' 44.5"). The atmospheric correction is 0.871 mGal at MSL, decreasing to 0.550 mGal at the summit of Aoraki/Mount Cook. Further work will investigate the use of more sophisticated atmospheric gravity corrections (e.g., Sjöberg, 2000).

The GNS gravity database also includes gravimetric terrain corrections, which have been computed from topographic maps using Hammer charts out to zones L-M (Reilly, 1972), which equates to a distance of 22.4 km, and using a topographic mass density of 2,670 kgm^{-3}. For the preliminary geoid model (described later), these terrain corrections are added to the free-air gravity anomalies to yield a crude approximation of Helmert gravity anomalies at the geoid. Future work will concentrate on the evaluation of detailed New Zealand-wide terrain corrections from the DEM (described later), as well as the downward continuation of gravity from the topography to the geoid (cf. Martinec et al., 1993). This will yield rigorous Helmert gravity anomalies on the geoid (cf. Vanicek et al., 1999) that appear theoretically more appropriate for regional gravimetric geoid computation.

Bouguer planar gravity anomalies were also recomputed from the above free-air gravity anomalies. Simple planar Bouguer anomalies use an infinite lateral plate of thickness equal to the observation height to model the gravitational attraction of the topography. Refined (or complete) Bouguer anomalies add the above terrain correction, which accounts for the departure of the actual topography about the simple Bouguer plate. There are also Spherical Bouguer anomalies that use a sphere instead of a plate. These different types of Bouguer gravity anomaly will be experimented with during the gridding of the land gravity data. This is because they are theoretically smoother than the free-air gravity anomalies and thus less sensitive to aliasing (e.g., Goos *et al.*, 2003), especially in the topographically rugged areas of New Zealand.

Other future work will concentrate on carefully validating the land gravity data. As pointed out by Featherstone *et al.* (2001), gravity data validation can be an extremely time-consuming part of regional geoid computation. The methods employed by Featherstone *et al.* (1997) will be used, together with those employed by the Bureau Gravimetrique International (BGI) in France, where nearby gravity anomalies are used to predict gravity at another location. At present, however, it will be assumed that the GNS validation of the land gravity data is satisfactory.

2.2.2 Ship-track Gravity Data

The ship-track gravity data in the marine regions around New Zealand were supplied to LINZ by GNS in 2001 and 2002. This database comprises 1,300,266 gravity anomalies bound by NZGD2000 coordinates 160°E ≤ λ ≤ 190°E and 25°S ≤ ϕ ≤ 60°S and auxiliary information (e.g., coordinates, gravity values and Eötvös corrections). These gravity anomalies had been sourced from a variety of different agencies, and their quality is largely unknown. However, the older observations are likely to be less accurate because of poorer gravity instrumentation and navigation, the latter of which affects the mapping, positioning for computation of normal gravity and Eötvös corrections. Therefore, all ship-track gravity data were compared with more homogeneous marine gravity anomalies derived from satellite-radar altimetry (described next).

2.2.3 Satellite Altimeter-derived Marine Gravity Anomalies

A homogeneous and complete spatial coverage of gravity anomalies is of prime importance for geoid determination, because in order to determine the geoid height at a single point, gravity data surrounding that point are required. Therefore, it is proposed that the land and ship-track gravity data will be supplemented with gravity anomalies derived from multi-mission satellite altimetry in open ocean areas.

There are currently three recent global grids of marine gravity anomalies derived from a combination of multi-mission satellite altimetry. Perhaps coincidentally, each has used each one of the above methods, but all are based on EGM96-implied gravity anomalies (Equation 2) in a remove-compute-restore procedure. They are:

- Sandwell's v9.2 (2001) global 2' × 2' grid of marine gravity anomalies, which was computed using Laplace's equation (c.f. Sandwell and Smith, 1997).
- The KMS01 global 2' × 2' grid of marine gravity anomalies, which was computed using the inverse Stokes integral (Andersen *et al.* 2001).
- The NCTU01 global 2' × 2' grid of marine gravity anomalies (Hwang *et al.*, 2002), which was computed using the inverse Vening Meinesz formula.

Though all these marine gravity anomalies are available at a 2' spatial resolution (~4 km at New Zealand latitudes), the spacing of the satellite altimeter ground tracks dictate that the highest resolution of the gravity anomalies is more realistically ~15-20 km.

2.2.4 Comparison between ship-track and altimeter gravity anomalies

Given the different altimeter-derived marine gravity anomalies available, it is important to first choose the most appropriate for the New Zealand geoid model. This is usually best done through comparisons with well-navigated ship-track gravity data. Therefore, the different grids were compared with the (corrected to GRS80 and IGSN71) ship-track gravity data supplied by GNS, described earlier. However, as will be seen, this approach proved more useful for identifying errors in ship-track gravity data (cf. Featherstone, 2003). The ship-track free-air gravity anomalies were visually and statistically compared with the above three altimeter grids using exactly the same procedures as explained in Featherstone (2003). The results are summarised in Table 2 and shown for the NCTU01 data in Figure 1.

grid	*Max*	*min*	*Mean*	*Std*
Sand v9.2	464.551	-366.510	-5.302	13.088
KMS01	467.513	-392.158	-4.912	12.434
NCTU01	465.705	-367.328	-4.960	12.655

Table 2. Descriptive statistics of the differences (in mGal) between altimeter-derived gravity anomalies and the 1,300,266 [GRS80, IGSN71 and atmospherically corrected] free-air gravity anomalies in the New Zealand ship-track gravity database.

From Table 2 and Figure 1 it can be seen that there are significant differences between the altimeter-derived gravity anomalies and the New Zealand ship-track data. The larger difference for some tracks is evidence that crossover corrections have not been applied to the ship-track data, which was verified by Woodward (2001 pers. comm.). Therefore, these data should not be used in the New Zealand geoid model until they are crossover-corrected. Future work will therefore compute crossover corrections to the ship-track gravity data. This will then allow the better altimeter-derived grid of gravity anomalies to be selected. More importantly, it will allow the well-known errors in altimeter-derived gravity anomalies near the coast (e.g., Andersen and Knudsen, 2000; Deng et al., 2002) to be corrected by warping the altimetry to fit the crossover-corrected ship-track data (cf. Kirby and Forsberg, 1998). This is particularly important because erroneous gravity data near the coast will propagate into the geoid model on land.

2.3 Digital Elevation Data

Auxiliary elevation data are necessary in gravimetric geoid determination because the gravitational effect of topographic masses

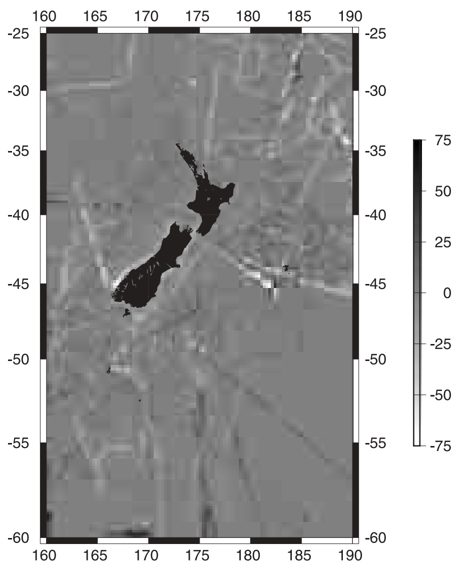

Figure 1. Difference between the NCTU01 altimeter grid and the 1,300,266 points in the New Zealand ship-track gravity database (Units in mGal, Mercator projection)

outside the geoid has to be mathematically condensed onto, or below, the geoid in order to satisfy the boundary-value problem of physical geodesy (e.g., Heiskanen and Moritz, 1967). The terrain effect on the gravimetric geoid is applied in two stages. Firstly, the gravimetric terrain correction is added to the terrestrial gravity anomalies. A co-geoid is computed from these gravity anomalies, which must then be converted to the geoid using a correction for the primary indirect effect of the terrain correction. Secondary indirect effects also come into play, but these will be neglected for the preliminary geoid models. Of course, later work will also consider these secondary indirect effects.

In addition to this theoretical demand, high-resolution terrain data can provide additional short-wavelength geoid information and to help smooth the gravity field prior to gridding. Removing high-frequency signals from the gravity anomalies makes the gridding process less sensitive to aliasing (e.g., Goos et al., 2003), where under-sampled high frequencies are incorrectly propagated into the low frequencies. A more significant effect is to use a digital elevation model (DEM) to 'reconstruct' mean free-air gravity anomalies (Featherstone and Kirby, 2000). This is necessary in areas of rugged and high terrain, where the practicalities of collecting gravity data in the field mean that gravity is generally observed in the more accessible lowland regions. It will be shown later that this reconstruction technique has a significantly positive effect in the Southern Alps.

A 0.0005-degree (1.8" or ~56m) resolution digital elevation model (DEM) was supplied by GeographX, which has been derived from LINZ topographic source data. The estimated precision of this DEM is ±20m horizontally and ±10m vertically. A more generalised DEM, at a 250m spatial resolution, was also supplied. This DEM was used in the preliminary geoid computations, because the 56 m DEM is too large to efficiently handle at the moment. Future work will use a more high-powered computer to compute topographic effects (i.e., terrain corrections and high-resolution free-air anomaly reconstruction) from the 56m DEM.

In immediate future work, the 56 m DEM will be used to compute gravimetric terrain corrections based on the two-dimensional fast Fourier transform (2D-FFT) implementation of Moritz's (1968) formula (Schwarz et al., 1990). Moritz's formula implicitly includes a downward continuation of free-air gravity anomalies to the geoid under the assumption of linear correlation between gravity and height (Martinec et al., 1993). However, a restriction with Moritz's formula is that it becomes numerically unstable for high-resolution DEMs close to the computation point (e.g., Martinec et al., 1996; Tsoulis, 2001), which requires regularisation (e.g., Schwarz et al., 1990) or alternative theories. Finally, the primary indirect effect of the terrain corrections must be consistent with the algorithm used; Wichiencharoen (1982) gives this for Moritz's (1968) formula. Future work will experiment with alternative theories for the computation of Helmert anomalies at the geoid, with their associated terrain corrections, downward continuation, and primary and secondary indirect effects (e.g., Vanicek et al., 1999).

However, for the preliminary geoid model presented in this paper, the terrain corrections supplied in the GNS database (described earlier) were used. As stated, these were computed from Hammer charts and the topographic maps available at the time. Therefore, they may not be ideally suited to gravimetric geoid computations; further work will ascertain this. Acknowledging this uncertainty, the co-geoid computed from Stokes's integration of the terrain-corrected free-air gravity anomalies must still be converted to the geoid via the primary indirect effect. To a first order approximation, the quadratic (first) term of Wichiencharoen's (1982) series expansion will be used; this is

$$N_i = \frac{\pi G \rho H^2}{\gamma} \tag{3}$$

where G is the Newtonian gravitational constant, which has the new value of 6.67310×10^{-11} kg^{-1}m^{3}s^{-2} (Mohr and Taylor, 2002), ρ is the bulk density of the topographic masses, which is taken to be a constant 2,670 kgm^{-3}, and H is the topographic height above the geoid. The 250m DEM was generalised (i.e., area-weight averaged) onto a 2' × 2' grid before computing the primary indirect effect so that it is identical to the spatial resolution of the computed co-geoid. The approximate primary indirect effect (Eq. 3) has a maximum value of 0.5 metres at the summit of Aoraki/Mount Cook.

2.4 GPS and Spirit Levelling Data

Relative carrier-phase GPS observations co-located with precise geodetic levelling can provide external control with which to test a gravimetric geoid on land, especially if the geoid model is to be used subsequently for the recovery of heights above MSL from GPS (cf. Featherstone, 1998). The normal-orthometric heights used to derive these geometrical geoid heights were observed by first-order spirit levelling and are based on the 13 different vertical datums in New Zealand. However, it is important to acknowledge that this does not yield estimates of the classical geoid because of

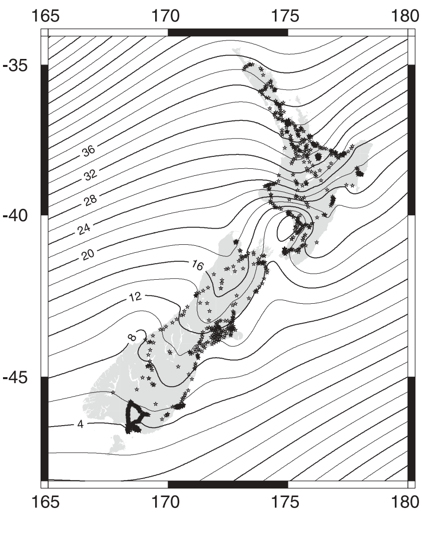

Figure 2. The geometrical 'geoid' (in metres) of New Zealand derived using the shown 1055 GPS-levelling heights (Mercator projection)

the error budget in the GPS data and the systematic errors in the levelling data, as well as the use of normal-orthometric corrections and 13 different tide gauges. Figure 2 shows a geometrical 'geoid' model that was derived from 1,055 GPS-levelling data points using the surface fitting algorithms of Smith and Wessel (1990). Of course, the extrapolated values offshore should not be relied upon.

Initially, these GPS-levelling data will be used to assess the precision of the gravimetric geoid model. These points will also be used to estimate offsets between the 13 different vertical datums used in New Zealand (cf. Amos and Featherstone, 2003a). The results presented later are only preliminary, so the computed offsets must be treated with caution.

3. A PRELIMINARY NEW ZEALAND GRAVIMETRIC GEOID MODEL

3.1 Theoretical Background

In modern gravimetric geoid determination, a GGM is combined with terrestrial gravity and terrain data surrounding each geoid computation point. When using this approach, one must avoid adding the long wavelength component of the gravity field to the geoid solution twice. Therefore, the gravity anomalies implied by a GGM (Eq. 2) are subtracted from the terrestrial gravity anomalies to produce residual gravity anomalies. These are then used to compute residual geoid heights based upon this GGM using some adapted implementation of Stokes's integral. The corresponding geoid component from the same degree of expansion of the same GGM (Eq. 1) is subsequently restored to produce the co-geoid. Generally, smaller corrections are then applied for the indirect effects (Eq. 3) to convert the co-geoid to the true geoid.

The equations used for the preliminary New Zealand geoid are based on the same theories and techniques that were used to compute the AUSGeoid98 gravimetric geoid model of Australia (Featherstone *et al.*, 2001). The contribution of the GGM is given by Eqs. (1) and (2), and the approximate contribution of the primary indirect effect is given by Eq. (3). The residual geoid undulations are given by an adapted Stokes formula with a deterministically modified integration kernel. The residual geoid is given by

$$N \approx N_M + \frac{R}{4\pi\gamma}\int_{\sigma 0} S_f^{\,L+1}(\psi,\psi_0)\,(\Delta g - \Delta g_M)\,d\sigma \tag{4}$$

where N_M is the geoid contribution of the GGM (Eq. 1),

R is the radius of a spherical Earth (6371005 m for GRS80; Moritz, 1980), σ_0 is the limited integration domain, which is taken as a spherical cap about each computation point, and Δg are the terrain-corrected free-air land and marine gravity anomalies.

The Featherstone et al. (1998) deterministically modified kernel ($S_f^{\,L+1}(\psi,\psi_0)$) requires that the kernel is zero at and beyond the truncation radius (y_0), and is given by

$$S_f^{L+1}(\psi,\psi_0) = S_V^{L+1}(\psi) - S_V^{L+1}(\psi_0)$$

for $0 \le y \le y_0$ (5)

where

$$S_V^{L+1}(\psi) = S^{L+1}(\psi) - \sum_{k=2}^{L}\frac{2k+1}{2}t_k(\psi_0)P_k(\cos\psi) \tag{6}$$

is the Vanicek and Kleusberg (1987) modified kernel, and $S^{L+1}(\psi)$ is given by

$$S^{L+1}(\psi) = S(\psi) - \sum_{n=2}^{L}\frac{2n+1}{n-1}P_n(\cos\psi) \tag{7}$$

where $S(\psi)$ is the spherical Stokes kernel (Heiskanen and Moritz 1967, p. 94) and ψ is the spherical distance between the computation point and the remote points in Eq. (4). The $t_k(\psi_0)$ modification coefficients in Eq. (6) were first derived by Vanicek and Kleusberg (1987) and can be evaluated from the solution of the following set of *L-1* linear equations once the spherical cap radius has been selected

$$\sum_{k=2}^{L}\frac{2k+1}{2}t_k(\psi_0)\,e_{nk}(\psi_0) = Q_n(\psi_0) - \sum_{k=2}^{L}\frac{2k+1}{k-1}e_{nk}(\psi_0) \tag{8}$$

where both of

$$e_{nk}(\psi_0) = \int_{\psi_0}^{\pi}P_k(\cos\psi)P_n(\cos\psi)\sin\psi\,d\psi \tag{9}$$

and

$$Q_n(\psi_0) = \int_{\psi_0}^{\pi}S(\psi)P_n(\cos\psi)\sin\psi\,d\psi \tag{10}$$

were computed using Paul's (1973) algorithms.

The use of Eq. (4) over a limited spherical cap σ_0 of radius ψ_0 about each geoid computation point leads to a truncation error term, which can sometimes be neglected when using a high-degree GGM with the modified spheroidal Stokes integral (hence the approximation in Eq. (4)). The reduction of this truncation error term is the primary aim of the deterministically modified kernel (Featherstone *et al.*, 1998). The final preliminary geoid is then given by the addition of Eqs. (1), (4) and (3). Further work will experiment with different modifications of Stokes's formula, as well as different integration radii.

3.2 Computations and Results

The preliminary gravimetric geoid of New Zealand and its surrounding seas was computed on a regular 2' by 2' grid in the region bound by NZGD2000 coordinates 160°E ≤ λ ≤ 190°E and 25°S ≤ ϕ ≤ 60°S. This gives a data array of 1051 rows by 901 columns, or a total of 946,951 geoid heights with respect to the GRS80 ellipsoid.

In order to demonstrate the effectiveness of the mean gravity anomaly reconstruction technique (described earlier), two types of gravity anomaly were extracted from the revised GNS gravity database together with their NZGD2000 latitude and longitudes. These were the 40,737 terrain-corrected, atmosphere-corrected, second-order free-air gravity anomalies and the refined Bouguer gravity anomalies, which also include the second-order free-air correction, the atmosphere correction and the terrain correction. Each gravity data set was arithmetically averaged into 2' × 2' cells and interpolated onto a regular grid using the tensioned-spline algorithm of Smith and Wessel (1990), which is conveniently included in the public domain GMT software (e.g., Wessel and Smith, 1991). A tension factor of 0.25 was used, since this is

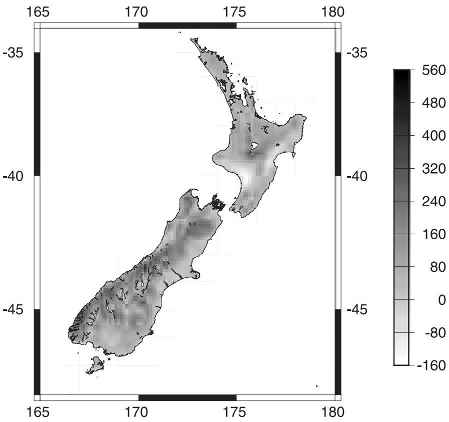

Figure 3. The reconstructed gravity anomalies over New Zealand (units in mGal relative to GRS80. Mercator projection)

best suited to gravity-related data (Smith and Wessel, 1990).

The 2' × 2' grid of mean refined Bouguer anomalies was then used to reconstruct mean free-air gravity anomalies by applying the 'reverse' Bouguer plate correction for the height of the DEM (Featherstone and Kirby, 2000). The DEM used for this purpose was the same 2' × 2' DEM used to compute the primary indirect effect. The reconstructed mean terrain-corrected free-air anomalies are given by (Featherstone and Kirby, 2000)

$$\Delta g_{FA} = \Delta g_{RBA} + 2\pi G \rho H_{DEM} \qquad (11)$$

where Δg_{RBA} is the mean refined Bouguer gravity anomaly and is the mean height of the topography in a cell as given by the DEM. The benefit of this approach is that it gives a more representative mean gravity anomaly and is also less subject to aliasing during the gridding process (cf. Featherstone and Kirby, 2000; Goos *et al.*, 2003).

No ship-track gravity observations were used in this preliminary gravimetric geoid solution because these have not yet been crossover corrected (see earlier). Therefore, marine gravity anomalies were taken from the 2' by 2' NCTU01 altimeter-derived gravity grid (Hwang *et al.*, 2002). This grid was chosen because it uses an arguably more rigorous theoretical basis than other approaches (Hwang, 1998), and also uses slightly more recent altimeter data. These reasons remain somewhat subjective, however, because the ship-track gravity data are not yet of sufficiently reliable quality to isolate the best altimeter data in the New Zealand region. In future, the crossover-corrected ship-track data will be used both in the geoid model and to select the most appropriate grid of altimeter-derived gravity anomalies around New Zealand. Moreover, they will be used to correct the well-known deficiencies of altimeter-derived gravity anomalies near the coast (cf. Kirby and Forsberg, 1998; Hipkin, 2000).

The gridded 2' by 2' land gravity anomalies and altimeter-derived gravity anomalies were concatenated and the land and marine areas sorted using the high-resolution shoreline database contained in the GMT software. The M = 360 gravity anomalies implied by the hybrid EIGEN-EGM GGM (described earlier) were computed on the same 2' by 2' grid, then subtracted from the terrestrial gravity anomalies to produce residual gravity anomalies. Figures 3 and 4 show the reconstructed gravity anomalies, and the residual gravity anomalies, respectively. The descriptive statistics of these gravity anomalies are given in Table

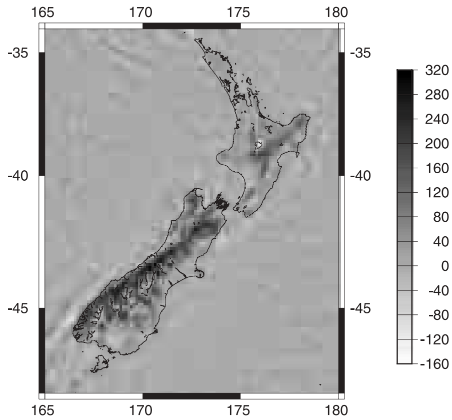

Figure 4. The residual reconstructed gravity anomalies over New Zealand (units in mGal relative to EIGEN-EGM. Mercator projection)

3. Also, the difference between the reconstructed and simple mean free-air anomalies is largest over the Southern Alps. This is as expected because the gravity observations are sparser in this area and are often located in the more accessible by road in the lowland areas.

These two types of residual gravity anomalies (i.e., simple mean and reconstructed mean) were then used to compute the residual geoid undulations using Eqs. (4) through (10) via the one-dimensional FFT technique (Haagmans *et al.*, 1993). Importantly, the 1D-FFT yields results that are identical to a quadrature-based numerical integration of the modified Stokes formula. The parameters chosen for the residual geoid computations were taken simply from those used for AUSGeoid98 (Featherstone *et al.*, 2001), specifically a spherical cap radius of ψ_0 = 1 arc-degree and L = 20. Future work will aim to optimise these parameters for the New Zealand data, as well as experimenting with other variants of the modified Stokes formula, many of which are cited in Featherstone *et al.* (1998). Table 4 gives the descriptive statistics of the residual geoid undulations computed from the modified Stokes formula (Eq. 4) for the respective simple mean and reconstructed gravity anomalies.

The two grids of residual geoid undulations were then added to the M = 360 EIGEN-

Grid	*max*	*min*	*mean*	*std*
Simple mean gravity anomalies	351.900	-292.800	1.748	34.359
Reconstructed gravity anomalies	555.187	-292.800	3.737	39.758
Residual simple mean anomalies	352.526	-358.800	-0.152	13.409
Residual reconstructed anomalies	539.182	-358.800	1.827	21.380

Table 3. Descriptive statistics of the 946,951 grided mean 2' by 2' gravity anomalies (units in mGal)

grid	*max*	*min*	*mean*	*std*
M = 360 EIGEN-EGM geoid	54.051	-46.607	5.981	28.258
Residual geoid undulations (from simple means)	2.208	-1.958	-0.019	0.226
Residual geoid undulations (from reconstruction)	15.355	-1.486	0.216	1.279
Approximate indirect effect	0.000	-0.499	-0.001	0.007
Preliminary geoid (from simple means)	54.131	-46.541	5.959	28.252
Preliminary geoid (from reconstruction)	54.169	-46.535	6.197	28.325

Table 4. Statistics of the various component contributions to the New Zealand geoid (units in metres. 946,951 points)

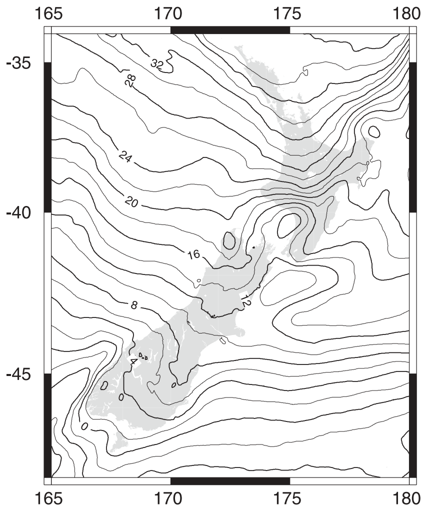

Figure 5. The geoid undulations of New Zealand computed from the simple mean free-air gravity anomalies (units in metres relative to EIGEN-EGM. Mercator projection)

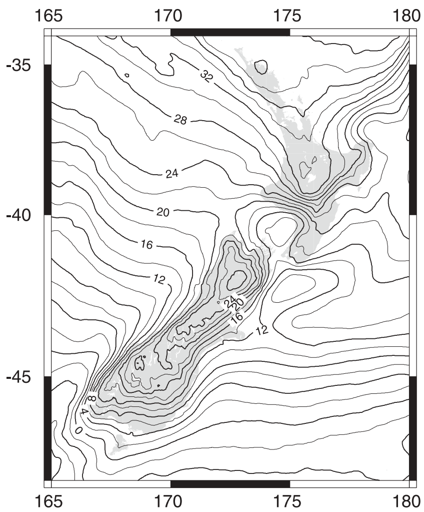

Figure 6. The geoid undulations of New Zealand computed from the reconstructed free-air gravity anomalies (units in metres relative to EIGEN-EGM. Mercator projection)

EGM-implied geoid undulations and the approximate primary indirect effect (Figure 1) to yield two preliminary geoid models of New Zealand (Figures 5 and 6). The descriptive statistics of each of the various geoid contributions are shown in Table 4.

It can be seen in Figures 5 and 6 that the 'reconstructed anomaly' residual geoid has much larger corrections in the areas of high topography, notably in the Southern Alps. This indicates that the reconstructed-anomaly geoid provides a better representation of the effect of topography on the geoid than the simple free-air model. However, this does not necessarily imply that the geoid computed from these reconstructed mean gravity anomalies is more precise. This is where the GPS-levelling data are of use.

The two preliminary geoid models were then compared with a set of 1055 points that have both GPS ellipsoidal and spirit levelled orthometric heights. As mentioned earlier, the levelled heights are based on 13 different vertical datums, which will bias the differences computed. Therefore, results for each local vertical datum will be presented later. It can be seen from Table 5 that the reconstructed gravity geoid has a significantly smaller standard deviation (~35 cm) than both the original GGM and the free-air gravity geoid. It should be noted that while this is a good indication of the general "fit" of the geoid to the levelling data, it still does not account for the 13 different vertical datums (shown later).

The two new preliminary geoid models were then compared with the GPS-levelling heights on a datum-by-datum basis. Table 6 shows the results of the differences between the GPS-geoid and levelled heights for both the reconstructed and simple mean gravity geoids, respectively. Though all the descriptive statistics are shown in Table 6, only the mean differences should be interpreted as the preliminary vertical datum offsets. It can be seen that the mean vertical datum offsets between the two models are often significantly different. In the case of the simple mean geoid, the offsets are often smaller than the (large) standard deviations. For reasons discussed in the next paragraph, it is expected that the offsets resulting from the reconstructed geoid are likely to be the more accurate preliminary values.

In general the standard deviations of the 'reconstructed gravity' geoid are lower than the 'simple mean' geoid. It is also notable that the vertical datums in the South Island (Bluff, Dunedin-Bluff, Dunedin, Lyttelton and Nelson) all show significant

geoid	*max*	*min*	*mean*	*std*
EIGEN-EGM GGM	3.4961	-1.3764	-0.0385	0.6057
Free-air gravity	4.6041	-0.4800	0.4358	0.7544
Reconstructed gravity	0.3090	-1.7118	-0.3518	0.3487

Table 5. Descriptive statistics of the comparison of geoid models with 1055 GPS-levelling points (units in metres)

Datum	*points*	*max*	*min*	*mean*	*std*
Auckland	84	-0.0063	-0.6116	-0.3731	0.1454
Bluff	91	0.2800	-0.0781	0.0503	0.0567
Dunedin-Bluff	170	0.3090	-0.0978	0.1144	0.0832
Dunedin	58	0.2339	-0.7779	-0.2879	0.1704
Gisborne	57	-0.5080	-0.7779	-0.6200	0.0800
Lyttelton	164	0.1938	-1.7118	-0.5555	0.2630
Moturiki	163	-0.0866	-0.6727	-0.3380	0.1461
Napier	26	-0.2974	-0.5512	-0.4380	0.0805
Nelson	46	-0.6897	-1.1852	-1.0210	0.0854
One Tree Point	34	0.0009	-0.2534	-0.1049	0.0646
Taranaki	57	-0.3139	-0.7255	-0.5347	0.1072
Tararu	13	-0.2209	-0.7747	-0.5298	0.2302
Wellington	67	-0.5395	-0.9638	-0.8164	0.1373

Table 6a. Descriptive statistics of the comparison of the "reconstructed mean gravity" geoid model with GPS-levelling points on the 13 vertical datums (units in metres)

Datum	*points*	*max*	*min*	*mean*	*std*
Auckland	84	0.1278	-0.3839	-0.1518	0.1203
Bluff	91	1.2101	0.2600	0.4484	0.1332
Dunedin-Bluff	170	3.5352	0.4857	1.1471	0.5321
Dunedin	58	4.4821	-0.2681	0.6866	1.3537
Gisborne	57	0.4822	0.1100	0.2196	0.0901
Lyttelton	164	4.6041	-0.2305	0.9991	0.9497
Moturiki	163	0.6750	0.2897	0.1466	0.2414
Napier	26	0.4925	-0.0549	0.0779	0.1165
Nelson	46	2.4749	-0.0327	0.4622	0.4971
One Tree Point	34	0.1658	-0.0782	0.0658	0.0708
Taranaki	57	0.1473	-0.4419	-0.1517	0.1586
Tararu	13	0.1517	-0.3557	-0.1260	0.2174
Wellington	67	-0.0398	-0.4800	-0.3267	0.1280

Table 6b. Descriptive statistics of the comparison of the "simple mean gravity" geoid model with GPS-levelling points on the 13 vertical datums (units in metres)

improvement (for example, ~0.4 and ~0.7 m in standard deviation for the Nelson and Lyttelton local vertical datums, respectively). For these two local vertical datums, the precise levelling lines, in general, travel through mountain passes in very rugged terrain. This means that the simple mean gravity anomalies computed in this region are not truly representative of the mean gravity anomalies over the topography in the vicinity, thus affecting the computed geoid. Therefore, the improvement in fit to the GPS-levelling is directly attributed to the reconstructed anomalies being a better representation of the actual mean gravity field than the simple mean anomalies.

SUMMARY AND CONCLUDING REMARKS

A preliminary gravimetric geoid model has been computed for New Zealand using a hybrid combination of the EIGEN-2 and EGM96 global geopotential models, a generalised digital terrain model, reconstructed gravity anomalies on land, and NCTU01 satellite altimeter-derived gravity anomalies at sea. The terrestrial gravity data have been partially reprocessed. It is not currently possible to recompute the terrain corrections, downward continuation and associated indirect effects due to current computational limitations; this will be the subject of future work.

It was found from this study that significant discrepancies exist between the marine gravity data and the three satellite altimetry grids, which is due to the marine gravity data not being crossover adjusted. A future task is to perform this adjustment. Therefore, for the purpose of these preliminary computations, the NCTU01 grid of satellite altimeter-derived gravity anomalies was used to provide coverage in the marine areas. Future studies will investigate optimal combination of the altimetry data with the adjusted marine gravity to obtain a better fit in the problematic near-shore areas.

The mean gravity reconstruction technique was used to account more correctly for undersampled gravity observations due to inaccessibility through the rugged terrain in New Zealand, notably the Southern Alps. This technique produced a preliminary geoid that yields a much better agreement with GPS-levelling points than previous models. In particular the fit to the South Island levelling datums has been significantly improved as a result of the better modelling of the effect of topography on the geoid.

The reconstructed-gravity geoid produced datum offsets that, in a number of cases, were significantly different to those produced by the simple-mean-gravity-geoid model. However, the high standard deviations of the simple mean model indicate that the offset values computed from the reconstructed model (Table 6) are likely to be much better preliminary vertical datum offsets. Importantly, these are only preliminary vertical datum offsets and must not be relied upon until the final, refined geoid model that takes into account the many approximations used to compute the preliminary geoid mode, are taken into account.

Finally, some significant progress has been made towards the computation of a high-precision national gravimetric geoid for New Zealand and its surrounding waters, and to the unification of the thirteen local vertical datums currently in use. While the preliminary geoid models presented in this paper represent improvements on previous attempts, it is anticipated that future studies will provide yet further refinement to these models, and ultimately produce a high-quality national vertical reference system for New Zealand.

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